Decision-support tool for assessing future nuclear reactor generation portfolios.

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Abstract

Capital costs, fuel, operation and maintenance (O&M) costs, and electricity prices play a key role in the economics of nuclear power plants, where especially capital costs are known to be highly uncertain. Different nuclear reactor types compete economically by having either lower and less uncertain construction costs, increased efficiencies, lower and less uncertain fuel cycles and O&M costs etc. The decision making process related to nuclear power plants requires a holistic approach that takes into account the key economic factors and their uncertainties. We here present a decision-support tool, that satisfactorily takes into account the major uncertainties in the cost elements of a nuclear power plant, to provide an optimal portfolio of nuclear reactors. The portfolio so obtained, under our model assumptions and the constraints considered, maximizes the combined returns for a given level of risk or uncertainty. These decisions are made using a combination of real option theory and mean-variance portfolio optimization.

1 Introduction

The global electricity demand is expected to double to over 30,000 TWh annually by the year 2030 and meeting this demand without substantially exacerbating the risks of climate change requires a solution comprised of a variety of technologies on both the supply and demand side of the energy system (Pacala and Socolow (2004), Holdren (2006) and European Commission (2007)). Nuclear power can play a key role in meeting the projected large absolute increase in energy demand while mitigating the risks of serious climate disruption. The fact that countries seem keen on building nuclear power stations suggests that their relative costs compared to low-carbon alternatives seem attractive to at least potential investors (Kessides, 2010). However, there are some concerns related to uncertainties underlying the various costs elements of nuclear power

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that are reflected in the wide range of cost estimates, cost overruns and schedule delays, for example of Finland's Olkiluoto and France's Flamanville nuclear power plants.

There have been numerous studies on the economics of nuclear power in recent years which use levelized cost [1] of electricity to compare the economics of different generation technologies. The levelized cost methodology used in these studies however does not address the role of risks and uncertainties involved. Methodologies that take into account the large and diverse set of risks characterizing investment in nuclear power are required. This paper concentrates on the effect of risks and uncertainties on investment decisions related to the nuclear industry and the use of diversification to mitigate some of these risks. Following Roques et al. (2008) and Fortin et al. (2007) we use a two-step approach, where first real options optimal investment decisions are taken at the plant level, and then mean-variance portfolio (MVP here after) theory is used to minimize the uncertainties of returns for a portfolio of nuclear reactors.

The seminal literature using MVP techniques in the power sector concentrated on fuel price risk, and focussed on minimizing generation cost, which, under ideal regulations of a vertically integrated franchise monopoly, should maximise social welfare. Awerbuch and Berger (2003) use MVP to identify the optimal European energy technology mix, considering not only fuel price risk but also Operation and Maintenance (O&M), as well as construction period risks, while Jansen et al. (2006) use MVP to explore different scenarios of the electricity system development in the Netherlands. Roques et al. (2008) applied the portfolio theory from a private investor perspective to identify optimal portfolios for electricity generators in the UK electricity market, concentrating on profit risk rather than production costs risk. Fortin et al. (2007) suggest the use of Conditional Value-at-Risk (CVaR) for portfolio optimization rather than mean-variance portfolio and provide a detailed review of the literature in this area.

Real options analysis (ROA) has been applied to the energy sector planning for years, since the special features of the electricity sector, such as uncertainty, irreversibility and flexibility to postpone investments, make standard investment rules solely relying on the net present value (NPV) not advisable as they ignore the options involved in a sequence of decisions. The real options approach for making investment decisions in projects with uncertainties was pioneered by Arrow and Fisher (1974).Using real options it's possible to value the option to delay, expand or abandon a project with uncertainties, when such decisions are made following an optimal policy.

Pindyck (1993) employs real options to analyse the decisions to start, continue or abandon the construction of nuclear power plants. There, uncertain costs of a reactor rather than expected cash flows are considered for making the optimal decisions. Rothwell (2006) uses ROA to compute the critical electricity price at which a new advanced boiling water reactor should be ordered in Texas. Naito et al. (2010) apply real options theory to determine the optimal timing for decommissioning of existing nuclear power plants and construction of their replacements. Zhu (2012) uses real options to evaluate the Sanmen nuclear

[1] The levelized cost of a project is equivalent to the constant euro price of electricity that would be required over the life of the plant to cover all operating expenses, interest and repayment obligations on project debt, and taxes plus an acceptable return to equity investors over the economic life of the project.

power plant in the Zhejiang province, China, taking into account factors such as uncertain construction and electricity costs. Gollier et al. (2005) evaluate projects where a firm needs to make a choice between a single high capacity reactor (1200 MWe) or a flexible sequence of modular SMRs (4×300 MWe) using real options. The authors in Jain et al. (2012) and Jain et al. (2013) study the value of modularity in nuclear power plants when decisions are to be made in *finite time horizon*. They show that the value of a modular project can be significantly affected by changing decision horizons, while taking into account factors such as learning, probabilistic lifetime extensions, and rare events can affect the operation of the power plant.

In this paper we concentrate on investment in nuclear power plants in a liberalized electricity market, where the energy utility diversifies into different nuclear reactor types as a strategy for reducing exposure to construction costs, fuel and electricity price risks. Mean-variance portfolio (MVP) theory is used to identify the portfolios that maximize the returns for given risk levels. The return distribution of individual nuclear generation types depends on the uncertainties in the costs and revenues of the plant. It is, however, also affected by decisions to continue or abandon a project, that may be taken based on evolution of construction costs and electricity prices. For example, if the construction costs become too high in the future, the management may decide to abandon a project. Using real options we compute the return distribution for each plant assuming the management makes optimal decisions in the future. The return distribution for each plant is then used to compute the mean-variance portfolio.

Real options in discrete finite time horizon can be priced using methods for pricing financial options with early exercise features. This paper uses a simulation based algorithm, called the Stochastic Grid Bundling Method (SGBM) (Jain and Oosterlee, 2012), for computing the return distribution for individual reactors. The simulation also computes the optimal policy to continue or abandon the project in order to maximize its expected cashflows.

The rest of the paper is structured as follows: Section 2 will be concerned with defining the portfolio optimization problem. Section 3 gives detailed account of the real options layer used for making optimal decisions at the individual plant level. In section 4 we validate our model against the results reported in (Pindyck, 1993). Section 5 illustrates the two steps involved when determining the optimal reactor order fractions through various numerical examples . Under our model assumptions, the sensitivity of reactor order fractions to a different choices of parameter values and constraints on the portfolio are also studied in this section. The final section will conclude the findings and interpret the general implications.

2 Mean Variance Portfolio

While selecting the generating technology, policy makers need to consider not only the cost of the generating technology but also uncertainties in the costs involved. Furthermore, in liberalized energy markets uncertainties are not only limited to the costs of the generating technology but also affect the revenues stream, as utilities are no longer able to pass on their prudently incurred investments costs to consumers. In order to systematically deal with uncertainties in the costs and revenues, we, like Awerbuch and Berger (2003), Roques *et al.*

(2008), employ the MVP theory [2] to find an optimal mix of generating technologies, that results in the highest expected return for a given level of uncertainty (or standard deviation) of the returns[3].

To compute the optimal reactor order fraction using MVP, the expected return distribution for individual reactors is required. One way of obtaining the return distribution is by simulating several samples of costs (like the fuel prices) and revenues (electricity prices) and then computing the return for each sample. This approach however does *not* address the effect of possible future decisions related to operation of the power plant (for example, abandoning the plant if the expected costs exceed expected revenues at a later date) on the return distribution. In order to include the effect of optimal decisions in the return distribution, first an optimal investment policy for each reactor type is computed. This policy is then applied to simulated paths to determine whether for a particular path there should be an early abandonment. Based on these decisions the costs and revenues for each sample path are computed, which then gives the optimal return distribution. The details for computing an optimal investment policy and the associated return distribution for individual plants are given in section 3.

