Computational Finance

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- Exam (in Delft): Successful completion of some exercises related to the material taught, and a computer exercise, handed out during the course, by means of a written report and a oral examination of the material afterwards.
- Credit: 6 ECTS points

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Course Outline

- Introduction to financial products, in particular to different options:
  - European options
  - American options
  - Exotic options
- Numerical techniques for pricing options:
  - Partial differential equations and numerical approximation
  - Discounted expected payoff and numerical integration
  - Monte Carlo methods
- Different stochastic processes for the underlying:
  - Geometric Brownian Motion
  - Include Jump processes (affine jump diffusion processes)
  - Stochastic Volatility (and even stochastic interest rate)

Recommended Book

$\Rightarrow$ Tools for Computational Finance (4rd ed.), by R. Seydel, Springer Verlag Berlin, 2008

Material

- Focus on Techniques from Numerical Mathematics:
  1. Modeling tools for financial options (Ch. 1, and Appendices)
  2. Generation of random numbers with specified distributions (Ch. 2)
  3. Monte Carlo simulation (Ch.3)

- Solution techniques for partial differential equations (PDEs)
  1. Finite differences and standard options (Ch.4, App.)
  2. Finite element methods (Ch.5)
  3. Pricing of exotic options (Ch.6)

The Netherlands

(Holland)

- Small country in Europe with an impressive past (trading, painters)
- Inhabitants are the Dutch

The Past

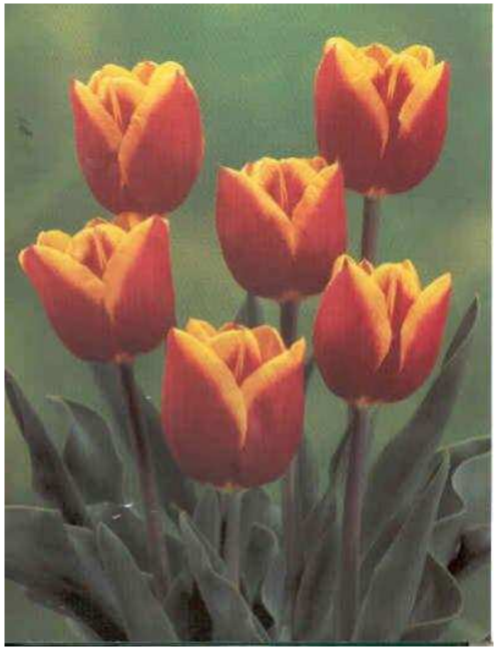
Trading

- In the year 1608, shares were issued to finance a company called the VOC: 'A united fleet to increase sailing to and trading activities with the East'
- A wealthy period in Dutch history started

The Present

Tulips

- The Netherlands is known for its tulips



Holland

The tulip and the bulb crash, 1634-1637

- Tulips were brought from Turkey to the Netherlands in 1593. After some time tulips contracted a nonfatal virus called 'mosaic', which did not kill the population but causing 'flames of colors'. This made the flower unique.
- Thus, tulips began to rise in price. Everyone began to deal in bubls, essentially speculating on the tulip market.
- The true bulb buyers filled up their inventories, so increasing scarity and demand.
- Soon prices were rising so fast and high that people were trading their land, life savings to get more tulip bulbs.
- The originally overpriced tulips enjoyed a twenty-fold increase in value - in one month.

The tulip and the bulb crash, 1634-1637

- The prices were not an accurate reflection of the value of a tulip bulb.
- This is called a speculative bubble, and typically some people decided to sell and crystallize their profits.
- A domino effect of progressively lower prices took place, as everyone tried to sell while not many were buying.
- Dealers refused to honor contracts and people began to realize that they traded their homes for some bulbs.
- The government attempted to step in and halt the panic and the crash, but that did not work out well.
- Even the people who locked in their profit early suffered under the following depression.

Interest Rate

Safe Money

The simplest concept in finance is the time value of money.

- €1 today is worth more than € 1 in a year's time.
- There are several types of interest
  - There is simple and compound interest. Simple interest is when the interest you receive is based only on the amount you initially invest, whereas compound interest is when you also get interest on your interest.
  - Interest typically comes in two forms, discretely compounded and continuously compounded.
- Invest €1 in a bank at a discrete interest rate of r (assumed to be constant), paid once per year. At the end of one year your bank account will contain $1 \times (1 + r)$.

- Now suppose you receive m interest payments at a rate of r/m per annum.

