Linear and non-linear morphological behaviour of coastal systems

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Introduction

Rhythmic features in the surf zone have often been observed at both planar and barred beaches. A number of theories have been developed to explain this rhythmicity. One of them explains surf zone rhythmicity as being forced by the hydrodynamics, either by a combination of two or more phase-coupled edge waves (Holman and Bowen, 1982) or by variations in the forcing on the scale of the wave groups (Reniers et al., 2004). Another possible mechanism resulting in surf zone rhythmicity is self-organization. This mechanism is the focus of this contribution. Self-organization is the positive feedback between the sea bed and the sediment transport patterns, leading to the growth of an initially small perturbation in the sea bed. The growth of very small perturbations can be studied with a linear stability analysis (LSA). LSA starts with a system that is in equilibrium with its forcing. In this study the system is an alongshore-uniform coast and the forcing is obliquely incident breaking waves. The system and the forcing are given a small perturbation. The LSA determines whether the system returns to its equilibrium state (negative feedback between the sediment transport patterns and the topography) or that the perturbation exponentially grows (positive feedback). Many studies using LSA have already been reported, exploring the linear stability characteristics of both planar beaches, e.g. Hino (1974), Christensen et al. (1994), Falqués et al. (2000) and Ribas et al. (2003), and barred beaches, e.g. Deigaard et al. (1999) and Klein et al. (2002).

LSA, however, only applies to small-amplitude perturbations, whereas coastal systems are highly non-linear in character. Another shortcoming of LSA's in general is that the governing equations have to be simplified to a large extent. Since the LSA presented in this paper has been set-up such that it uses a numerical model to compute the perturbed velocity field given a perturbed sea bed, these simplifications do not have to be made. A second advantage of this method is that the same model can also be applied to morphodynamic computations, exploring the non-linear regime. A study that covers both the linear and the non-linear regime with comparable models has been performed by Damgaard et al. (2002) in the case of perpendicular incident waves. They showed that the modes found in the LSA correspond well with the modes found in the non-linear regime. They also showed that different initial perturbations yielded different dominant wavelengths.

This study discusses the linear stability of double-barred beaches under the forcing of obliquely incident waves. Also the non-linear morphological development of these same beaches under the same forcing is studied. By choosing the bed change per time step sufficiently small, also the linear regime is covered with the morphodynamic model. This is done to verify the results of the LSA. Then the non-linear experiments are discussed with the focus on the dependence of the morphological development on the initial perturbation.

Numerical models and linear stability analysis

The numerical model, that has been used to perform the LSA, is based on the depth-averaged shallow water equations. The water motion is solely forced by obliquely incident breaking waves. The wave forcing is computed with the 2^{nd} generation wave model HISWA, see Holthuijsen et al. (1989). The Engelund and Hansen (1967) total load formula has been used to compute the sediment transport rates, meaning that the sediment transport is proportional to the velocity to the power five.

The use of a numerical model also means that another method of determining the eigenvalues and the eigenfunction has to be used, which normally follow from solving the set of simplified and linearized governing equations. The method that has been used determines the eigenvalues and eigenfunction for only one alongshore wavelength. Moreover, it only finds the eigenfunction with the largest growth rate corresponding to the wavelength under consideration. Thus per considered wavelength, the method finds one solution. Applying this method to a range of wavelengths, the growth rate can be plotted versus the wavelength and the wavelength with the largest growth rate can be identified. The solution with this wavelength is designated as the fastest growing mode (FGM).

The method starts with an initial guess for the bed perturbation h':

$$h' = \beta(x)\exp[iky] + c.c. \tag{1}$$

In this equation, x is the cross-shore coordinate, β(x) is the complex, cross-shore amplitude function, i the imaginary unit, k the considered alongshore wave number and y the longshore coordinate. The Rayleigh quotient R has been used to computed the eigenvalues, see Griffel (1985) and is defined as

$$R = -i\omega = \frac{\int \beta^* \Lambda dx}{\int \beta^* \beta dx} \tag{2}$$

in which $\omega=\omega_r + i\omega_i$ is the complex eigenvalue with ω_i the growth rate and ω_r the migration rate. Furthermore, Λ=Λ(x) is the complex, cross-shore amplitude function obtained from the bed changes corresponding to the wavelength under consideration. Fourier decomposition is used to retain Λ(x) associated with that wavelength. Finally, * indicates the complex conjugate.

Although the system is (quasi-)linear by choosing the amplitude of the bed perturbation sufficiently small (about 1% of the water depth), non-linearity is still present in the system. Therefore the above mentioned procedure is repeated a number of times. The (estimate of the) eigenfunction Λ(x) is imposed as a bed perturbation (β(x)= Λ(x)) in a new flow, sediment transport and bed change computation. With these new bed changes R and Λ(x) are again determined, being a better estimate of the eigenvalue and -function, etc. This iteration process will finally converge to the eigenvalue and eigenfunction of the linear system, since with every iteration non-linear effects are removed from the solution. This method is known as the power method.

