Mechanism for the Initial Formation of Rhythmic Coastlines

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Introduction

- Along the Dutch coast rhythmic coastline variations on length scale λ~1-10 km are observed. They are called shoreline sand waves. Time-scale is in the order of 50-100 years. Migration speed is tens of meters per year to the north (see figure 1).
- Influence of tides on formation of rhythmic coastline variation is not considered so far.

Research questions

- What is the possible role of tidal motion on the formation of rhythmic coastlines?
- What is the underlying physical mechanism?

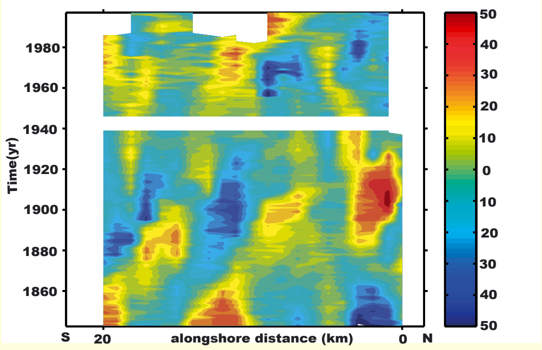
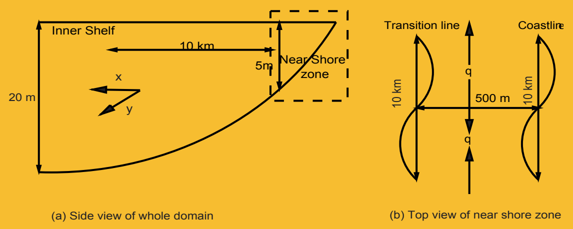


Figure 1: Space-time diagram showing the rhythmic pattern of accretion and erosion along a stretch of 20 km along the Dutch coast. Red colors indicate accretion (in meters), blue colors erosion. (adapted from Ruessink&Jeuken, 2002)



(a) Side view of whole domain

(b) Top view of near shore zone

Figure 2: Geometry of the model. Hydrodynamics described on inner shelf, sediment transport q in near shore zone.

Modelling approach

Geometry:

- Two regions: Inner shelf and near shore zone divided by transition line, see Figure 2.

Assumptions:

- Hydrodynamics described on inner shelf. Sediment transport only in the near shore zone.
- Width of near shore zone is constant. Variations of coastline results in variations of transition line and vice versa.
- Hydrodynamics described by 2-DH Shallow Water Equations. Only M_2 tidal forcing.
- Sediment transport q (m^3s^{-1}), linear in residual velocity on the transition line, $q=\beta\langle u\rangle$. Parameter β is estimated on base of wave-driven sediment transport.
- Convergence of sediment transport results in a change of coastline position.

Linear stability analysis:

- Basic state: straight coastline, shore-parallel tidal M_2 flow. Basic state has vorticity.
- Perturbation of coastline results in perturbed velocity by boundary conditions.
- Periodic solutions in y-direction (alongshore). Model calculates complex growth rate $\Gamma=\Gamma_{re}+i\Gamma_{im}$ where Γ_{re} describes the growth of the perturbation and Γ_{im} the migration.

Results of linear stability analysis

Real part of growth rate (Figure 3):

- Perturbations linearly stable for wave lengths larger than 8 km.
- Growing perturbations for wave lengths smaller than 8 km.

Imaginary part of growth rate:

- All perturbations migrate to the north.

Residual flow determines sediment transport and thereby growth:

- In case of no Coriolis force: only growth, no migration (see figure 4a).
- In case of realistic Coriolis force: growth and migration (see Figure 4b).

The critical wave length scales with e-folding length scale of bathymetry

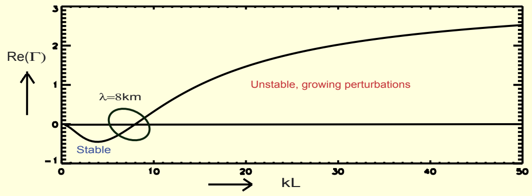


Figure 3: Real part of growth rate as a function of kL, where L is the length scale of 10 km. A growth rate of Γ=1 corresponds to a time scale of 1500 years. A critical wave length of 8 km occurs. For smaller wave lengths the perturbations are unstable.

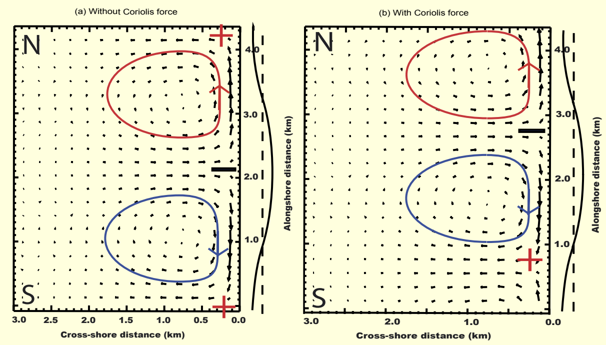
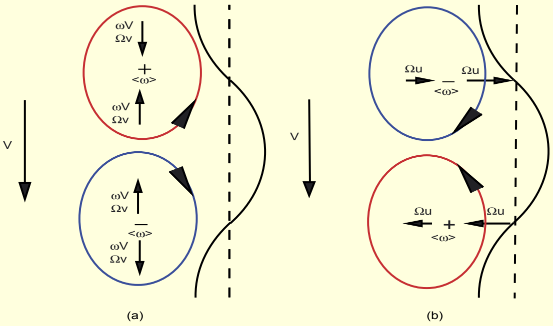


Figure 4: Residual flow on inner shelf in the case of no Coriolis force (a) and with Coriolis force (b). Coastline perturbation is shown on the right-hand side of each subplot.



(a)

(b)

Figure 5: Explanation of physical mechansim. There is a competition between the convergence of the alongshore vorticity fluxes (a) and the cross-shore vorticity fluxes (b).

Physical Mechanism

Analyze the tidally-averaged vorticity balance without Coriolis force:

$$\langle\Omega u\rangle_x + \langle\Omega v\rangle_y + \langle\omega V\rangle_y = -r\langle\omega\rangle/H + rH_x\langle v\rangle/H^2$$

Herein is Ω basic state vorticity, ω the vorticity of the perturbed velocity field, V the basic state alongshore velocity, u and v the perturbed velocity in cross-shore and alongshore direction, r the friction parameter and H the bathymetry.

In figure 5 the vorticity fluxes are plotted:

- In alongshore direction (5a) the convergence of the vorticity fluxes, ωV and Ωv, results in residual circulation cell that act destabilizing.
- In cross-shore direction (5b) the convergence of the vorticity flux, Ωu, results in residual circulation cells that act stabilizing.

Conclusions

- For small wave lengths (λ < 8 km) perturbations can grow.
- These perturbations migrate to the north due to Coriolis force.
- Mechanism can be understood in terms of competition between convergence of alongshore and cross-shore vorticity fluxes.
- No damping mechanism at this stage for small-scale perturbations, hence no preferred mode.
- With introduction of diffusive terms a preferred mode will occur.