



Tidal and Morphologic Properties of Embayments: Effect of Sediment Deposition Processes and Length Variation

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Abstract. An idealised morphodynamic model is analysed to study the properties of both the vertical and horizontal tide in a rectangular embayment with an erodible bottom. Forcing is due to a prescribed tidal constituent at the entrance. The model is based on the shallow water equations, sediment is transported as suspended load and the divergence of the tidally averaged sediment flux determines the bottom evolution. It is further assumed that the embayment is in morphodynamic equilibrium, i.e., the bottom profile is steady. The main results are that the tidal and morphologic properties strongly depend on the sediment deposition process and on the embayment length. A depth-dependent deposition term causes bed profiles to become convex. With increasing embayment lengths the profiles become concave and tidal resonance is found at a length which is slightly smaller than a quarter of the frictionless tidal wave-length. This shift is less than in case of a fixed bed because water depths increase with increasing embayment lengths. The morphodynamic equilibria do not exist for lengths larger than the frictional length scale of the tide.

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1 Introduction

The propagation and distortion of tides in semi-enclosed embayments has been the subject of intense research. Many studies (cf. Friedrichs and Aubrey, 1994; Lanzoni and Seminara, 1998) are based on the cross-sectionally averaged shallow water equations in order to gain further understanding of observed tidal records and of net water and material fluxes. Important aspects turn out to be the geometry of the embayment (in particular the convergence of the cross-section in the landward direction), the formulation of bottom friction, the length of the embayment and the presence of tidal flats.

A common characteristic of these models is that the bottom is taken to be fixed, whereas many estuaries and tidal embayments have a sandy bed. Consequently, net erosion, transport and deposition of sediments will take place, resulting in an evolving bottom. Often in such areas it is observed that water depths decrease in the landward direction and superimposed on this a pattern of channels and shoals exists. Examples are the embayments located in the Wadden Sea (Ehlers, 1988) and the Western Scheldt, located in the southwest of The Netherlands, see Van den Berg et al. (1996). An important distinction is that the Western Scheldt has a length of about 160 km, which is of the order of the tidal wave-length, whereas the former embayments are much shorter (typically 20 km). This results in a different hydrodynamic and morphodynamic behaviour.

In a study by Schuttelaars and De Swart (1996) the properties of tides have been investigated for a rectangular tidal embayment with an erodible bottom. The embayment length was taken to be small compared with the tidal wave-length, no cross-channel variations were considered and the sediment deposition process was independent of the water depth. They found that solutions of this system tend to a unique, stable morphodynamic equilibrium in which no net sediment fluxes occur. This equilibrium is characterized by a vertical and horizontal tide which are both spatially uniform and the bottom profile has a constant slope. These results appear to be consistent with field observations, such as those discussed in Friedrichs (1995).

The aim of the present paper is to extend the work of Schuttelaars and De Swart (1996) in two ways. The first is to account for the dependence of the sediment deposition process on the local water depth. The second is to study the change in tidal and morphologic properties when the embayment length is increased and the condition of morphodynamic equilibrium is imposed. In section 2 the model will be introduced and the solution method will be outlined. The effect of the depth-

dependent deposition term is discussed in section 3 and the results for the variation of the embayment length are presented in section 4. We end with conclusions and an outlook.

2 Model geometry and equations

The embayment is schematized as a rectangular basin, consisting of three non-erodible walls, an erodible bottom and an open connection to the adjacent sea, see figure 1. Typical lengths have been discussed in the introduction and characteristic widths are a few kilometers.

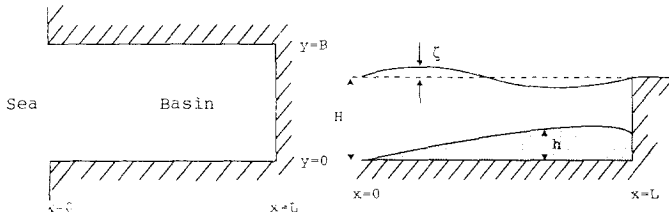


Fig. 1. Topview (left) and cross-section (right) of model basin.

The model consists of the depth-averaged shallow water equations for a homogeneous fluid (1),(2), a depth-integrated concentration equation (3) and a bottom evolution equation (4). In this work only embayments containing fine sediment are considered, which means that bedload transport is neglected. In one dimension these equations read

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(\zeta + H - h)u] = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{ru}{(\zeta + H - h)} = -g \frac{\partial \zeta}{\partial x} \quad (2)$$