  After one year you will have

$$(1+\frac{r}{m})^m.$$

  Suppose these interest payments come at increasingly frequent intervals, but at an increasingly smaller interest rate (we will take the limit $m \to \infty$). This will give a continuously paid rate of interest.

  The expression above become

$$(1+\frac{r}{m})^m = e^{m \log{(1+\frac{r}{m})}} \sim e^r.$$

  That is how much money you will have in the bank after one year if the interest is continuously compounded. And similarly, after a time t you will have an amount

$$e^{rt} \quad .$$

- Suppose $M(t)$ in the bank at time t, how much does this increase with time?

  If you check your account at time t and again a short period later, time $t + dt$, the amount will have increased by

$$M(t + dt) - M(t) \approx \frac{dM}{dt}dt + \ldots, \quad \text{(Taylor series expansion).}$$

  The interest you receive must be proportional to the amount you have, M, the interest rate r and the timestep, dt. Thus,

$$\frac{dM}{dt}dt = rM(t)dt \text{ giving } \frac{dM}{dt} = rM(t) \quad .$$

  If you have €M(0) initially, then the solution is $M(t) = M(0)e^{rt}$.

  Conversely, if you know you will get 1€ at time T in the future, its value at an earlier time t is simply

$$e^{-r(T-t)} \quad .$$

Financial instruments

Equities

A basic of financial instruments is the equity, stock or share.

- This is the ownership of a small piece of a company. The price is determined by the value of the company and by the expectations of the performance of the company.
- These expectations are seen in the bid and ask behavior in the market.
- The expectations give an uncertainty to the future price development of the stock. The exact profit is known at the date of selling.
- The real value of the stock is sometimes a bit higher, sometimes a bit lower than the expected value. The amount in which the stock price development can differ from the expected value is determined by the so-called volatility.

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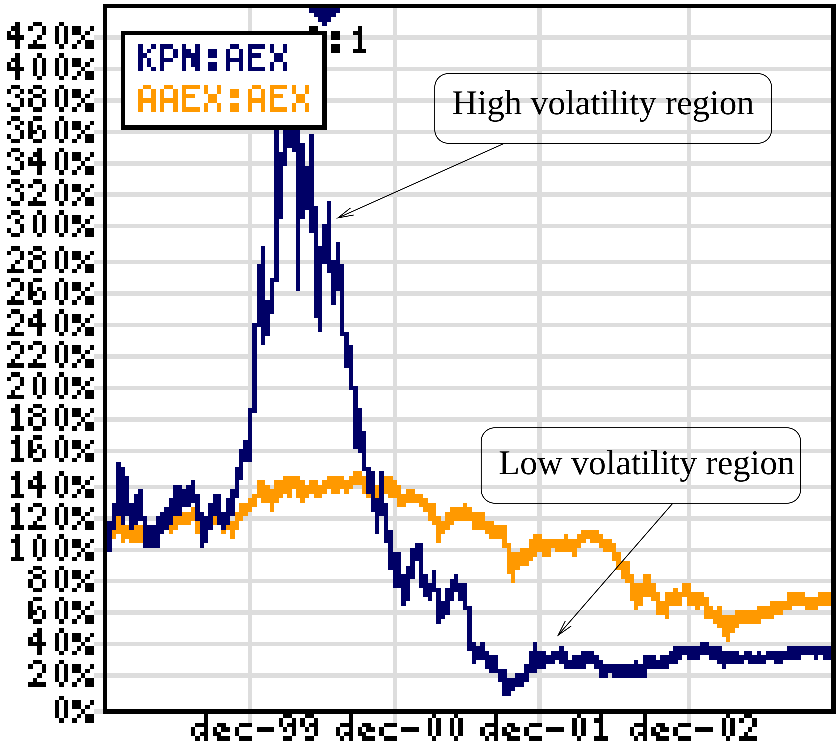
Volatility

- What does it mean?
  - A statistical measure of the tendency of a market or security to rise or fall sharply within a period of time.
- Volatility is typically calculated by using variance of the price or return. A highly volatile market means that prices have huge swings in very short periods of time.
- Security: An instrument representing ownership (stocks), a debt agreement (bonds), or the rights to ownership (derivatives).
- Return: The gain or loss of a security in a particular period. The return consists of the income and the capital gains relative on an investment. It is usually quoted as a percentage.