Suppose an investor has a certain wealth to invest in a set of J reactors. Let the return from operation of reactor i be denoted by random variable R_i, and let w_i represent the proportion of the total investment to allocate in the i-th reactor. The expected return of this portfolio is given by:

$$\mathbb{E}[R_p] = w_1\mathbb{E}[R_1] + \ldots + w_J\mathbb{E}[R_J]. \tag{1}$$

The portfolio variance, in turn, is calculated by

$$Var(R_p) = \mathbb{E}\left[\left(\sum_{i=1}^{J} w_i R_i - \mathbb{E}\left(\sum_{i=1}^{J} w_i R_i\right)\right)^2\right]. \tag{2}$$

So,

$$Var(R_p) = \sum_{i=1}^{J}\sum_{j=1}^{J} \mathbb{E}\left[(R_i - \mathbb{E}[R_i])(R_j - \mathbb{E}[R_j])\right] w_i w_j. \tag{3}$$

Representing each entry i, j of the covariance matrix Q by

$$q_{ij} = \mathbb{E}\left[(R_i - \mathbb{E}[R_i])(R_j - \mathbb{E}[R_j])\right], \tag{4}$$

one has

$$Var(R_p) = \mathbf{w}^\top Q \mathbf{w},$$

where $\mathbf{w} = (w_1, \ldots, w_J)^\top$.

As w_i represents the weight of reactor i, the weights are required to satisfy an additional constraint:

[2] MVP is one of the possible ways for portfolio optimization, based on how the risk is expressed, which in the case of MVP is the standard deviation of the returns. Others like Szolgova *et al.* (2011) , Fuss *et al.* (2012) use Conditional Value at Risk (CVar) for portfolio optimization.

[3] See Awerbuch and Berger (2003) and Jansen et al. (2006) for a discussion of the assumptions and limitations affecting the application of MVP theory to power generation assets.

$$\sum_{i=1}^{I} w_i = 1.$$

As we deal with a portfolio of nuclear reactors additional conditions on the weights, like that they cannot be negative, need to be applied. Additionally, weights of individual reactors might be constrained by an upper and lower bound, for example, if the utility decides that the new portfolio should not excessively deviate from the existing one. In general, we can state that:

$$L_i \leq w_i \leq U_i, \; i = 1, \ldots, J,$$

for given lower L_i and upper U_i bounds on the weights.

MVP theory does not prescribe a single optimal portfolio combination, but rather a *range* of efficient choices for each level of return, which form a *Pareto efficient frontier* composed of non dominated points. This means that a rational investor should use an external criterion to choose a portfolio out of the set at hand. Investors will choose a risk-return combination based on their preferences and risk aversion. By solving the mean-variance optimization problem we identify a portfolio for given risk tolerance, λ, of the investor, of minimum variance amongst all that provide a return equal to $R_{\min}$, or, in other words, minimize the risk for a given level of return. The formulation can be written as:

$$\begin{aligned}
\min_{\mathbf{w}} \quad & \frac{1}{\lambda}\mathbf{w}^\top Q\mathbf{w}, \\
\text{subject to:} \quad & \mathbb{E}[R_p] = R_{\min}, \\
& \sum_{i=1}^{J} w_i = 1, \\
& L_i \leq w_i \leq U_i, \; i = 1, \ldots, J.
\end{aligned} \tag{5}$$

Equation (5) is a convex quadratic programming problem for which the first-order necessary conditions are sufficient for optimality. The classical Markowitz mean-variance model can be seen as a way of solving the bi-objective problem, which consists of simultaneously minimizing the portfolio risk (variance) and maximizing the portfolio return (profit), i.e.

$$\begin{aligned}
\min_{\mathbf{w}} \quad & \frac{1}{\lambda}\mathbf{w}^\top Q\mathbf{w}, \\
\max_{\mathbf{w}} \quad & \mathbb{E}[R_p], \\
\text{subject to:} \quad & \sum_{i=1}^{J} w_i = 1, \\
& L_i \leq w_i \leq U_i, i = 1, \ldots, J.
\end{aligned} \tag{6}$$

The solution of equation (6) is non-dominated, efficient or Pareto optimal for equation (5). Efficient portfolios are thus the ones which have the minimum variance among all that provide a certain expected return or, in other words, those that have maximal expected return among all upto a certain variance.

3 Plant level optimization using real options

The real option valuation of nuclear power plants should take into account the major uncertainties that affect the decision making process associated with them. Of the several risks involved in the life cycle of nuclear power plants (see Kessides (2010) for a comprehensive review), the following have been identified as significant from the perspective of economic risks and are taken into account in our model.

- *The construction or capital costs, and the speed to build:* The length of the pre-construction period and the time it takes to construct the plant are highly uncertain as there are several factors that make forecasting nuclear plant construction costs difficult. As pointed out by Kessides (2010) one of the reasons for this is that new nuclear plants require a significant amount of on-site engineering, which accounts for a major portion of the total construction cost (Thomas, 2005). It is generally difficult to manage and control the costs of large projects involving complex on-site engineering. While major equipment items (turbine generators, the steam generators, and the reactor vessel) can be purchased on turnkey terms, it would difficult for the entire nuclear plant to be sold on turnkey terms precisely because of the lack of confidence on the part of vendors that they can control all aspects of the total construction costs. Additionally, governmental licensing and certification procedures can add up significantly to construction costs and delays.
- *The O&M and fuel costs:* The O&M component includes expenses related to health and environmental protection and accumulation of funds for spent-fuel management and for eventual plant decommissioning. It also includes the cost for insurance coverage against accidents. Thus, several potential externalities are internalized in O&M costs.
- *The price of electricity:* Electricity prices are highly uncertain and vary significantly not just between different seasons but also during a single day. Thus, the revenues generated by a power plant are uncertain and an important parameter for making optimal decisions.

3.1 Modelling uncertain construction costs

Construction or capital costs constitute almost 60% of the total costs associated with nuclear power plants and are the major source of uncertainty when it comes to a comprehensive cost-benefit analysis of nuclear power. An economic assessment that reflects on the uncertainty in construction costs by employing probabilistic scenario analysis can help making economic decisions related to NPPs. To capture the uncertainties associated with the construction costs and their effect on the decision making process we follow the model proposed by Pindyck (1993) for irreversible investment decisions when projects take time to complete and are subject to uncertainties over the cost of completion.

Expenditure of nuclear power plants are sunk costs that cannot be recovered should the investment turn out, *ex post*, to have been an unfavourable one, i.e. the firm cannot disinvest and recover the money spent. Cost uncertainties have implications for irreversible investment decisions. The uncertainties in construction costs of nuclear power plants can be classified into two different types. The first, as Pindyck (1993) states, is *technical uncertainty*, that relates to the technical difficulties associated with the completion of the nuclear power plant, i.e. if the cost of raw materials, labour etc. are fixed then the uncertainty reflects how much time, effort and material will ultimately be required. Technical uncertainties involved in the construction of the plant can be resolved only by undertaking the project which unfolds the actual costs and construction time as the project proceeds.

The second type of uncertainty that affects the construction costs is external or independent of what the firm does and is called *input cost uncertainty.* Input cost uncertainty arises when the prices of labour, land, materials needed to build the plant fluctuate unpredictably, or when there are unpredictable changes in government regulations (for example a change in the required quantities of construction inputs or certification time). As prices and government regulations change irrespective of whether or not the construction of a plant has already begun, input costs uncertainties affect the expected plant costs.

Consider the expected cost of completion of a nuclear power plant to be a random variable K, then, following Pindyck (1993), the stochastic differential equation (SDE) governing the dynamics of K_t can be written as:

$$dK_t = -Idt + \beta(IK_t)^{\frac{1}{2}}dW_\beta + \gamma K_t dW_\gamma, \tag{7}$$

where I is the rate of investment. When the construction of a nuclear power plant has begun the expected change in K_t over an interval dt is $-Idt$, but the realized change can be greater or less than this due to the random fluctuations in the cost to completion of the project. The term $\beta(IK_t)^{\frac{1}{2}}dW_\beta$ constitutes a part of the fluctuation in the project cost due to the technical uncertainty, where the noise is introduced by the Wiener process W_β and the amplitude of the noise depends on the remaining expected costs of the project and the rate of investment I, and β. When the firm is not investing, i.e., I is zero the project cost is not influenced by technical uncertainties. The term $\gamma K_t dW_\gamma$ constitutes the part of the fluctuation in the project costs due to input cost uncertainty. As discussed before, this uncertainty affects the cost of the plant irrespective of I, i.e. whether the firm is investing or not. Higher values of parameters β and γ, result in greater uncertainties in realized construction costs of the power plant. The time for completion of the power plant is a stochastic variable $\tilde{T}$ and is the time when K_t falls to zero. W_β and W_γ are uncorrelated Wiener processes, with W_β being also uncorrelated to the economy and the stock market, while W_γ may be correlated with the market.

We assume that the firm invests in the project at a constant rate (i.e. I is constant), also observed in practice as shown in Table 1, where the fraction of the overnight costs[4] for the construction of a power plant in different countries incurred each year is almost equal.