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x} \left[uC - \mu \frac{\partial C}{\partial x} \right] = \alpha u^2 - \gamma \beta(\zeta, h)C \quad (3)$$

$$\frac{\partial h}{\partial t} = -\frac{1}{\rho_s(1-p)} \langle \alpha u^2 - \gamma \beta(\zeta, h)C \rangle \quad (4)$$

where ζ is the free surface elevation, u the depth-averaged velocity, C the depth-integrated concentration and h the bed height with respect to the reference depth H (see figure 1); typically $H \sim 10$ m. The brackets in (4) denote an average over the tidal period. Furthermore, g is the acceleration due to gravity, α ($\sim 10^{-2}$ kgsm $^{-4}$) is an erosion coefficient (for sediment with a grain size of $2 \cdot 10^{-4}$ m), γ ($\sim 4 \cdot 10^{-3}$ s $^{-1}$) a deposition coefficient, μ (~ 100 m 2 s $^{-1}$) a horizontal diffusion coefficient, r ($\sim 5 \cdot 10^{-5}$ ms $^{-1}$) a linear bottom friction parameter and ρ_s (~ 2650 kgm $^{-3}$) the density of the individual grain particles. The bed porosity is denoted by p (~ 0.4).

It is assumed that the bed evolves on a much longer timescale than the water motion. This allows the bed to

be taken fixed on the short timescale. Solving equations (1)-(3) then yields the tidally-averaged sediment flux, from which the bottom changes are calculated.

Near the bottom an equilibrium is assumed between deposition and whirling up of sediment, yielding a depth-dependent deposition term $\beta = (1 - e^{-w_s(\zeta + H - h)/\kappa})^{-1}$. Here w_s ($\sim 2 \cdot 10^{-2}$ ms $^{-1}$) is the settling velocity of sediment particles and κ ($\sim 10^{-1}$ m 2 s $^{-1}$) a vertical diffusion coefficient. The depth-dependence becomes important when the thickness of the sediment boundary layer is of the same order of magnitude as the total water depth, which often occurs near the landward boundary. Here $\beta = 1$ refers to depth-independent deposition and $\beta > 1$ to depth-dependent deposition.

The boundary conditions are no water flux and sediment flux through solid boundaries, no diffusive boundary layer in the fluctuating part of the concentration at the entrance of the embayment and a fixed bottom at $x = 0$. The system is forced by a prescribed M_2 tide at the entrance with frequency $\sigma = 1.4 \cdot 10^{-4}$ s $^{-1}$ and amplitude a (~ 1 m). For further details on the equations and boundary condition see Schuttelaars and De Swart (1996).

There are three methods that can be used to obtain a morphodynamic equilibrium. First of all, it is possible in some cases to solve the equilibrium condition $\frac{\partial h}{\partial t} = 0$ analytically. This has been done in the case of short embayments with depth-independent deposition. The second possibility to find a steady bed is by using time integrations of (1)-(4). This has been done in the case of depth-dependent deposition, starting from a flat bottom. Thirdly, continuation methods can be used: starting from a known equilibrium a model parameter is varied, thereby yielding new equilibria. In our experiments continuation in the embayment length has been applied.

3 Short embayments: the effect of deposition parametrization

First we consider embayments with length scales much smaller than the tidal wave-length, which excludes the possibility of tidal resonance. Parameter values used are representative for the Frisian Inlet situated in the Dutch Wadden Sea and taken from Oost (1995). The embayment length in this case is 20 km. Furthermore, frictional effects are moderate because of the relatively large water depths considered. The prescribed M_2 tide at the entrance then results in a spatially uniform vertical tide (pumping mode).

The results for the bottom and the horizontal tide are shown in figure 2. In the case of depth-independent deposition the equilibrium bottom profile can be determined analytically. The constantly sloping bottom yields a horizontally uniform tide. If the depth-dependent deposition term is used the equilibrium condition can be

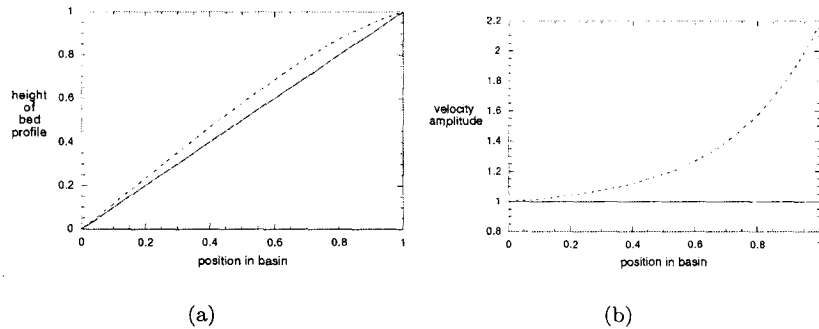


Fig. 2. Solid line represents depth-independent deposition, dashed line represents depth-dependent deposition. a. Bottom elevation h , scaled by reference water depth H (see figure 1), as a function of the dimensionless distance x/L to the seaward boundary in case of a short embayment and for different sediment deposition parametrizations. b. Tidal velocity amplitude, scaled by $U = (a/H)\sigma L$, as a function of x/L for the same situation as in a; here $H = 10$ m, $L = 20$ km and $U = 0.3$ ms⁻¹.

solved numerically and a convex bottom profile is found. This results in a horizontal velocity profile which increases in the landward direction. This behaviour stems from the dominant balance in concentration equation (3), which is between erosion and deposition. Since the morphodynamic equilibrium in short embayments is characterised by a spatially uniform time-mean concentration (see Schuttelaars and De Swart, 1996), the velocities in case of $\beta > 1$ must increase towards the land such that erosion can compensate for the increased sediment deposition.