Volatility


KPN:AEX
AAEX:AEX
High volatility region
Low volatility region
420%
400%
380%
360%
340%
320%
300%
280%
260%
240%
220%
200%
180%
160%
140%
120%
100%
80%
60%
40%
20%
0%
dec-99
dec-00
dec-01
dec-02

Exchanges

- Some shares are quoted on a regulated stock exchange, so that they can be bought and sold freely.

- Beurs van Berlage (Amsterdam), Beursplein 5 (Euronext)

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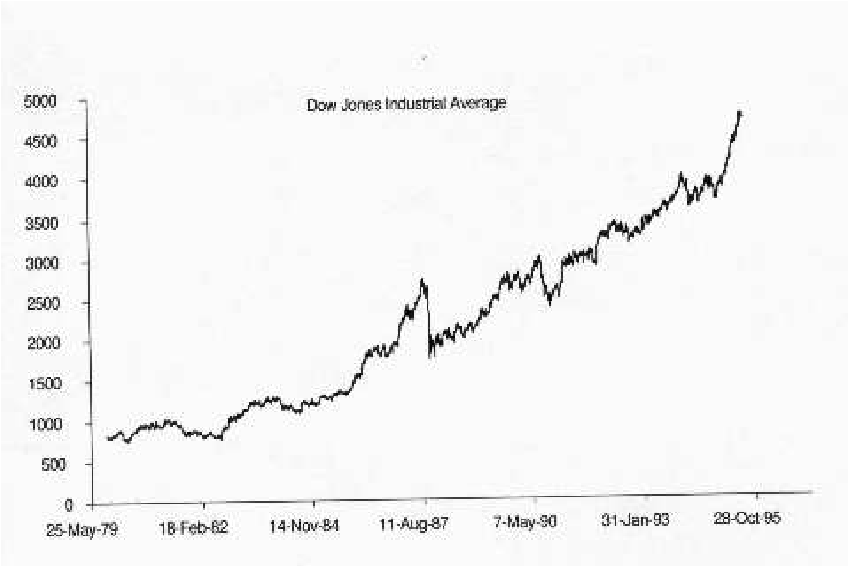
Dow Jones Index

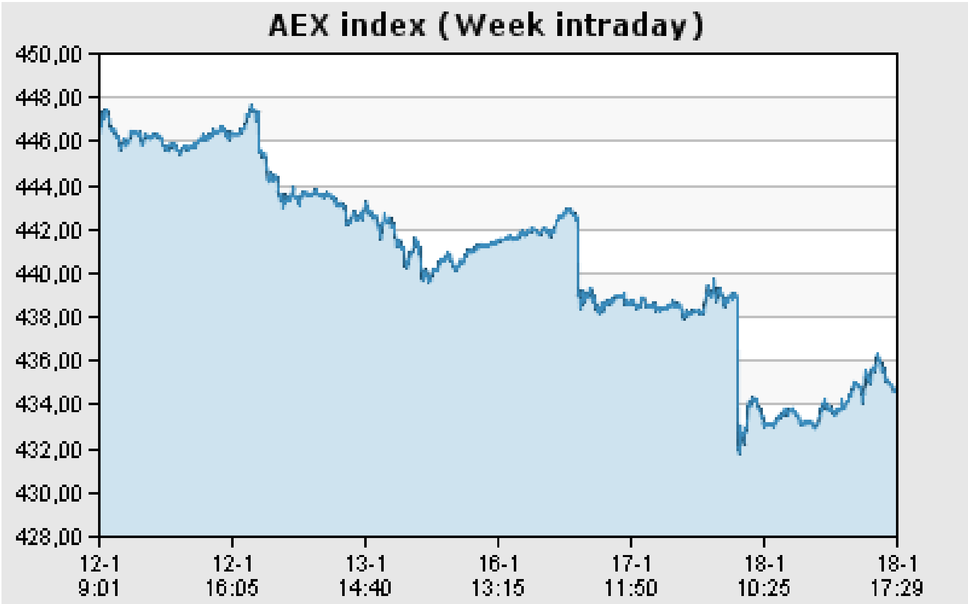
- Prices have a large element of randomness. This does *not* mean that we cannot model stock prices, but it does mean that the modelling must be done in a probabilistic sense.
- A model for generating asset prices, a stochastic differential equation, is the geometric Brownian motion.



- Dow Jones, 15 years time, 30 leading stocks

AEX

- AEX Index, 25 leading Dutch stocks
- 1 day, 1 week, 1 year



Commodities

- Commodities are usually raw products such as precious metals, oil, food products etc.

