Future Energy Resources: Mathematical Modeling of Decisions under Uncertainty

Applicants

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Project Description

Summary

When and by how much should the Dutch dikes be increased under the expected rise of the sea level? When could a mobile phone company invest best in a new network? Which energy resources will dominate in the future energy supply? These are three questions on investment decisions under uncertainty of future developments. One possible direction to model such questions mathematically is by so-called *Real Options* [3, 9]. As there is a direct link of the real option models to the pricing of financial derivatives [10], the research theme represents a generalization of our expertise [1, 2, 8, 4] in modeling and computing of financial derivatives towards decision making for classical engineering applications.

Modeling the Future

There are several realistic scenarios for the next generation of powerplants (think of from the year 2030 onwards). We can distinguish between green energy from solar or wind, sustainable energy from gas, coal or nuclear based energy generating technologies. Most likely, we will deal with a combination of several of the energy sources. At NRG in Petten there is an interest in an economic assessment of the profitability of the future sources of energy, in particular of clean and safe versions of nuclear energy [5, 6, 7].
Some insight in the cost and safety aspects, based on an advanced mathematical model, which includes stochastic differential equations to model future uncertainty, can be interesting for future investment decisions even with respect to R&D budgets. Optimal decision in this context may depend on the future commodity prices, like the oil or uranium prices, but even on the political and public opinion on energy issues. In the broad context of future investment decisions we will extract some questions about future generators of energy, that are relevant for NRG (in close cooperation) and treat them in detail.

Questions of investment under uncertainty in an economic context can be modeled with the help of the theory of financial option pricing. The right - not the obligation - to undertake a business decision can be modeled in the context of the real options approach [3, 9]. A company may have the option to abandon a project completely or partly during its life. This amounts to the model of a put option on the remaining cash flows associated with the project.

An option to use different inputs to produce the same output (i.e. energy) is known as an *input mix option*, whereas the option to choose when to start a project is a *deferment option*. This latter kind of option is particularly valuable as it gives a value for the decision of a company to delay building a particular powerplant until market conditions are more favorable. A company may thus have the option to make strategic changes to investment decisions.

With the real option's approach one puts an emphasis on explicitly modeling future flexibility. This leads one to favor those decisions which preserve this flexibility. Ignoring the value of these real options (as in standard Net Present Value computations) may lead to incorrect investment evaluation.

Energy Research

The potential contribution of nuclear energy in improving the security (and independence) of the energy supply, in minimizing greenhouse gas emissions from the electricity sector and a low sensitivity towards uranium-fuel price increase have been recognized as important drivers for its recent worldwide revival. However, the decision to deploy different forms of energy demands a holistic view on a wide variety of aspects covering short-term but especially long-term periods. Advanced decision making theory considering long term uncertainties is needed to properly handle the economic challenges of such capital-intensive industrial investment. The timing and the decision making process to select between different technologies and the risk involved can be supported. In this respect, one can observe that the electricity industry learned fast how to handle price and volume risk via long term contracting, customized financing and diversification through the entire generation and product chain as have other industries which need to mitigate risk.

In order to tackle socio-political concerns regarding nuclear energy in today's and the future generating mix, governments push for a transition into the next generation of sustainable nuclear systems, the so-called Generation IV nuclear powerplants (NPPs). Deploying these sustainable energy systems will, however, essentially depend on finding the path forward in using the synergy between local competitiveness of the power plants and global sustainability via the nuclear fuel cycle. Utilities essentially look into the local competitiveness of power plants in various energy markets and will therefore be hesitant at least to invest in less-proven and more capital intensive Gen-IV NPPs. On the other hand, only through an appropriate mix of various sources of energy and the reprocessing fuel cycle, sustainability may be achieved and this is conflicting with the profit maximization strategy of a single utility. Here, governments may play an important role in setting the framework to guide the market, and thus utilities, to invest in specific types of energy without jeopardizing too much the (local) competitiveness.

Main goal and emphasis of the thesis is the development of a model that is able to point out possible transition paths of various types of energy and corresponding fuel cycle strategies in dynamic energy markets and to simulate the decision making process of stakeholders in the energy businesses, especially with respect to investment in new NPPs and fuel cycle facilities.