3.2 Modelling uncertain O&M , fuel and electricity prices

During a nuclear power operation period, the *generating costs* consist of operational and maintenance cost, back-end and front-end fuel cycle costs. Following

[4]Overnight cost is the cost of a construction project if no interest was incurred during construction, as if the project was completed "overnight."

Year	CAN	USA	FIN	NLD	CHE	JPN	ROU
-8							16.5
-7							12.5
-6		10			3	5	12.5
-5	8	20	10	20	19	15	12.5
-4	22	20	22	20	19.5	20	12.5
-3	29	20	28	20	19.5	20	16.5
-2	21	20	20	20	19.5	18.5	12.5
-1	12.5	10	20	20	19.5	21.5	4.5
1	7.5						

Table 1: Expense schedule for nuclear power plant construction from country to country expressed as percentage of total overnight construction cost per year. Source: OECD (2005), CAN: Canada, FIN: Finland, NLD: The Netherlands, CHE: Switzerland, ROU: Roumania. Year stands for number of years before the plant becomes operational.

Rothwell (2006) and Zhu (2012) we model the uncertain generation costs by Geometric Brownian Motion (GBM). The dynamics of the generation costs are described by the following SDE:

$$dC_t = \mu_c^* C_t dt + \sigma_c C_t dW_C, \tag{8}$$

where C_t is the instantaneous cost of generation in € per kWh, μ_c^* is a risk adjusted drift[5] of the generation costs and σ_c is the volatility of the generation costs. W_C is a Wiener process which may be correlated to the market.

Modelling *electricity spot prices* is difficult primarily due to factors like:

- Lack of effective storage, which implies that electricity needs to be continuously generated and consumed.
- The consumption of electricity is often localized due to constraints of the grid connectivity.
- The prices show other features like daily, weekly and seasonal effects, that vary from place to place.

Models for electricity spot prices have been proposed by Pilipovic (1997), Lucia and Schwartz (2002) and Barlow (2002), where the latter develops a stochastic model for electricity prices starting from a basic supply/demand model for electricity. These models are focused on *short term* fluctuations of electricity prices which helps better pricing of electricity derivatives.

As decisions for setting up power plants look at *long term evolution* of electricity prices, we, like Gollier *et al.* (2005), use the GBM as the electricity price process. However, it should be noted that within our modelling approach we can easily include other price processes. The dynamics of electricity prices in our model are now described by

[5] if μ_c is the true drift of generation cost then the risk adjusted drift is $\mu_c^* = \mu_c - \eta$, assuming that the Intertemporal Capital Asset Pricing model of Merton (1973) holds, the risk premium η is equal to the β^* of the successful project times the risk premium of market portfolio: $\eta = \beta^*(r_m - r_f)$.

$$dP_t = \mu_p^* P_t dt + \sigma_p P_t dW_P, \quad (9)$$

where P_t is the instantaneous cost of electricity in € per kWh, μ_p^* is the risk adjusted drift of electricity price process and σ_p gives the volatility of electricity prices.

3.3 Value of the power plant after it becomes operational

When the construction of a power plant is finished, i.e. $K_t = 0$, the value of the project depends only on the net cashflow to be generated from the operation of the power plant. Let $h_t(P_t, C_t)$ be the value of the power plant, once it becomes operational, at time t when the instantaneous cost of electricity is P_t € per kWh and the combined O&M and fuel cycle costs are C_t € per kWh. Let $\tilde{t}_S$ denote the time when the plant starts its operation, i.e. $\tilde{t}_S$ is the first instance when $K_t = 0$. Then, the time when it will be decommissioned, $\tilde{t}_f$, is equal to,

$$\tilde{t}_f = L + \tilde{t}_S,$$

where L is the designed lifetime of operation for the power plant and $\tilde{t}_S \leq t \leq \tilde{t}_f$. The expected discounted stream of future differences in cash flows at time t, under the risk neutral measure $\mathbb{P}$, from the remaining operation of the power plant, assuming the plant is decommissioned only after completing its designed lifetime, is then a function of its current state, P_t, C_t, and is equal to:

$$\begin{aligned} h_t(P_t, C_t) &= \mathbb{E}\left[\int_t^{\max(\tilde{t}_f, t)} e^{-r_f \tau}\left(P_\tau - C_\tau\right) d\tau | P_t, C_t\right] \\ &= e^{-(r_f - \mu_p^*)t} P_t \frac{1 - e^{-(r_f - \mu_p^*)(t_f - t)^+}}{r_f - \mu_p^*} \\ &\quad - e^{-(r_f - \mu_c^*)t} C_t \frac{1 - e^{-(r_f - \mu_c^*)(t_f - t)^+}}{r_f - \mu_c^*}, \end{aligned} \quad (10)$$

where r_f is the risk free discount rate and $(t_f - t)^+$ is used to denote $\max(t_f - t, 0)$.

3.4 Real option value of the power plant

The option value of the power plant before it becomes operational depends on the electricity price, P_t, combined fuel cycle and O&M costs, C_t, that would be incurred if the plant becomes operational and on the expected cost of completion, K_t of the power plant. The option value, $V_t(P_t, C_t, K_t)$, of the plant can be computed using Ito's lemma to obtain the differential equation for dV :

$$\begin{aligned} dV &= \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial P} dP + \frac{\partial V}{\partial C} dC + \frac{\partial V}{\partial K} dK \\ &\quad + \frac{1}{2}\frac{\partial^2 V}{\partial^2 P} dP^2 + \frac{1}{2}\frac{\partial^2 V}{\partial^2 C} dC^2 + \frac{1}{2}\frac{\partial^2 V}{\partial^2 K} dK^2 \\ &\quad \frac{1}{2}\frac{\partial^2 V}{\partial P \partial C} dP dC + \frac{1}{2}\frac{\partial^2 V}{\partial P \partial K} dP dK + \frac{1}{2}\frac{\partial^2 V}{\partial K \partial C} dK dC, \end{aligned}$$

and substituting equations (7), (8), (9) into the corresponding Bellman equation for optimality (see Pindyck (1993)) with the final condition :

$$V_{\tilde{t}_S}(P_{\tilde{t}_S}, C_{\tilde{t}_S}, K_{\tilde{t}_S}) = \max(h_{\tilde{t}_S}(P_{\tilde{t}_S}, C_{\tilde{t}_S}), 0). \qquad (11)$$

Here $h_{\tilde{t}_S}(P_{\tilde{t}_S}, C_{\tilde{t}_S})$ is given by equation (10).

Solving the partial differential equation so obtained can be cumbersome due to the free boundary condition, as the date at which the power plant starts its operation, $\tilde{t}_S$, is a random variable. The problem we consider has a dimensionality of three, but in practice it can be even higher, which makes the use of finite difference based methods for solving the above PDE cumbersome. We, like Schwartz (2004), use a simulation-based approach to solve the optimal investment decision problem.

3.5 Computing the real option value using simulation

We assume a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and finite time horizon $[0, T]$, with Ω the set of all possible realizations of a stochastic economy between 0 and T. The information structure in this economy is represented by an augmented filtration $\mathcal{F}_t : t \in [0, T]$, and $\mathbb{P}$ is the probability measure on elements of $\mathcal{F}$. We assume that the state of economy is represented by an $\mathcal{F}_t$-adapted Markovian process (P_t, C_t, K_t), i.e. the electricity price rate, the generation cost rate and the expected cost of completion of the power plant, respectively, at time t. The state space is generated at discrete time steps and for simplicity the time horizon is divided into M equal parts, with $t \in [t_0 = 0, \ldots, t_m, \ldots, t_M = T]$. The length of each time step is equal to

$$\Delta t = \frac{T}{M}.$$

The simulation begins by generating N stochastic paths for the remaining expected construction cost K_t, generation cost C_t and electricity price rate P_t. The vector $P_{t_m}(n), C_{t_m}(n), K_{t_m}(n)$, where $n \in [1, \ldots, N]$ and $m \in [0, \ldots, M]$, defines a unique state at time step t_m. We simulate the random cost of completion paths using the following discrete approximation to equation (7).

$$K_{t_{m+1}}(n) = K_{t_m}(n) - I\Delta t + \beta(IK_{t_m}(n))^{\frac{1}{2}}(\Delta t)^{\frac{1}{2}}X_\beta + \gamma K_{t_m(n)}(\Delta t)^{\frac{1}{2}}X_\gamma, \qquad (12)$$

where X_β, X_γ are uncorrelated standard normal variates. Time point $\tilde{t}_S(n)$ is the first time step at which $K_t(n)$ reaches a value less than or equal to zero and $K_t(n)$ is set to zero for all $t \geq \tilde{t}_S(n)$. Figure 1 shows a few of the scenario paths obtained using equation (12), and Figure 2 gives an example of the distribution of the total construction time. The generation cost rate C_t and the electricity price rate P_t paths are simulated as:

$$C_{t_{m+1}}(n) = C_{t_m}(n)e^{\left(\mu_c^* - \frac{1}{2}\sigma_c^2\right)\Delta t + \sigma_c\sqrt{(}\Delta t)X_C}, \qquad (13)$$

$$P_{t_{m+1}}(n) = P_{t_m}(n)e^{\left(\mu_p^* - \frac{1}{2}\sigma_p^2\right)\Delta t + \sigma_p\sqrt{(}\Delta t)X_P}, \qquad (14)$$

where X_γ, X_C and X_P are standard normal variates that can be correlated.