Results of LSA

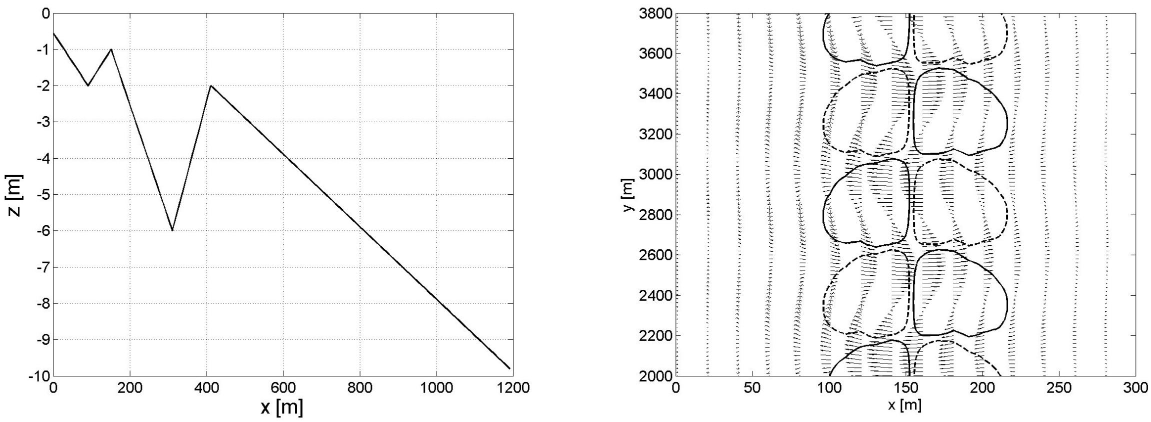


Figure 1. Cross-shore profile of the alongshore-uniform equilibrium beach (left panel) and bed and flow perturbations corresponding to the FGM with a wavelength of 900 m (right panel).

A LSA has been performed for a double-barred beach. The cross-shore equilibrium profile of this coast is presented in the left panel of Figure 1. In Klein et al. (2002) also LSA's of double-barred beaches have been performed. The difference between the present study and the former is the sediment transport formulation and the inclusion of the bed and water level perturbations in the computation of the wave forcing, which have been neglected in Klein et al. (2002). The FGM found in this study therefore differs from the one found in Klein et al. (2002).

From the left panel of Figure 2, which displays the growth rate versus the wavelength, it is clear that the mode with an alongshore wavelength of 900 m is the FGM. The right panel of Figure 1 presents the bed and flow perturbations of the FGM. The spatial structure of the bed perturbation is the typical form of a rip channel system, with alternating channels (dashed lines) and shoals (solid line) along the crest of the inner breaker bar. The present system and forcing did not yield any unstable bed features around the crest of the outer bar.

The right panel of Figure 2 sketches the migration rate as a function of the wavelength. It clearly demonstrates that the migration rate increases with decreasing wavelength.

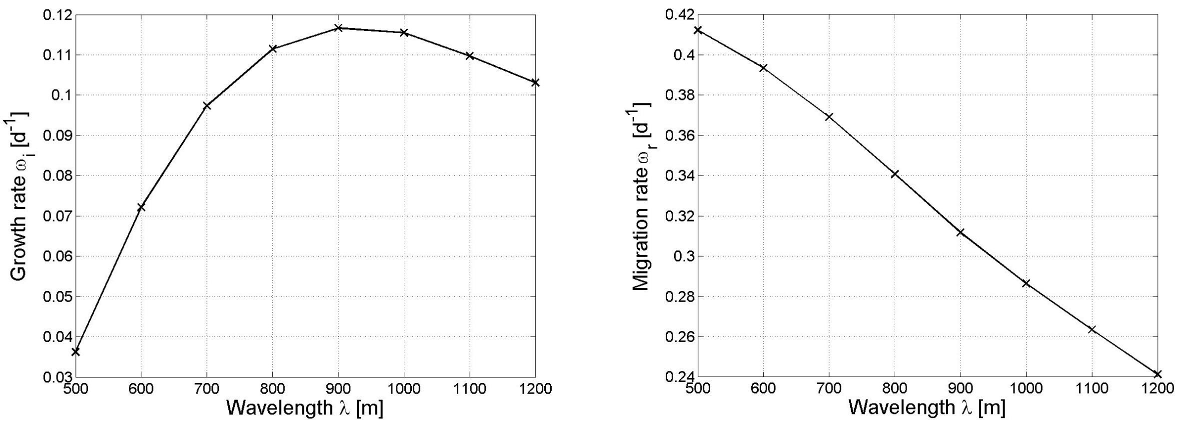


Figure 2. Growth rate (left panel) and migration rate (right panel) as a function of the alongshore wavelength.