- The prices of these products are unpredictable but often show seasonal effects. Scarcity of the product results in higher prices.

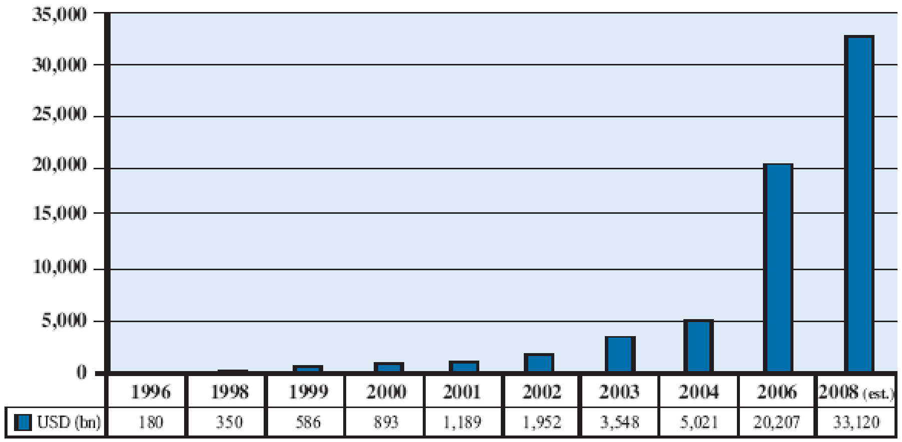
- Most trading is done on the futures market, making deals to buy or sell the commodity at some time in the future.

Currencies

- The exchange rate is the rate at which one currency can be exchanged for another: This is the world of foreign exchange or FX.
  Some currencies are pegged to another, others are allowed to float freely.
- There must be consistency throughout the FX world: If it is possible to exchange dollars for euros and then euros for yen, this implies a relationship between the dollar/euro, euro/yen and dollar/yen exchange rates.

  If this relation moves out of line, it is possible to make arbitrage profits by exploiting the mispricing.
- Central banks can use interest rates as a tool for manipulating exchange rates, but only to some degree.

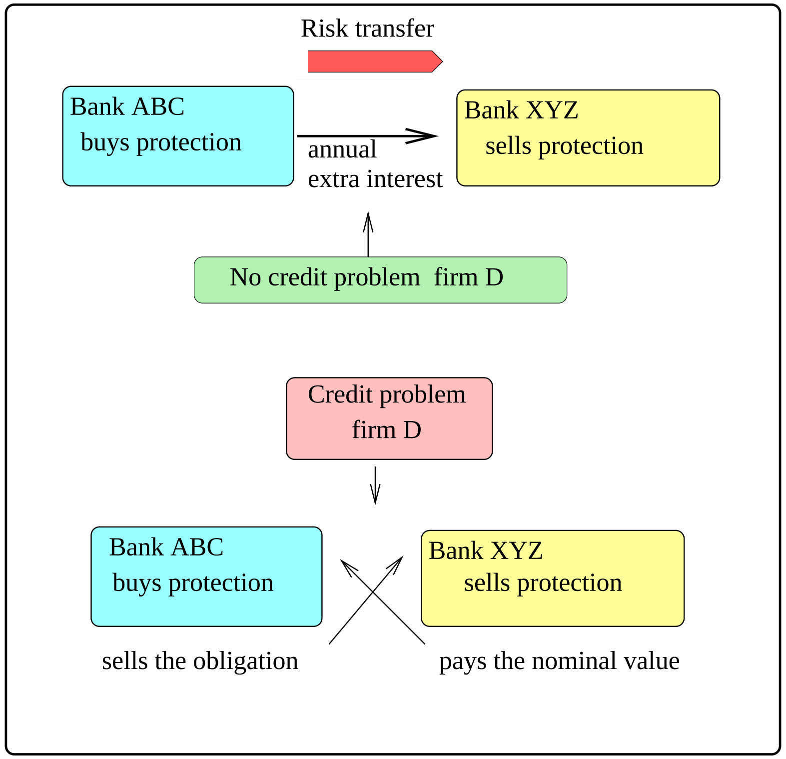
Credit



- The credit market is 3 times bigger than the stock market
- 30 trillion US dollar bussiness volume (30 000 000 000 000 \$)
- Common products are the Credit Default Swap (CDS) and the Collateralized Debt Obligation (CDO)