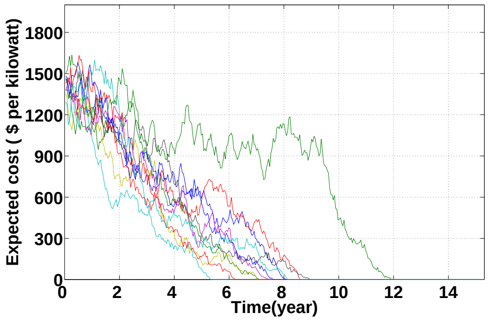


Figure 1: Sample paths for expected cost of completion at different time steps

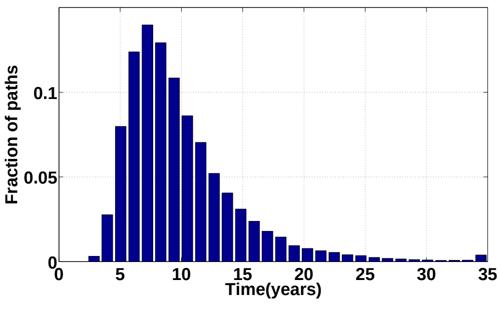


Figure 2: Distribution of construction time when construction costs are uncertain.

Time horizon T is taken sufficiently long, so that the construction of the plant is almost surely finalized before T, i.e. $\tilde{t}_S < T$ with very high probability.

The real option value problem, like its financial counterpart the Bermudan option, is solved backwards in time, starting from the final time step, $t_M = T$. For those paths where the construction of the plant is finalized the option value at any time step is given by equation (10). Particularly, the option value at the time point at which the plant becomes operational is given by:

$$\begin{aligned} V_{\tilde{t}_S}(P_{\tilde{t}_S}(n), C_{\tilde{t}_S}(n), 0) &= e^{-(r_f-\mu_p^*)\tilde{t}_S} P_{\tilde{t}_S}(n) \frac{1-e^{-(r_f-\mu_p^*)L}}{r_f-\mu_p^*} && (15) \\ & \quad - e^{-(r_f-\mu_c^*)\tilde{t}_S} C_{\tilde{t}_S}(n) \frac{1-e^{-(r_f-\mu_c^*)L}}{r_f-\mu_c^*}, && (16) \end{aligned}$$

where $n \in [1, \ldots, N]$ and L is the designed lifetime of the plant.

For those paths where investment is still ongoing the optimal decision to continue the investment is based on the *continuation value* $Q_{t_m}(P_{t_m}, C_{t_m}, K_{t_m})$, which is given by:

$$Q_{t_m} := e^{-r_f \Delta t} \mathbb{E}\left[V_{t_{m+1}} | P_{t_m}, C_{t_m}, K_{t_m}\right], \tag{17}$$

where the simplified notations Q_{t_m} and $V_{t_{m+1}}$ are used for $Q_{t_m}(P_{t_m}, C_{t_m}, K_{t_m})$, and $V_{t_{m+1}}(P_{t_{m+1}}, C_{t_{m+1}}, K_{t_{m+1}})$, respectively. It is optimal for the firm to continue with the investment, when the construction is not yet finalized, i.e. if $Q_{t_m}(n) \geq I\Delta t$, and abandon it otherwise. More intuitively, irrespective of how much the firm has already spent on the construction of the power plant, the optimal decision at a given state point is just based on whether the net future expected revenues are greater than zero. The option value at a state described by path n, at time step t_m, is then:

$$V_{t_m}(n) = \max(Q_{t_m}(n) - I\Delta t, 0). \tag{18}$$

Once the option value has been computed for all paths at t_m, the above process (17,18) is followed recursively moving backwards in time until we reach the starting time t_0. The main challenge here is to efficiently compute the continuation value given by equation (17), for which we use the *Stochastic Grid Bundling Method* (SGBM), details of which are discussed in Jain and Oosterlee (2012) .

The policy for continuing or abandoning the construction of the plant obtained above is used to compute the real option value, i.e. the expected discounted cashflow, and the distribution of the net cashflow obtained following the optimal policy. The mean and the distribution of the optimal cashflow are required as inputs for the portfolio optimization step described in section 2. To compute them we generate another set of N_l paths[6] and apply the policy computed above to continue or abandon the construction of the plant. If the n-th path enters the critical zone, i.e. reaches a state $(P_t(n), C_t(n), K_t(n))$ where it is optimal to abandon, the plant is abandoned for that path and revenues for

[6] Fresh paths are generated as using the same set of paths that were used to obtain the optimal policy may result in an option value which is biased high, due to perfect foresight (or over-fitting).

the path are set to zero, i.e. Revenue(n) = 0. The costs incurred until the plant was abandoned are discounted to time t_0 to :

$$\text{Cost}(n) = \sum_{t=t_0}^{\tilde{t}_a(n)} e^{-r_f t} I \Delta t,$$

where $\tilde{t}_a$ is the first time the path enters the abandonment region. For those paths whose construction is successfully completed (i.e. the paths which never enter the abandonment region), the revenues as seen at time t_0 are:

$$\text{Revenue}(n) = e^{-r_f \tilde{t}_S} V_{\tilde{t}_S}(P_{\tilde{t}_S}(n), C_{\tilde{t}_S}(n), 0), \tag{19}$$

and the costs of construction of the plant, discounted to time t_0, are:

$$\text{Cost}(n) = \sum_{t=t_0}^{\tilde{t}_S} e^{-r_f t} I \Delta t, \tag{20}$$

where $\tilde{t}_S(n)$ is the time when the plant starts its operation along the n-th scenario path. The real option price or the net expected cash-flow following the optimal policy of the power plant is then given by

$$V_{t_0}(P_{t_0}, C_{t_0}, K_{t_0}) = \frac{1}{N_l} \sum_{n=1}^{N_l} (\text{Revenue}(n) - \text{Cost}(n)). \tag{21}$$

The option price so obtained is a *lower bound*[7] of the true price as the policy used is generally sub-optimal due to numerical errors involved.

4 Validation: A Case from Pindyck

Pindyck (1993) examined the decision to start or continue building of a nuclear power plant. To apply the model the estimates of the expectation and variance of the cost of building a kilowatt of nuclear generating capacity are used. The variance is decomposed into two parts to obtain estimates for technical uncertainty and input cost uncertainty. The survey of individual nuclear power plant costs published by the Tennessee Valley Authority (1977 to 1985) was used, which provided data on expected cost of a kilowatt of generating capacity on a plant-by-plant basis. A cross-section regression analysis over time was employed to estimate the expected costs and variance of a power plant. The variance of the costs and their decomposition were estimated from time-series and cross-sectional variations of the data, using the fact that the variance of cost due to technical uncertainty is independent of time, whereas the variance due to input cost fluctuations grows with time. Based on these estimates the technical uncertainty parameter β in (Pindyck (1993)) is found to vary from 0.24 to 0.59, while γ in (Pindyck (1993)) varies between 0.07 to 0.2. In this analysis an instant revenue as soon as the construction is finalized was considered.

As a first validation experiment, like Pindyck (1993) we use the parameter set given in Table 2.[8] Table 3 compares the values reported by Pindyck with

[7] Lower bound implies that if the same Monte Carlo simulation is performed several times, with different initial seeds, the mean of V_{t_0} so obtained would be lower than V_{t_0}.

[8] Note that prices are in USD here in accordance to the reference values from the literature.