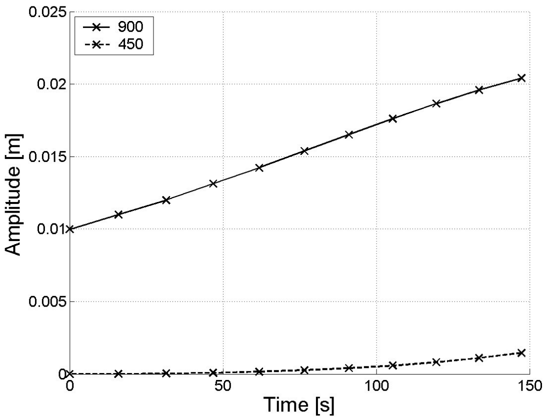
Results of morphodynamic modelling

The same coastal system with the same forcing as studied with the LSA is now studied with the numerical model in a morphodynamic sense. First it is verified whether the morphodynamic modelling applied in the linear regime yields the same results as the LSA. The equilibrium bathymetry is given an initial perturbation that is equal to the eigenfunction of the FGM obtained in the LSA (maximum amplitude is 0.01 m). It is also given the same wavelength, viz. 900 m. After one morphological loop -the time step has been chosen such that the maximum absolute bed change did not exceed 0.01 m- the growth and migration rates have been determined. They compare quite well with the growth and migration rates determined with the LSA viz. 0.123 vs. 0.117 d^{-1} for the growth rate and 0.230 vs. 0.239 m/d for the migration rate. The differences between these numbers are well within the accuracy with which these values can be determined. Repeating these experiments for a range of wavelengths, it appeared that the FGM of 900 m could also be distinguished with this model set-up.

Next, the influence of the initial perturbation has been examined. The model has, for the time being, been run for ten time steps. Two different initial perturbations have been used. The first initial perturbation is the spatial structure (both cross-shore structure and alongshore wavelength) corresponding to the FGM. The second one is a random perturbation throughout the whole modelling domain with an amplitude varying between -0.01 and 0.01 m. The temporal development of the amplitudes of most important modes has been plotted in Figure 3. It demonstrates that starting with the bed perturbation of the FGM already very early in the temporal development other modes are becoming important, although the FGM still dominates (left panel of Figure 3). Note that only a limited number of wavelengths has been assessed, viz. λ=2700/N with N=1,2,...135.

The right panel of Figure 3 shows that, when starting with a random perturbation, a multitude of modes are important. It also shows that, although the mode with a wavelength of 900 m is significant, other modes than the one found when starting with a periodic perturbation appear.

These two experiments demonstrate that different initial perturbation can trigger different temporal morphological behaviour. They also demonstrate that the FGM mode, determined with a LSA, is not necessarily the FGM when starting with a random perturbation. Future research will structurally explore the transition from the linear via the weakly non-linear to the non-linear regime for different initial perturbations.



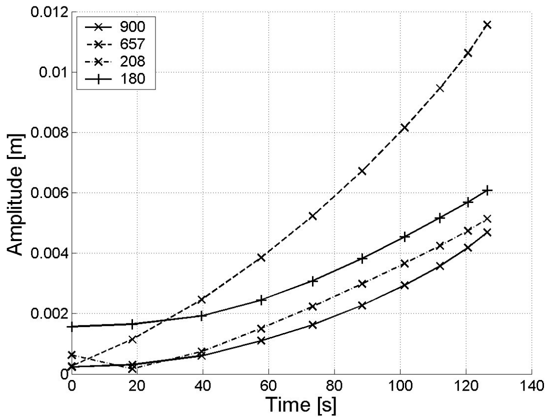


Figure 3. Effect of initial perturbation on the temporal development of most important modes. Initial perturbation is the spatial structure of the FGM (left panel) and initial perturbation is a random perturbation (right panel).

Conclusions and future research

This paper explores the linear morphological regime with both a LSA and morphodynamic modelling utilizing a small morphological time step. It is shown that the growth and migration rates found in the LSA are also found with the morphodynamic model. Furthermore, also the FGM itself is selected by the morphodynamic model. Extending the computations into the non-linear regime it has been shown that other modes than FGM are becoming important, but not (yet) dominant. Also the initial perturbation is of importance for the selection of the modes.

The selection of the modes for different initial perturbations will be structurally explored. Starting with a nearly stable system, parameters controlling the stability will be increased. It is likely that more modes will appear. The temporal development of each of these modes will be followed in order to see in which equilibrium state they result.

Furthermore, the non-linear morphological behaviour under different hydrodynamic forcings and for different equilibrium profiles will be studied. The analysis of the results will focus on the temporal development of different modes. These results will also be compared with the results of LSA's performed for identical forcings and equilibrium profiles.

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