

Morphodynamic Equilibria in Tidal Embayments With Decreasing Cross-Section.

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February 28, 2003

Abstract

The geometrical shape of many tidal embayments is characterized by a strongly decreasing cross-sectionally area from the entrance to the landward side. Furthermore a strong sea-ward river flow is often observed. To get a better understanding of the nature of this equilibrium shape and the influence of river inflow a one-dimensional model of a highly schematized tidal embayment is developed and its possible morphodynamic equilibria are analyzed.

The new aspects in this model are the incorporation of a variable embayment width and the influence of a river inflow.

1 Introduction

Using idealized models based on cross-sectionally averaged shallow water equations, tidal dynamics in estuaries have been studied by many authors (c.f. Speer & Aubrey (1985); Friedrichs & Aubrey (1994)). In these contributions they demonstrated the importance of inertial terms, bottom friction,

*This research was supported by the National Institute for Coastal and Marine Management at The Hague, The Netherlands

hypso-metric effects and channel convergence for the propagation of the principal tide and the generation of overtides and residual components.

Idealized models for the morphodynamics of tidal embayments were studied by Schuttelaars & De Swart (1996); Schuttelaars & De Swart (2000). The embayments studied were rectangular, i.e. width variations were neglected. Furthermore, it was assumed that river inflow was negligible compared to the tidal water fluxes. For sufficiently strong amplitudes of the ratio M_4/M_2 , multiple morphodynamic equilibria were found. These equilibria had completely different characteristics.

The frequent occurrence of strongly convergent estuaries and estuaries with large river inflow (see Friedrichs & Aubrey (1994)) motivated us to investigate their impact on morphodynamic equilibria of tidal embayments with arbitrary lengths. This problem will be studied with an extended version of the nonlinear 1D model of Schuttelaars & De Swart (2000).

2 Model Description

The embayments under study are linearly converging embayments. The choice of linear convergence is motivated by our hypothesis that the observed exponential convergence of the width is a result of the internal dynamics of the embayment. This is already shown in case of a short embayment where the initial geometry was rectangular.

The water motion is forced at the seaward entrance by a prescribed sea surface variation ζ of tidal origin. The boundary on the landward side is either closed or a finite depth and velocity are prescribed. The reference undisturbed water depth is denoted by H , the bed elevation by h and u is the horizontal tide. The embayment width at the entrance is denoted by W_E , at the river side by W_R . If a river inflow is prescribed, a water depth H_R at the river side is prescribed.

The set of equations describing the water motions, sediment transport and bed evolution are discussed in Schuttelaars & De Swart (2000). The water motions can be described by the cross-sectionally averaged shallow water equations for a homogenous fluid. Due to the converging channel geometry, the system is described in polar coordinates instead of cartesian. Hence cross-sectional averaging means averaging over the polar angle. The sediment in the embayment consists of noncohesive material with only one grain size and is mainly transported as suspended load. This transport is

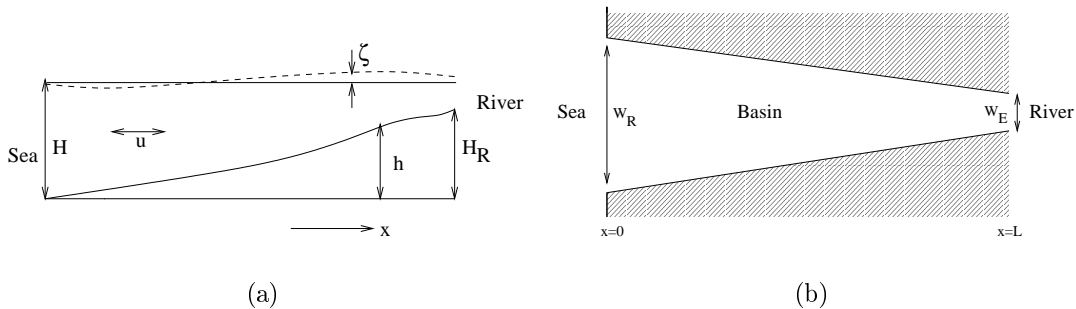


Figure 1: Topview and side view of an idealized embayment. For explanation of the symbols see the text.

described by a depth-integrated concentration equation and the bottom evolution equation is derived from continuity of mass in the sediment layer.