Initial expected cost K_0	\$ 1435 per kilowatt
Investment rate I	\$ 144 per annum
Discount factor r	0.045
Life Time of reactor	40 years
Revenue	\$ 2000 per kilowatt or 1.23 cents per kWh

Table 2: Parameter set used for validation case

	Pindyck		SGBM		LSM	
β	V_{t_0}	K_0^*	V_{t_0}	K_0^*	V_{t_0}	K_0^*
0	121	1550	120.64	1550.5	120.64	1550.5
0.24	131	1609	128.89	1582	128.75	1612
0.59	215	1881	211.46	1798	210.36	1887

Table 3: The real option value and critical expected construction cost for different levels of technical uncertainties. K_0^*, is the critical expected construction cost at time t_0, above which the project should not be undertaken.

those obtained using the simulation method SGBM as well as the *least squares method* (LSM) (see Longstaff and Schwartz (2002), Schwartz (2005) for details on LSM), for different levels of technical uncertainty. It can be seen that without uncertainties in the construction costs the closed-form solution and results from simulations are almost identical, where a minor difference is due to the discretization of equation (7). When technical uncertainty, β, is non-zero the real option values from simulation are slightly lower than the closed form values from (Pindyck, 1993), as simulation results are biased low. The option values obtained using SGBM are slightly higher than those obtained using the *least squares method* for the same set of paths, which implies that in the discrete time version the critical costs for abandonment, K_0^*, obtained using SGBM are more accurate.

We would like to emphasize the role of real options in computing the net expected cashflow and its distribution when a firm is flexible to take decisions during the course of construction and operation of the reactor. If the underlying stochastic factors like expected cost of completion turn unfavourable in the future the firm uses its discretion to abandon the project[9] in such a way that the net expected cashflow is maximized. Figure 3 compares the cashflow distribution when (a) the firm doesn't have the flexibility to change its decision in the future and continues with the construction of the reactor irrespective of whether the scenario is favourable or not, (b) the firm has the flexibility to change its decision and continues or abandons the project following the policy computed using SGBM. It can be seen that the option to abandon the project under unfavourable price scenarios reduces the possibility of extreme losses. Table 4 compares the expectation and standard deviation of the net cashflow for the above two cases.

[9] It is assumed that the firm behaves rationally throughout the life cycle of construction and operation of a nuclear power plant, although there is some empirical evidence which suggests that management might act otherwise when sunk costs are involved, for example see "Throwing good money after bad ? : Nuclear power plant investment decisions and the relevance of sunk costs"by Bondt and Makhija (1988) .

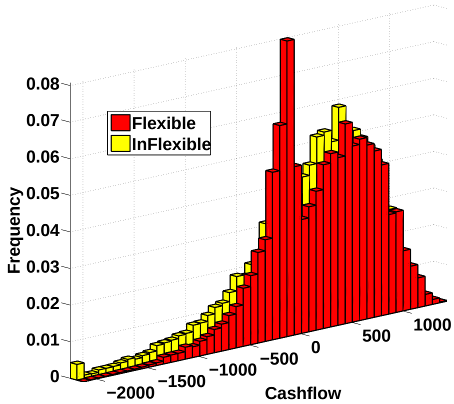


Figure 3: Distribution of net cashflow when the firm has flexible and inflexible decision to abandon the project in future. The policy, when early abandonment is possible, is computed using SGBM. Same set of scenario paths are used for the two cases.

	Inflexible Case	Flexible case (SGBM)
Expected net cashflow (\$/kWe)	186	221
Standard deviation	600	500

Table 4: The expected value and standard deviation of the net cashflow corresponding to the distribution in Figure 3. The reactor parameters are taken from Table 2 and $[\beta, \gamma]$ values are [0.59 0.07], respectively.

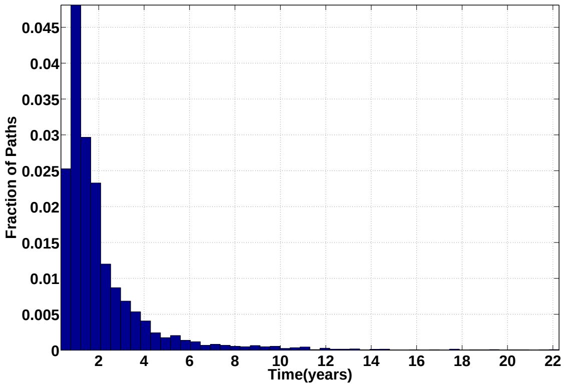


Figure 4: Fraction of paths abandoned at different time steps, when the policy from SGBM is followed, corresponding to the case considered in Table 4.

Figure 4 shows the fraction of scenario paths for which the project is abandoned at different time steps when the policy from SGBM is followed for the above case. It's more likely for a project to be abandoned in its early phases than in later stages. As the project commences the remaining expected construction costs (due to the ongoing investment) and also the remaining expected time to finish the construction reduce while the anticipated revenues increase (as the revenues are expected to start flowing in relatively sooner, which implies they are discounted less), which reduces the chance of the project being abandoned.

5 Numerical Examples

In this section we illustrate by various examples the two steps involved in deciding the optimal mix of NPPs for a power utility (or country or otherwise). We consider a more realistic case, where not only the costs are uncertain but also the market price of electricity. We analyze the real option value, optimal decision rules to start, continue or abandon the construction of a reactor, distribution of costs and cashflows obtained following the optimal policy for different reactors. Finally, based on the expected net cashflow and its distribution, we find the optimal reactor order fraction for the different reactors considered.

5.1 Choice of nuclear power plants

In this section we discuss the economics of different nuclear reactors we consider for determining an optimal portfolio in energy generation planning. Here, not only the expected costs of completion of the reactors are uncertain, but also the source of revenues, i.e. the electricity prices. The optimal decisions do not only depend upon the expected costs of the reactor but also on the present market price of electricity. Under our model assumption, the construction of an unfinished reactor continues as long as the expected cost of completion is below some critical value and the electricity prices are above the corresponding threshold electricity price. For a given expected construction cost if the present electricity price (per annum) falls below a threshold the expected net cashflow

	Power (Thermal) MW	Power (Electric) MW	Efficiency (%)	Capacity Factor (%)	Life Time (years)
Gen III	4500	1600	35.5	90	60
FR	3600	1500	42	85	60
HTR	500	(200 +100)	(40 + 20)	90	60
SCWR	2300	1000	43.5	90	60

Table 5: The specification of the reactors considered.

would be negative and hence the construction of the plant is discontinued in our model. Similarly for a given electricity price if the expected cost of completion increases above a threshold price the construction of the plant will be abandoned in our model.

For our analysis we consider the following types of reactors for the portfolio.

- *Generic Gen III type Light Water Reactor (LWR):* The light water reactor (LWR) is a type of thermal reactor that uses water as its coolant and a neutron moderator and solid compound of fissile elements as its fuel. Thermal reactors are the most common type of nuclear reactor, and light water reactors are the most common type of thermal reactor.

- *Fast Reactors (FR):* Fast reactors or fast neutron reactors are a category of nuclear reactors in which the fission chain reaction is sustained by fast neutrons. They are considered an attractive option because of their potential to reduce actinide wastes, particularly plutonium and minor actinides which eliminate much of the long-term radioactivity from the spent fuel. Fast reactors with closed fuel cycle allow a significantly improved usage of natural uranium. The Sodium Cooled Fast Reactor (SFR), Lead Cooled Fast Reactor (LFR) and Gas Cooled Fast Reactor (GFR) are examples of fast reactors featured in the Generation IV roadmap (2002) (Gen IV, 2002).

- *High Temperature Reactor (HTR):* Also featured in the Generation IV roadmap, HTRs are graphite-moderated nuclear reactors with a once-through uranium fuel cycle. The high temperatures enable applications such as an emission-free process heat or hydrogen production, which effectively increase the efficiency of the reactor by as much as 20% (Generation IV roadmap (2002)).

- *Super Critical Water Reactor (SCWR):* Featured in the Generation IV roadmap, SCWRs resemble light water reactors (LWRs) but operate at higher pressure and temperature, with a direct once-through cycle like a boiling water reactor (BWR), with the water always in a single fluid state like the pressurized water reactor (PWR). The SCWR is an advanced nuclear system because of its high thermal efficiency of 45% vs. 33% for current LWRs, and simple design (Generation IV roadmap (2002)).

The size, efficiency and capacity factors of the reactors considered, taken from (Roelofs et al., 2011), are given in Table 5.