The results show that the tidal and morphologic properties are rather sensitive to the parametrization of the deposition process. It is also important to remark that, within the context of our model, morphodynamic equilibria can exist in the absence of sea level rise. This aspect needs further attention, because it is often stated (Oost, 1995) that tidal inlet systems will fill up with sediment if sea level rise is too slow.

4 Longer embayments

We now investigate the changes in tidal and morphologic properties if the embayment length is varied, subject to the condition of morphodynamic equilibrium. In these experiments the deposition term is independent of the depth. Parameter values are chosen such that they are representative for the conditions in the Western Scheldt, in particular $a = 1$ m, $H = 15$ m. We remark that this estuary is only weakly convergent, moderately dissipative and that the influence of river discharge is small.

Solutions have been obtained by means of the continuation method, starting from the known solution for the short embayment. In figure 3(a) a contour plot is shown of the equilibrium bottom profile as a function of the dimensionless position x/L to the seaward boundary and the length scale ratio L/L_g . Here $L_g = \sqrt{gHT}$ is the frictionless tidal wave-length and T the tidal period; in this case $L_g \sim 550$ km. It appears that longer embayments have more concave bottom profiles with a maximum water depth near the seaward boundary. This is caused by the resonance properties of the tide: with increasing lengths the amplitude of the vertical tide will

increase in the landward direction. This requires increasing divergences of the water flux which, due to the condition of morphodynamic equilibrium, are only possible if water depths become larger.

For reasons of brevity and because of their relevance for sediment transport only the results for the horizontal tide are shown. In figure 3(b)-3(e) contour plots are shown of the amplitude and phase of the principal (M_2) component of the horizontal tide and its first overtide (M_4 constituent) as a function of x/L and L/L_g . The M_4 tide is generated internally in the system by advection and (depth-dependent) friction terms in the shallow water equations. It can be seen that with increasing embayment lengths the character of the tide changes from a standing wave to a travelling wave. This is a consequence of the stronger influence of bottom friction. The tidal velocity amplitude attains a maximum for an embayment length which is slightly smaller than a quarter of the frictionless tidal wave-length. This is in accordance with standard theory (cf. Prandle, 1991). However, the shift towards smaller lengths, caused by bottom friction, is smaller than in case that the embayment would have a fixed reference water depth H . This is because the sandy bed adjusts to morphodynamic equilibrium which causes water depths to become larger with increasing embayment lengths.

The amplitude of the M_4 tide reaches a maximum close to the landward side of the embayment. This indicates that bottom friction, which is more effective in shallower depths, is very important for the generation of overtones and related tidal asymmetry.

Finally we remark that no equilibria exist if $L > L_f$, where L_f appears to be the frictional length scale of the tide; in this case $L_f \sim 220$ km. For larger lengths the velocities near the landward side are too small to maintain the morphodynamic equilibrium condition. The result that L_f determines the maximum embayment length for morphodynamic equilibria was also found by De Swart and Blaas (1998) for short, frictionally dominated tidal embayments.