An important modeling issue is the question to the appropriate size of the future *discount rate*. By using a low discount rate, we are more likely to recommend spending money today. We are asking current generations to invest today to preserve opportunities for future generations. In decisions that will affect future generations, most people would take a different perspective than that implied by market rates. This is the view taken in the Stern review, see `http://en.wikipedia.org/wiki/Stern_Review`. One could try to model the discount rate as a function which depends, for example, on the global temperature, but these details are to be discussed with the partners at NRG.

Numerical Mathematics: Solving the HJB Equation

In the context of making decisions under uncertainty, each decision will lead to some outcome in some outcome set. By assigning a *utility* to every outcome, we can define the utility of a particular decision and an optimal decision as one that maximizes the cost functional of interest.

In the present project, it is not possible to determine the optimal solution with certainty, so a probabilistic approach should be followed. We can then formulate the problem mathematically as: Given a decision, model by means of a conditional probability density the distribution for the possible outcomes and compute the expected utility as a discounted expectation. The optimal decision within this model is the one that maximizes the utility function.

The flexibility to defer investment including multiple decision time points can be dealt with in this context by means of the *dynamic programming technique* from control theory. The Hamilton-Jacobi-Bellman (HJB) equation, a partial differential equation which describes the optimal cost for a given scenario, needs to be solved in the present project.

The function of interest is V, which represents the cost from starting in state x at time t and "decide within the scenario optimally" from then until time T. The HJB equation needs to be solved backwards in time, starting from $t = T$ and ending at $t = 0$. If we can solve for V, we can determine a control u that achieves the minimum cost. At each time step an optimal decision has to be made, out of a variety of decisions, which makes the numerical solution process computationally intensive. There is a need for highly efficient pricing approaches.

The HJB equation does in general not admit a classical solution, and one usually computes a so-called viscosity solution. The viscosity solution guarantees the convergence of the numerical solution to its continuous counterpart, however, unfortunately only low accuracy solutions can be obtained (first order accurate). Recently, however, at CWI, post-doc S. van der Pijl managed to achieve higher order numerical schemes for the HJB equation, based on Essentially Non-Oscillatory (ENO) high order schemes. We would also like to investigate the quality of these novel schemes in the present project.

Relation to Option Pricing

The modeling approach and computational technique described above are related to financial options that gives the opportunity of early exercise. Early-exercise can be done either *at pre-specified points in time*, prior to the exercise date (the Bermudan option) or at *any* time before the exercise date, in which case we deal with an American option. We have gained a lot of experience with dynamic programming in the financial context, by pricing a variety of these early-exercise options. We have worked both on the PIDE approach [1, 2], which is closely related to the HJB equation discussed above, and on an approach based on numerical integration, computing the so-called "discounted expected payoff" [8, 4]. It is a challenge to generalize the solution methods developed previously to the dynamic programming situation that we encounter with the real options. As there is an optimization involved in the numerical solution of the HJB equation, it is important to solve the dynamic programming problem as efficient as possible.

As an example of a highly efficient solution method, we briefly explain here the COS method [4], based on Fourier cosine expansions, which we developed for Bermudan options. The error convergence is exponential for processes characterized by very smooth ($C^\infty[a, b] \in \mathbb{R}$) transitional probability density functions. The computational complexity is $O((M-1)N\log N)$ with N a (small) number of terms from the series expansion, and M, the number of early-exercise/monitoring dates. The method can readily be applied to solving problems under different stochastic models for the asset price, as long as the characteristic function (i.e., the Fourier transform of the transitional probability density function) is available.