Notice that HTRs have an efficiency of 40% + 20%, as not only would the reactor produce 200 MW of electricity, but also 100 MW of process heat. We incorporate this in our model by assuming that the cost of electricity is 2.32

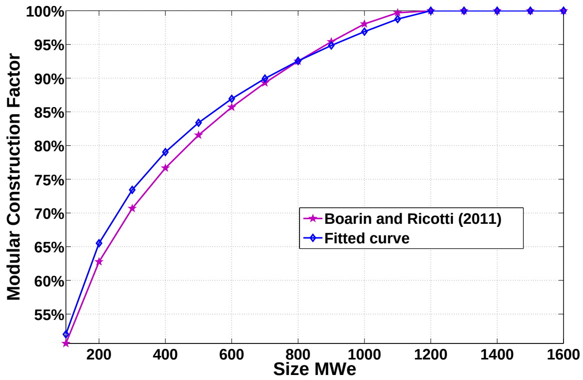


Figure 5: Modular construction factor as a function of size of the reactor. The reference power plant size is 1100MWe.

times the process heat costs, as in Gandrik (2012), which results in a revenue for this reactor equal to $1.21 \times P_t$.

We take the reference values for the expected construction costs, fuel cycle costs, operation and maintenance costs and also the confidence interval or standard deviation of these costs from van Heek *et al.* (2012), Roelofs *et al.* (2011). These are engineering cost estimates as there isn't sufficient experience to estimate these values from historical data. Table 6 reports the expected construction costs and fuel, operation and maintenance costs as derived from the values in van Heek *et al.* (2012) and Roelofs *et al.* (2011). In the case of the HTR we additionally include the benefits of modular construction (increased standardisation and faster learning curves), different from van Heek *et al.* (2012). We follow the analysis of Boarin & Ricotti (2011), where four effects of modular construction are distinguished:

1. *Learning factor:* The number of similar plants constructed world-wide will lead to increased experience in construction and therefore in decreased costs;

2. *Modularity factor:* The modularization factor assumes a capital cost reduction for modular plants, based on the reasonable assumption that the smaller the plant size, the higher the degree of design modularization;

3. *Multiple units factor:* The multiple units saving factor shows a progressive cost reduction due to fixed cost sharing among multiple units at the same site;

4. *Design factor:* The design factor takes into account a cost reduction by assumed possible design simplifications for smaller-sized reactors.

Figure 5 shows the curve constructed when all these separate effects are combined. A fitted curve that gives the modular construction factor is then given by,

$$mcf = \min\left(0.195 \ln\left(\frac{\text{Power}_{mod}}{100}\right) + 0.63 - 10^{-4} \times \text{Power}_{ref}, 100\%\right), \quad (22)$$

Reactor	Expected construction cost €/kWe	Expected Fuel and O&M cost €/kWe/a
Generic Gen III LWR	2900 (320)	140 (35)
FR	4600 (580)	185 (35)
HTR	3600 (750)	165 (35)
SCWR	3400 (400)	140 (33)

Table 6: Expected construction, fuel and O&M costs for different reactors considered. The values in brackets are standard deviations of these costs.

where Power$_{ref}$ is 1100 MWe and Power$_{mod}$ is on the x-axis of figure 5. Following equation (22) based on the assessment of Boarin and Ricotti (2011), the modularity construction factor would be 65.5% for a Power$_{mod}$ = 200 MWe HTR, which brings down the expected costs of construction of HTRs from 6100 to 3600 €/kWe.

As the values reported in Table 6 are "engineering estimates" the uncertainty in these values can primarily be attributed to technical uncertainty. When only technical uncertainties are involved the variance of the expected cost of construction is given by (see Pindyck (1993)) :

$$\mathrm{Var}(K) = \left(\frac{\beta^2}{2-\beta^2}\right) K^2;$$

a relation we use to compute the corresponding value of β for different reactors in the portfolio. The Brownian motions driving the input cost uncertainties (see section 3.1) for different reactors can be correlated to each other (and the economy), as raw material required and government regulations are similar for different reactors, while technical uncertainties for different reactors are assumed to be uncorrelated. Table 7 summarizes the parameter choices related to Table 6.

The real option value of the reactors and the distribution of the net cashflow under optimal policy for construction and operation of the reactors, depends on, amongst others, the expected growth rate for electricity prices (μ_p^*), uncertainty in electricity prices (σ_p), and the discount rate used (r)[10]. Table 8 gives values considered for these parameters. For the base case, values corresponding to the row 'Medium' in Table 8 are taken, and the initial price of electricity P_{t_0} is set to 8.5 cents/kWh.

[10] As the Brownian motions dW_γ, dW_C, dW_P may be correlated with the market, we cannot use the risk-free interest rate for discounting, especially if spanning is not possible. We instead consider different discount rates which represent different levels of risk premiums added to the risk-free rate.

Generic GenIII	K_0 = 2900 (€/kWe), γ=0.07, β = 0.15, expected construction time = 5 years, C_0 = 1.36 (cents/kWh), σ_C = 0.25;
Generic Fast Reactor	K_0 = 4600 (€/kWe), γ =0.07, β = 0.18, expected construction time = 7 years, C_0 = 1.95 (cents/kWh), σ_C = 0.19;
HTR	K_0 = 3600 (€/kWe), γ=0.07, β = 0.17, expected construction time = 4 years, C_0 = 1.70 (cents/kWh), σ_C = 0.22;
SCWR	K_0 = 3400 (€/kWe), γ = 0.07, β = 0.16, expected construction time = 5 years, C_0 = 1.43 (cents/kWh), σ_C = 0.24;

Table 7: Initial expected cost of completion, input cost uncertainty parameter γ, technical uncertainty parameter β, expected construction time, present value of combined O&M and fuel charges C_0 and the corresponding volatility for different reactors. For all cases considered we assume the correlation coefficient ρ between W_P and W_C to be 0.5 and the growth rate in O&M costs, μ_C^* to be 0. The rate of investment I for each reactor is taken as their initial expected construction costs divided by their expected construction times.

	Growth rate μ_p^* (% per annum)	Uncertainty σ_p (% per annum)	Discount rate r (% per annum)
Low	0	10	6
Medium	3	20	8
High	5	30	10

Table 8: Values of electricity price growth rate μ_p^*, uncertainty in electricity prices, σ_p and discount rate r considered in various examples.

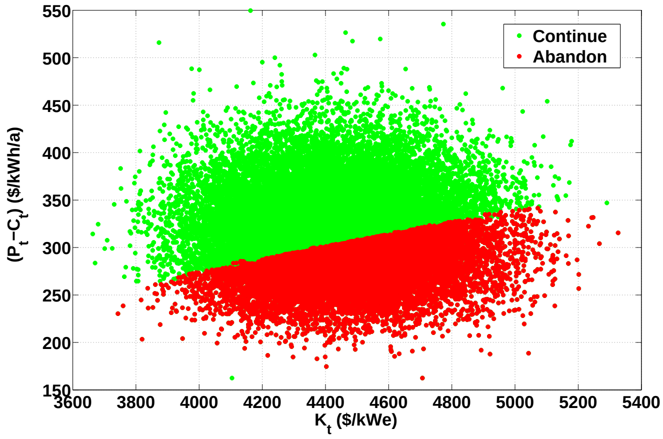


Figure 6: Simulated K_t, P_t, C_t paths at a particular time step. Green represents the states where it is optimal to continue and red where it is optimal to abandon the project.

5.2 Real option value analysis

We use real option analysis to determine the optimal policy to start, continue or abandon the construction of a project, so that the net expected discounted cashflow is maximized. As stochastic construction costs K_t, combined O&M and fuel cycle costs C_t, and cost of electricity P_t, are considered, the optimal decision will depend on these three state variables. It will be optimal to abandon the project, if :

- the expected cost of completion is too high,
- the O&M and fuel cycle costs are too high,
- the electricity prices are too low.

Figure 6 shows the early abandonment region at an intermediate time step of the simulation. Here the $x-$ axis represents the expected costs of completion of the reactor and the $y-$ axis represents the cost of electricity minus fuel and O&M costs. The red coloured grid points represent the states at which the construction of the reactor should be abandoned, while green colour represents the ones for which the construction should continue.

Table 9 reports the critical price of electricity above which each of these reactors should be ordered and their real option values when the initial price of electricity equals $P_0 = 8.5$ cents/kWh. Reactor specific parameters are taken from Table 7. The same set of simulated electricity paths should be used for different reactors.

Under our model assumptions and parameter choices, we see that the HTRs, despite their high expected capital costs, appear economically most attractive, primarily due to their higher efficiencies. The Gen III LWRs have the lowest critical electricity price above which they can be ordered, while the fast reactors seem economically least viable in our model settings.