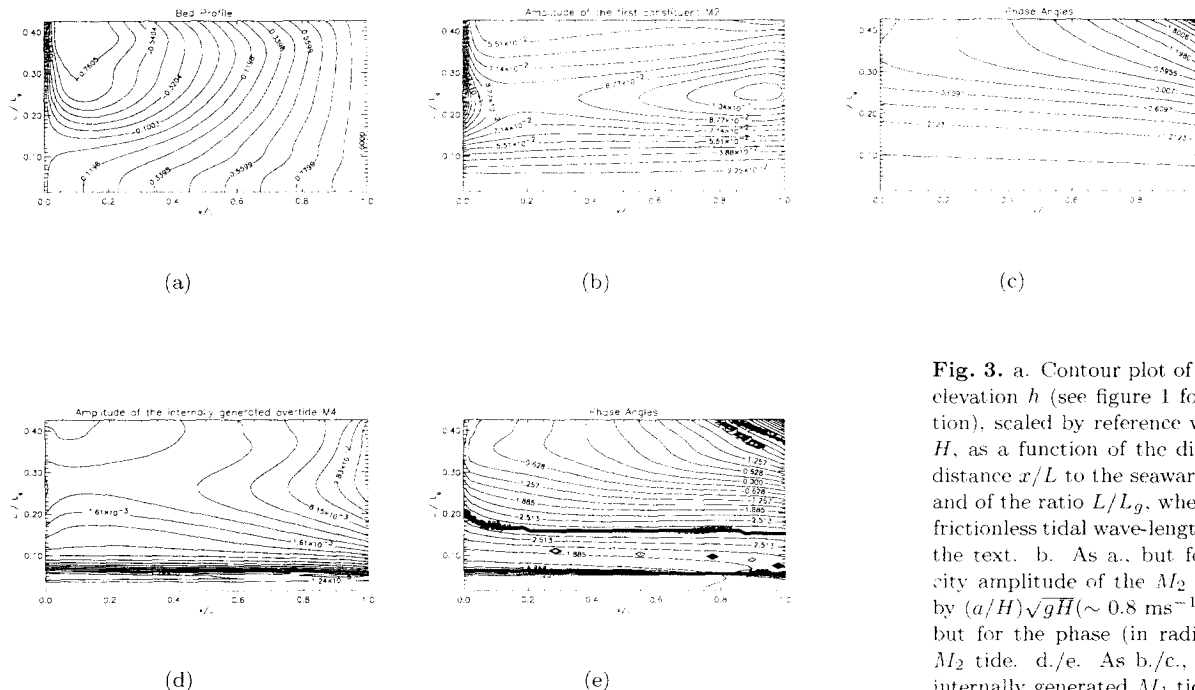


Fig. 3. a. Contour plot of the bottom elevation h (see figure 1 for its definition), scaled by reference water depth H , as a function of the dimensionless distance x/L to the seaward boundary and of the ratio L/L_g , where L_g is the frictionless tidal wave-length defined in the text. b. As a., but for the velocity amplitude of the M_2 tide, scaled by $(a/H)\sqrt{gH}$ ($\sim 0.8 \text{ ms}^{-1}$). c. As a., but for the phase (in radians) of the M_2 tide. d./e. As b./c., but for the internally generated M_4 tide.

5 Conclusions and outlook

In this paper it has been shown that for fixed boundary conditions (in particular no sea level rise) a morphodynamic equilibrium can exist. In case of a short embayment and depth-independent sediment deposition the result is spatially uniform tide over a constantly sloping bottom. A depth-dependent deposition causes the bottom profile to become convex and the tidal velocity amplitude increases in the landward direction. It has also been demonstrated that for embayments with longer lengths the bottom profiles tend to become concave. Tidal resonance occurs for an embayment length which is slightly smaller than a quarter of the frictionless tidal wave-length. For lengths which are smaller (larger) than the resonance length the water motion is characterized by a standing (travelling) tidal wave. The maximum embayment length, beyond which morphodynamic equilibria cease to exist, is determined by the frictional length scale of the tide.

The presently ongoing research is focused on the stability properties of morphodynamic equilibria using a 2DH model. The latter allows bottom perturbations to have both a longitudinal and a transverse structure. The objective is to gain further understanding about the role of residual circulations and tidal asymmetry on the formation of channels and shoals induced by tide-topography interaction.

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References

- De Swart, H. E. and Blaas, M., Morphological evolutions and a 1D model for a dissipative tidal embayment, in *Physics of estuaries and coastal seas*, edited by J. Dronkers and M. B. A. M. Scheffers, pp. 305–314. Balkema, Rotterdam, 1998.
- Ehlers, J., *Morphodynamics of the Wadden Sea*, Balkema, Rotterdam, 1988.
- Friedrichs, C., Stability shear stress and equilibrium cross-sectional geometry of sheltered tidal embayments. *Coastal Res.*, 11, 1062–1074, 1995.
- Friedrichs, C. and Aubrey, D., Tidal propagation in strongly convergent channels. *J. Geophys. Res. C*, 99, 3321–3336, 1994.
- Lanzoni, S. and Seminara, G., On tide propagation in convergent estuaries. *J. Geophys. Res. C*, 103, 30793–30812, 1998.
- Oost, A., Dynamics and sedimentary development of the Dutch Wadden Sea with emphasis on the Frisian Inlet, *PhD thesis, Utrecht University*, 1995.
- Prandle, D., Tides in estuaries and embayments, in *Tidal hydrodynamics*, edited by B. B. Parker, pp. 125–152, John Wiley & Sons, New York, 1991.
- Schuttelaars, H. M. and De Swart, H. E., An idealized long-term morphodynamic model of a tidal embayment, *Eur. J. Mech., B/Fluids*, 15, 55–80, 1996.
- Van den Berg, J., Jeuken, C., and der Spek, A. V., Hydraulic processes affecting the morphology and evolution of the Westerschelde estuary, *Estuarine shores, evolution, environments and human alterations*, pp. 157–184, 1996.