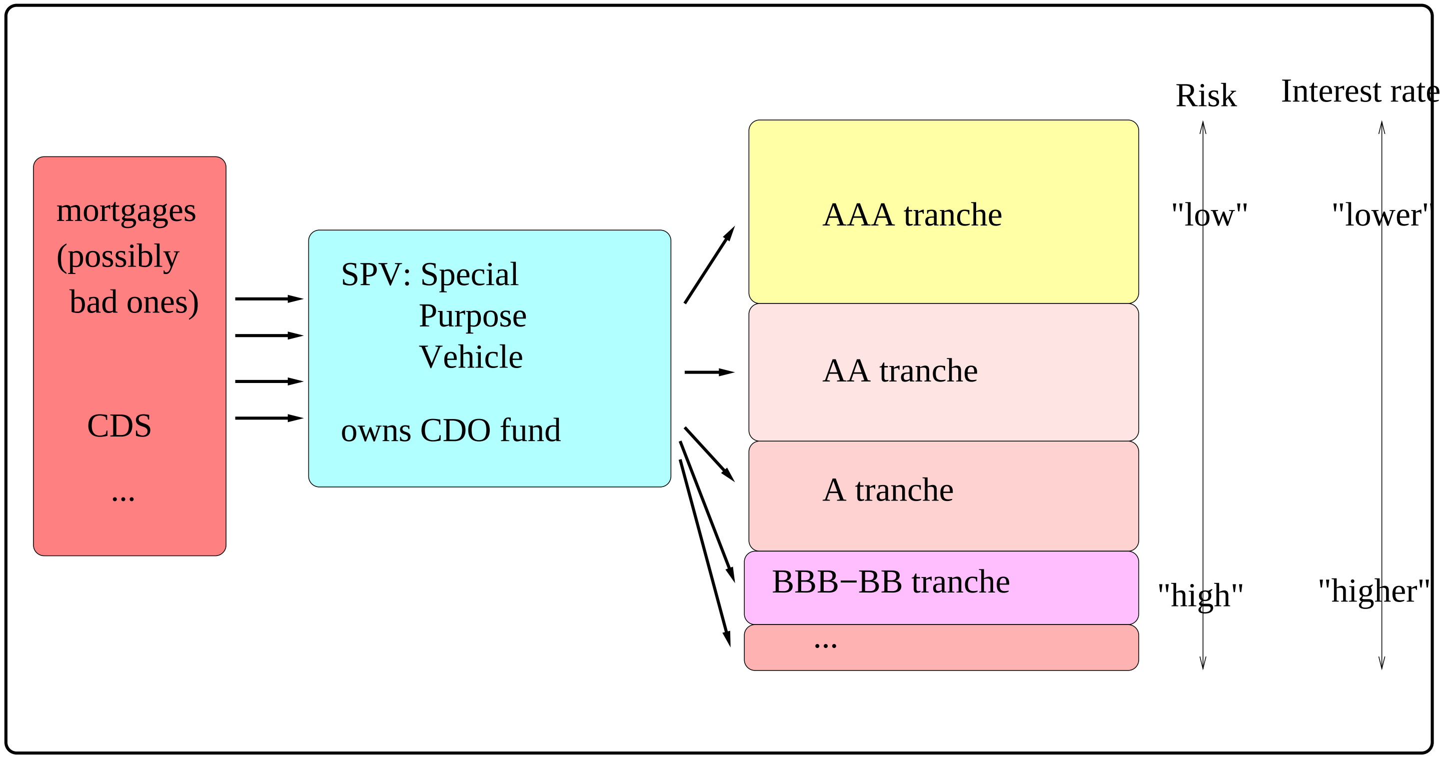
Credit Default Swap

CDS



Collateral Debt Obligation

CDO



Derivatives

Options

- It was only on 26th April 1973 that options were first officially traded on an exchange. It was then that The Chicago Board Options Exchange (CBOE) first created standardized, listed options.

  Initially there were just calls on 16 stocks. Puts weren't introduced until 1977.

- In the US options are traded on CBOE, the American Stock Exchange, the Pacific Stock Exchange and the Philadelphia Stock Exchange. Worldwide, there are over 50 exchanges on which options are traded.