Type	Critical electricity price P_0^*	Option value $P_0 = 8.5$ cents/kWh
Gen III LWR	4.0	3100
FR	6.25	875
HTR	4.5	3500
SCWR	4.7	2650

Table 9: Critical price of electricity $P_{t_0}^*$ in (euro cents/kWh) above which the reactors should be ordered and their option values (in €/kWe) when the initial price of electricity is 8.5 euro-cents/kWh. The reactor parameters are taken from Tables 7 and 8.

5.3 Optimal portfolio analysis

If a firm has to choose amongst the above reactors, solely based on their capital costs (Table 7), then their portfolio would contain only Generic Gen III type LWRs, something also observed in practice. However, such a portfolio excludes the role of uncertainties of cashflows for these reactors. Application of MVP theory takes into account not only the expected returns but also the uncertainties or risks associated with these returns.

An efficient frontier gives the optimal reactor order fraction for a portfolio designed to meet a given expected return while minimizing the uncertainties of these returns. In order to determine the efficient frontier the expected returns and the covariance matrix of the returns from the reactors considered are required. The distribution of returns for each reactor optimally constructed is sampled by computing the returns along each simulated path.

The following constraints on the portfolio are considered:

- *Budget constraint:* Under a budget constraint, the optimal reactor order fraction for every euro spent is computed. Returns corresponding to a euro spent on a reactor are given by,

$$R_i(n) = \frac{\text{Revenue}_i(n) - \text{Cost}_i(n)}{\text{Cost}_i(n)}, \tag{23}$$

  and the constraint for the portfolio optimization problem is then:

$$\sum_{i=1}^{J} w_i = 1,$$

  $n = 1, \dots, N$ being the path index and $i = 1, \dots, J$ indicate the different reactors considered. The weights correspond to the fraction of money invested in different reactors, which is then used to compute the reactor order fraction (per kWe) by taking into account the expected construction costs as reported in Table 7.

- *Capacity constraint:* Under a capacity constraint, the optimal reactor order fraction for every kWe of capacity ordered is computed. Returns corresponding to a kWe ordered are given by,

$$R_i(n) = \text{Revenue}_i(n) - \text{Cost}_i(n),$$

	Gen III	FR	HTR	SCWR
Expected Return	1.3376	0.1863	1.0645	0.9744
Stdev Return	1.2356	0.9046	0.9926	1.0527

Table 10: The expected return and its standard deviation per euro spent for the base case.

and the constraint for the portfolio optimization problem is:

$$\sum_{i=1}^{J} w_i = 1,$$

$n = 1, \ldots, N$ being the path index, and $i = 1, \ldots, J$ indicate the different reactors considered. The constraint implies here that reactor order fractions should add up to a kWe.

For both constraints, the weights are additionally bounded as,

$$0 \leq w_i \leq 1,\ i = 1, \ldots, J,$$

which comes naturally from the fact that short selling is not possible here, and thus the weights cannot be negative.

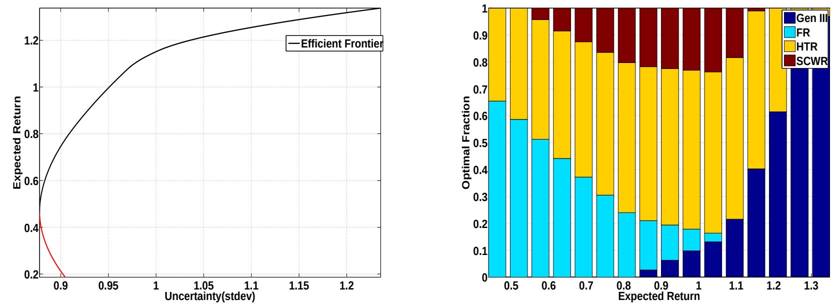
The quadratic programming problem expressed by equation (5) is solved using the optimization toolbox of MATLAB, which solves general problems of the kind:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \frac{1}{2}\mathbf{w}^\top Q w + f'\mathbf{w}, \\ \text{such that:} \quad & A\mathbf{w} \leq a, \\ & B\mathbf{w} = b, \\ & \mathbf{L} \leq \mathbf{w} \leq \mathbf{U}, \end{aligned} \tag{24}$$

using the command w = quadprog(Q,f,A,a,B,b,L,U).

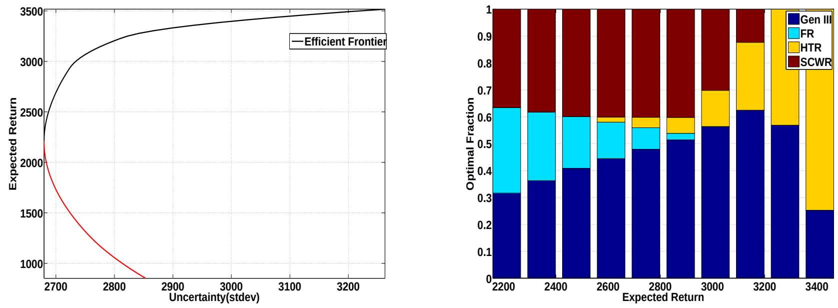
Figure 7 displays the efficient frontier and the corresponding optimal reactor order fraction when the portfolio has the *budget constraint*. The mean and standard deviation of the simulated returns for the individual reactors are reported in Table 10. Under our model assumptions and choice of parameter values, the GenIII LWRs have the highest expected returns (based on equation (23), while the FRs have the lowest returns per euro spent. However, the uncertainty of returns for FRs is lower than that for GenIII LWRs. An investor who wants to minimize the uncertainty of returns and is willing to take a lower expected return in order to do so, will choose a portfolio with more Gen IV type reactors. An investor who wants higher returns and is indifferent to the uncertainty of returns, will hold a portfolio with more Gen III type reactors.

Figure 8 shows the efficient frontier and optimal reactor order fraction corresponding to points on the optimal frontier, when the portfolios have the *capacity constraint*. Expected returns and their standard deviations per kWe of reactor ordered are reported in Table 11. We see that unlike the case with the budget constraint, where portfolios with high returns were dominated by Gen III LWRs, here portfolios with higher expected returns are dominated by *both* HTR and GenIII LWRs. This difference can be explained as the returns in equation (23) are scaled by the individual reactor costs.



(a) (b)

Figure 7: (a) Efficient frontier for the base case when portfolios have the budget constraint and (b) reactor order fraction corresponding to points on the efficient frontier.



(a) (b)

Figure 8: (a) Efficient frontier for the base case when there is a capacity constraint and (b) the reactor order fractions corresponding to points on the efficient frontier.

	Gen III	FR	HTR	SCWR
Expected Return (€/kWe)	3100	880	3500	2625
Stdev Return	2825	2870	3225	2850

Table 11: The expected returns and their standard deviations per kWe of reactor ordered for the base case.

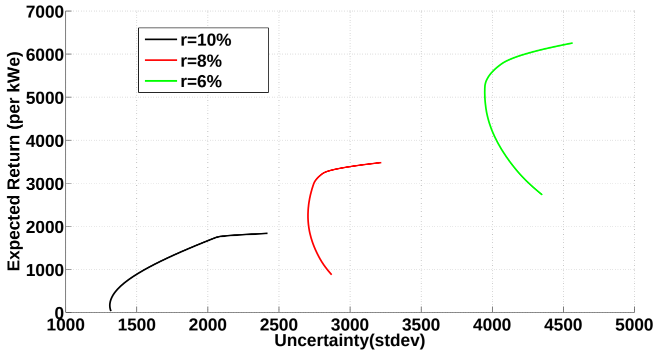


Figure 9: Efficient frontiers for varying discount rates. Parameter values are taken from Tables 7 and 8.

5.4 Portfolio sensitivity

In addition to the constraints on the portfolio, the choice of parameter values affects the structure of the optimal portfolio. We study the optimal portfolio for varying parameter values, which gives an intuition about the portfolio's sensitivity with respect to these parameters. In particular, we consider the following cases:

- Different discount rates r, with other parameters constant.
- Varying electricity price growth rates μ_p^*, with other parameters constant.
- Varying uncertainties in electricity prices σ_p, with other parameters constant.

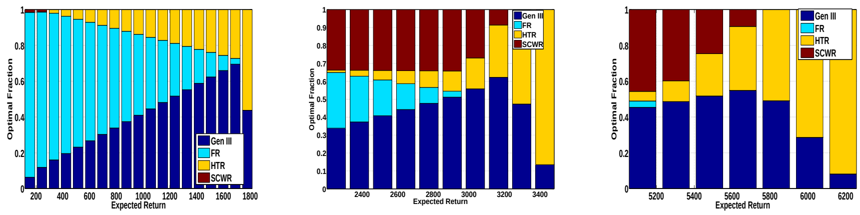
From here on, we only consider portfolios that have capacity constraints.