3 Results

In the experiment presented here, the ratio W_R/W_E is varied. The length of the embayment is set to 110 km, the width at the entrance is 6 km and the width at the river-side is varied from 6 km to 0.6 km. No river inflow is taken into account ($H_E = 0$) and the amplitude of the M_2 and M_4 tidal forcing at the entrance are 1.75 m ($\equiv A$) and 0.12 m respectively. In figure 2 the equilibrium bed profile and corresponding phase of the M_2 horizontal tide is plotted as a function of W_R/W_E . It is observed that by increasing the channel convergence the type of morphodynamic equilibrium changes: for $W_R/W_E \sim 1$ the equilibrium is characterized by a weakly concave bottom profile and a travelling tidal wave. The amplitude of the externally prescribed M_4 -tide is negligible. When the width convergence increases, the externally prescribed external overtide is amplified and another type of morphodynamic equilibrium is found: the bottom profiles are strongly concave, with locally large water depths, and the water motion resembles a standing tidal wave.

In the second experiment the depth H_E is set to 5 m and the ratio $W_R/W_E = 0.2$. The embayment length is 90 km. The forcing at the entrance is kept constant and the river inflow is increased from $0 \text{ m}^3 \text{ s}^{-1}$ to $3000 \text{ m}^3 \text{ s}^{-1}$ (so the river inflow was increased from 0 to approximately 0.05% of the tidal water flux). In morphodynamic equilibrium the water motion is

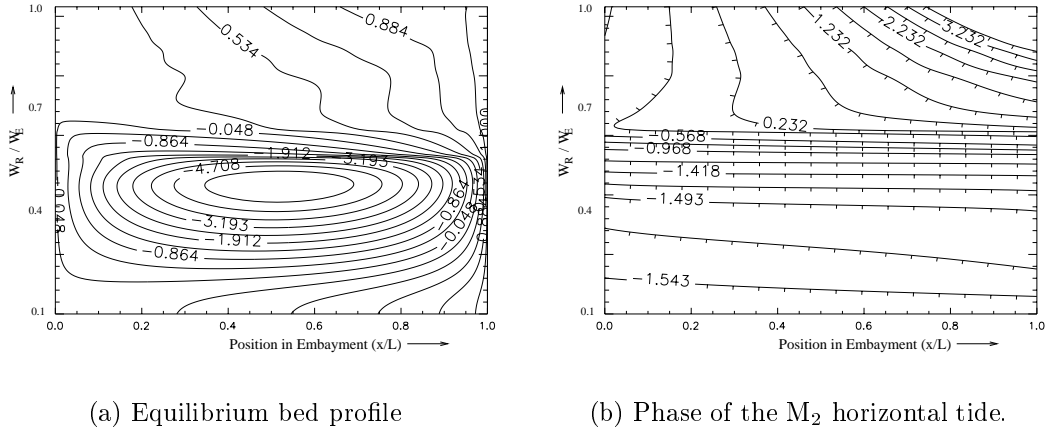


Figure 2: Contour plots of the equilibrium bed profile and phase of the M_2 horizontal tide in morphodynamic equilibrium as a function of the dimensionless coordinate x and the ratio W_R/W_E . Velocities are scaled with $A/H\sqrt{gH}$. Here $H = 10$ m, $A = 1.75$ m and $L = 110$ km. The small ticks in the contour lines point to smaller contour levels.

a standing wave and concave bed profiles are observed. However, for negligible river inflow, the maximum water depth is reached near the entrance of the estuary. If the river inflow increases, this maximum depth near the estuary mouth decreases and a new maximum is formed near the end of the embayment.

4 Discussion

In this contribution the existence of morphodynamic equilibria in width-averaged convergent tidal embayments has been investigated. The water motion is described by a principal M_2 tide and an external M_4 tide prescribed at the open sea boundary. Sediment is transported as suspended load. A morphodynamic equilibrium is defined by the condition that the bottom profile is steady. In the present model this is equivalent to a zero tidally averaged sediment flux in the entire embayment.

In the experiment discussed in this contribution the ration W_R/W_E was decreased. It was shown that if the convergence was strong enough the

characteristics of the morphodynamic equilibrium changed. For an almost rectangular geometry the water motion has the characteristics of a travelling wave and the bed profiles are quite shallow. If the convergence increases enough ($W_R/W_E \sim 0.5$), the water depths for the equilibrium bed are much larger than in the previous case: the maximum value is about five times the depth near the entrance. The tide behaves like a standing wave and a zero net sediment flux is maintained by a balance between advective contributions related to velocities and concentrations generated by external overtides and by nonlinear interactions, respectively.

When incorporate river inflow it was found it had to be quite substantial compared to tidal fluxes to give a significant change in the results discussed above. In the cases studied so far, the type of equilibrium did not change by increasing river inflow but the position of maximal embayment depth shifted towards the end of the embayment.

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