A Bermudan option can be exercised at pre-specified dates before final time. The holder receives the exercise payoff when she decides to exercise the option. Let t_0 denote the initial time and $\mathcal{T}\{t_1,\cdots,t_M\}$ be the collection of all exercise dates with $\Delta t := (t_m - t_{m-1})$, $t_0 < t_1 < \cdots < t_M = T$. The pricing formula for a Bermudan option with M exercise dates then reads, for $m = M, M-1, \ldots, 2$:

$$\begin{cases} c(x,t_{m-1}) &= e^{-r\Delta t}\int_{\mathbb{R}} v(y,t_m)f(y|x)dy, \\ v(x,t_{m-1}) &= \max\left(g(x,t_{m-1}), c(x,t_{m-1})\right), \end{cases} \tag{1}$$

followed by

$$v(x,t_0) = e^{-r\Delta t}\int_{\mathbb{R}} v(y,t_1)f(y|x)dy. \tag{2}$$

Here x and y are state variables, defined as the logarithm of the ratio of the asset price S_t over the strike price K, $v(x,t)$, $c(x,t)$ and $g(x,t)$ are the option value, the continuation value and the payoff at time t, respectively. The probability density function of y given x under a risk-neutral measure is denoted by $f(y|x)$ in (2), and r is the (deterministic) risk-neutral interest rate.

The COS method is based on the insight that the Fourier-cosine series coefficients of $f(y|x)$ are closely related to its conditional characteristic function, $\phi(\omega;x)$, defined as,

$$\phi(\omega;x) := \int_{\mathbb{R}} f(y|x)e^{i\omega y}dy. \tag{3}$$

We replace the density function by its Fourier-cosine series expansion on truncated interval $[a,b]$,

$$f(y|x) = {\sum_{k=0}^{\infty}}' A_k(x)\cos\left(k\pi\frac{y-a}{b-a}\right), \tag{4}$$

where $\sum'$ indicates that the first term in the summation is multiplied by 1/2. The series coefficients $\{A_k(x)\}_{k=0}^{\infty}$ are defined by

$$A_k(x) := \frac{2}{b-a}\int_a^b f(y|x)\cos\left(k\pi\frac{y-a}{b-a}\right)dy, \tag{5}$$

Coefficients $A_k(x)$ can now be written as

$$A_k(x) = \frac{2}{b-a}\mathrm{Re}\left\{e^{-ik\pi\frac{a}{b-a}}\int_a^b e^{i\frac{k\pi}{b-a}y}f(y|x)dy\right\}. \tag{6}$$

where $\mathrm{Re}\{\cdot\}$ denotes taking the real part. The finite integration in (6) can be approximated by

$$\int_a^b e^{i\frac{k\pi}{b-a}y}f(y|x)dy \approx \int_{\mathbb{R}} e^{i\frac{k\pi}{b-a}y}f(y|x)dy =: \phi\left(\frac{k\pi}{b-a};x\right).$$

As a result, $A_k(x)$ can be approximated by $F_k(x)$ with

$$F_k(x) := \frac{2}{b-a}\mathrm{Re}\left\{\phi\left(\frac{k\pi}{b-a};x\right)e^{-ik\pi\frac{a}{b-a}}\right\}, \tag{7}$$

which gives the COS formula for pricing options,

$$\hat{c}(x,t_{m-1}) := e^{-r\Delta t}{\sum_{k=0}^{N-1}}' \mathrm{Re}\left\{\phi\left(\frac{k\pi}{b-a};x\right)e^{-ik\pi\frac{a}{b-a}}\right\}V_k(t_m). \tag{8}$$

Here the function $\hat{c}(x, t_{m-1})$ represents the approximation of the continuation value $c(x, t_{m-1})$. This forms the basis for the fastest pricing method around for this kind of options. It is our aim to define similar solution methods for relevant real options questions from the energy sector.

Cooperation

Besides various teams on technical disciplines, like reactor physics, thermal hydraulics and fuel chemistry, NRG is building up a team on Nuclear Business Studies. Its field of study is the future market for the innovative nuclear systems to whose development NRG is contributing on the technical side. For this purpose, NRG has the tool DANESS at its disposal, a code that builds nuclear park scenarios from input data on reactors and fuel cycle facilities like power level, fuel composition and costs.
The PhD student could start as soon as possible, as there would be an overlap with a PhD student at NRG that gained some initial experience with decisions under uncertainty.
If you are interested in this PhD position, please send your application letter, inclusing a CV, a list of three references, and an overview of relevant courses which you took during your MSc period, to c.w.oosterlee@cwi.nl

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