Varying discount rates

For our reference case, we considered a discount rate of 8% per annum. We examine the portfolios sensitivity to varying discount rates. A change in discount rate affects the expected revenues, costs and the optimal investment strategy, which in turn affects the returns. This makes the discount rate an important parameter while computing the efficient frontier and corresponding optimal reactor order fractions.

Figure 9 shows the efficient frontier for low, medium and high discount rates, with corresponding values taken from Table 8. Lowering the discount rate can help realize higher expected returns, although at increased uncertainty (variance) in returns. Although both the expected returns and the variance of returns increases, however, the increase in the expected returns is more significant than increase in the variance of returns. Therefore, reactors with higher expected returns would then be more favoured in the mean-variance portfolio.

The optimal reactor order fractions corresponding to the points on the efficient frontier are shown in Figure 10. Under our model assumptions and parameter choices, we see that lowering discount rates results in a portfolio dominated by reactors having greater expected returns, while higher discount rates result in a portfolio where reactors with lower uncertainties dominate.



(a) (b) (c)

Figure 10: Optimal reactor order fractions when (a) $r = 10\%$, (b) $r = 8\%$, and (c) $r = 6\%$. Parameter values are taken from Tables 7 and 8.

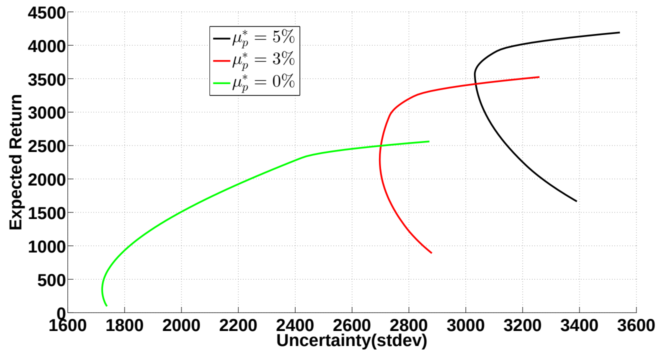


Figure 11: Efficient frontier for varying electricity price growth rate, where the reactor specific parameters are taken from Table 7, and economic parameters from Table 8.

Varying electricity price growth rates

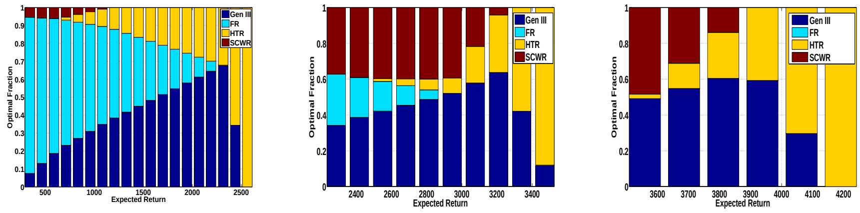
Long term growth rates of electricity prices are difficult to predict. A sensitivity analysis of the optimal portfolio with respect to different electricity price growth rates is then essential. We do an optimal portfolio analysis for low, medium and high growth rate scenarios for electricity prices.

Figure 11 shows the efficient frontiers corresponding to different electricity price growth rates. A higher growth rate in electricity prices results in portfolios which can achieve greater expected returns.

The optimal reactor order fractions corresponding to the points on the efficient frontiers for different electricity price growth rates are shown in Figure 12. Under our model assumptions, we see that a higher expected growth rate in electricity prices leads to portfolios that are dominated by reactors with higher expected returns (HTR and GenIII), while for low growth rate scenarios optimal portfolios can have reactors with lower returns (like FRs).

Varying uncertainty in electricity prices

Uncertainty in electricity prices affects the expected return and its distribution for different reactors.We study the mean-variance portfolio for low, medium and high uncertainty in electricity prices, with the corresponding values for σ_p taken from Table 8. Figure 13 plots the efficient frontiers for the three



(a) (b) (c)

Figure 12: Optimal reactor order fractions when (a) $\mu_p^* = 0\%$, (b) $\mu_p^* = 3\%$. (c) $\mu_p^* = 5\%$. Parameter values are taken from Tables 7and Table 8.

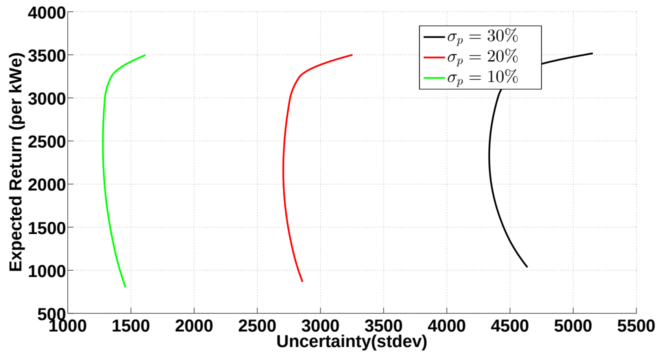


Figure 13: Efficient frontier for varying uncertainty in electricity prices, where parameter values are taken from Tables 7 and 8.

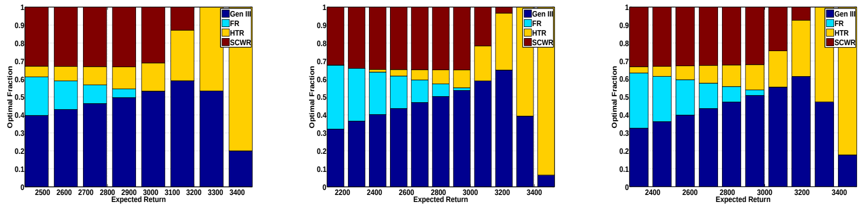
different scenarios considered. With increasing uncertainty in electricity prices, the uncertainty in returns of the optimal portfolio increases for a given level of expected returns.

The optimal reactor order fractions corresponding to the points on the efficient frontiers for the three scenarios considered are presented in Figure 14. Under our model assumptions, the reactor order fractions seem less sensitive to uncertainty in electricity prices, when compared to their sensitivity to discount rates or electricity price growth rates.

6 Conclusion

While the future of nuclear power depends on resolving the issues of safety of operations, safe management of radioactive wastes and measures to prevent proliferation (MIT, 2003), in a deregulated electricity market, the economics of NPPs will be the most important determinant of nuclear energy's role in the future global energy mix. A decision-support tool, which takes into account major factors and their uncertainties for studying the economics of individual reactors as well as for a portfolio of reactors has been presented here.

Specifically, we have used real option analysis and portfolio optimization to study optimal reactor order fractions within the nuclear sector. A two-step approach is proposed, where first optimal decisions are taken at the plant level,



(a) (b) (c)

Figure 14: Optimal reactor order fractions when (a) $\sigma_p = 10\%$, (b) $\sigma_p = 20\%$, and (c) $\sigma_p = 30\%$. Parameter values are taken from Tables 7 and 8.

and then the resulting distribution of returns for each reactor-type are used as inputs to a portfolio optimization problem solved using MVP theory. The main contribution on the methodological side can be stated as:

- The method adequately accounts for uncertain reactor construction costs and schedule, and reflects their effect on the return distribution for different reactors.
- An optimal policy for continuing the construction or abandoning the project is computed taking into account the uncertainties in construction costs, electricity prices and O&M and fuel cycle costs involved.
- A detailed study on optimal portfolios based on MVP theory is conducted.
- The effect of different constraints on portfolio diversification is studied.
- The sensitivity of the optimal portfolio with respect to electricity price growth rates, uncertainty in electricity prices and discount rates is studied.

It should be emphasized here that, although careful attention has been paid to choose realistic parameter values for the reactors considered, however, the main focus of the paper is to illustrate a methodology that accounts for the various economic uncertainties related to nuclear power plants. Under our model assumptions, it has been shown that certain scenarios lead to portfolios that are dominated by Generation IV type reactors, while others result in conventional Gen III type LWRs being the dominant ones. Following the methodology described here can be useful when decisions related to reactor order fraction need to be made.

As possible future direction of research, the portfolio optimization step should in addition to the variance of the returns also consider other risk measures, such as, value at risk and conditional value at risk. The resulting portfolios will not only minimize the variance of the returns but will also avoid reactors which are likely to be abandoned in the future.

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