Standard options:

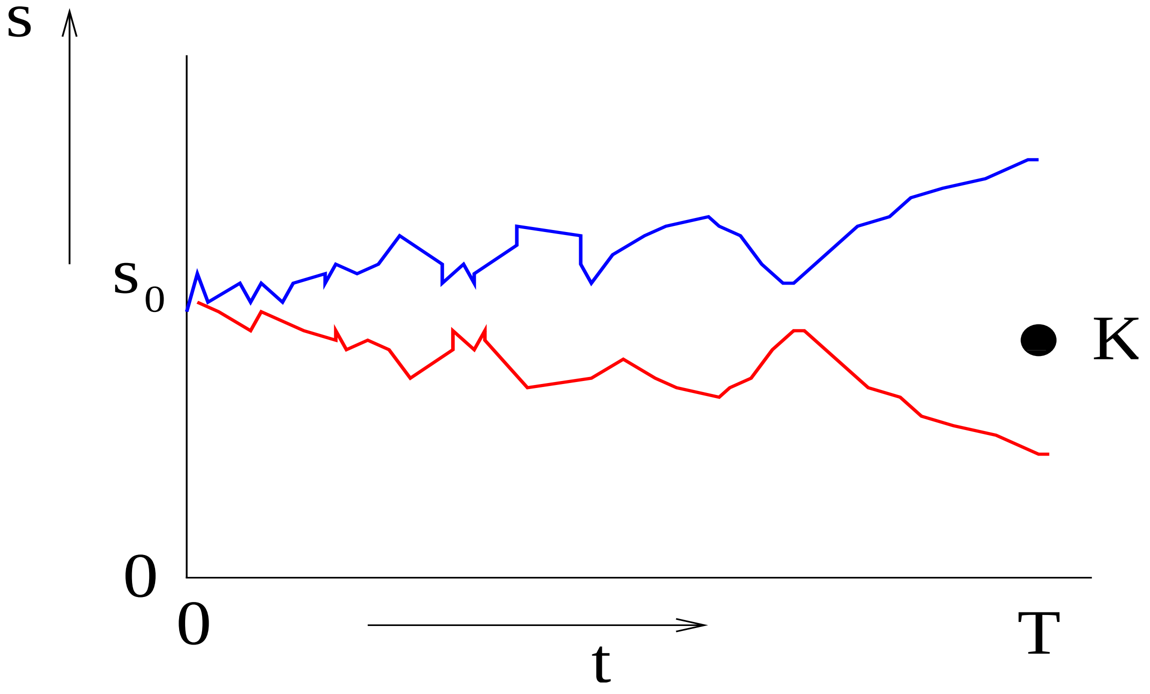
Call

A call option gives the holder the right to trade in the future at a previously agreed price but takes away the obligation. If the stock falls, we don't have to buy it.

- European Call option: At a prescribed time in the future, maturity or expiry date, the holder of the option may purchase a prescribed asset (shares, stocks) for a prescribed amount (the exercise or strike price).
  The other party of this contract (the writer) must sell the asset, if the holder decides to buy it.
- Financial Engineering: Choosing suitable options in an optimal portfolio.
- Portfolio: Set of securities (shares, options, ...).
- Main interesting questions:
  How much would one pay for this right ? (What's the value of an option ?)
- How can the writer minimize the risk associated ?

Options

- Schematic picture



Speculation

- Options have two main uses: speculation and hedging.

- Example:
  Portfolio consisting of several stocks is worth €30 000.
  Suppose we buy the portfolio and see how much gain we make in 1 year. If its value goes up to €60 000, we make a profit of €30 000.
  If it goes down (by €2000), the loss goes simultaneously (by €2000)

- Suppose we bought instead 6 call options priced €5000 to buy 6 portfolios in 1 year for €30 000. Then, with the value at €60 000, we exercise our 6 options (cost €180 000) and sell the stocks immediately (get €360 000) !
  The loss, if the portfolio would decrease its value is, of course, €30 000: One would not exercise the option.

$\Rightarrow$ Options can be a relatively cheap way of leading a portfolio to a large risk.

Notation

- The price of an asset (stock, share) is $S(t)$
- Value of an option is $V(S(t), t)$ sometimes $C(S, t)$ for a call and $P(S, t)$ for a put option
- Its strike price is K
- The time to maturity is T, so $0 \le t \le T$
- The interest rate r, (here assumed to be constant),
  This assumption is reasonable: a typical option has a lifespan of about nine months.
  $r = 0.05$ means 5 % interest per year
- The volatility σ, indicates the randomness of the asset price,
  $\sigma = 0.2$ means 20 % volatility