Summary
We derive an approximate integral representation to model and image near-surface scattering. This formulation forms the basis of a prediction-and-subtraction algorithm to reduce near-surface scattering on seismic data. We investigate how well this approximate equation reconstructs the scattered field obtained by full elastic 3D modeling. The approximate equation characterizes the scattering in the immediate subsurface and allows one to model near-array scattering on seismic records. The equation is accurate when the scattering takes place immediately under the receivers which is consistent with the assumptions made.

Introduction
To compensate for variations in reflectors due to the overburden, a whole range of near-surface corrections has been proposed. Most of these methods are single channel amplitude or time corrections, imposing strong restrictions on the near-surface model. However, when the overburden is strongly heterogeneous, these restrictions have to be relaxed. Especially to include scattering in near-surface compensation schemes, requires multi-channel and wave-field based methods (Baeten et al., 2001). In Campman et al. (2003) such a method is presented and validated on laboratory data. In the present paper, we derive the basic integral equation that forms the basis of the method described in Campman et al. (2003), explicitly stating assumptions. We evaluate the sensitivity of the approximate scattering equation with respect to two major assumptions using data modeled with an accurate 3-D elastic scheme (Riyanti and Herman, 2003).

Approximate integral representation for scattering and inverse scattering in the near-surface
Consider the elastic wave equation for the vertical component \( u_3 \) of the particle displacement in a (laterally invariant), isotropic, elastic half space in the frequency domain, due to a vertical, impulsive point source \( f_3 \):

\[
(\lambda + \mu) \partial_3 \partial_k u_k(x, \omega) + \mu \partial_k \partial_3 u_3(x, \omega) + \rho(x) \omega^2 u_3(x, \omega) = -f_3(x, \omega).
\]  

(1)

Here, \( \omega \) is the angular frequency; \( u_k \) denotes the components of particle displacement (for \( k = 1, 2, 3 \)). Position is denoted by the coordinate \( x = (x, y, z) \) and differentiation with respect to position is denoted by \( \partial_i \). The density is denoted by \( \rho \) while we have taken the Lamé parameters \( \lambda \) and \( \mu \) constant here. Eq. (1) is supplemented with the stress-free boundary condition at the surface \( S \):

\[
\mathbf{n} \tau_{ij}(\mathbf{x}) = 0, \quad \mathbf{x} \in S,
\]

where \( \tau_{ij} \) are the components of the stress tensor and where \( \mathbf{n} \) is the unit vector along the normal pointing away from the medium. Riyanti and Herman (2003) use a domain type integral equation to solve the scattering problem given by Eq. (1), leading to accurate and efficient calculations of near-surface scattering in a layered half space. We use this method here to calculate the test data. Our objective here is to derive an approximate integral representation for the scattered field near a receiver array, that allows one to characterize the immediate subsurface and to model near-array scattering in an efficient way. Because most of the surveys are still conducted only with
vertical geophones—especially when sampling is dense—we only consider a representation for the vertical component of the velocity. For upcoming reflections, we assume that the horizontal components are negligibly small with respect to the vertical component and we obtain:

\[
\frac{c_p^2}{c_s^2} \partial_3 \partial_3 + \partial_1 \partial_1 + \partial_2 \partial_2 \right] u_3(\mathbf{x}, \omega) + \frac{\omega^2}{c_s^2} u_3(\mathbf{x}, \omega) = -\frac{1}{\mu} f_3(\mathbf{x}, \omega). \quad (2)
\]

Here, \( c_p = \sqrt{\frac{\lambda+2\mu}{\rho}} \) is the compressional wave velocity and \( c_s = \sqrt{\frac{\mu}{\rho}} \) is the shear wave velocity.

This type of assumption also arises in soil-mechanics, where it was first used to derive closed form expressions for the vertical component of stresses in a horizontally constrained medium (Harr, 1966). Obviously, this wave equation exhibits anisotropy, but it is readily removed by introducing a scaled vertical coordinate \( \zeta = \frac{c_p}{c_s} z \), yielding a scalar Helmholtz equation for \( u_3 \). The free-surface boundary condition can be written as \( \partial_\zeta u_3 = 0, (\zeta = z = 0) \). We consider perturbations in the velocity about a homogeneous background medium: \( \frac{1}{c_s(\mathbf{x})} = \frac{1}{c_0^s} + \zeta(\mathbf{x}) \). From this, a representation for the scattered field, \( u_3^s \), can be derived using the Green’s function \( u_3^G \). Taking into account the boundary condition of a traction-free half space for \( u_3 \) and \( u_3^G \) respectively and conditions at infinity, yields

\[
u_3^1(\mathbf{x}, \omega) = -\omega^2 \int_{\mathbf{x}' \in D} u_3^G(\mathbf{x} - \mathbf{x}', \omega) \xi(\mathbf{x}') u_3(\mathbf{x}', \omega) d\mathbf{x}'. \quad (3)
\]

Here, \( D \) is the domain occupied by the scatterers. Now, we consider the scattering medium as a stack of thin, homogeneous layers with thickness \( \Delta z \). Then, on account of the mean value theorem, the integral over \( z \) in Eq. (3) can be approximated by a sum over all layers. Blonk and Herman (1994) show, that for scattering of surface waves, it suffices to take only one term of the sum in account. Although, in our case, the scattered surface waves are in general converted from upcoming body waves, we use this result as an ansatz and thus express scattering close to the surface in terms of a surface impedance distribution:

\[
u_3^1(\mathbf{x}, z_0, \omega) = \int_{\mathbf{x}' \in \Sigma} u_3^G(\mathbf{x} - \mathbf{x}', \Delta \zeta, \omega) \sigma(\mathbf{x}', \zeta_1, \omega) u_3(\mathbf{x}', z_1, \omega) d\mathbf{x}', \quad (4)
\]

where we have included \( -i\omega^2 \Delta z \) into the impedance function, i.e. \( \sigma = -i\omega^2 \Delta z \xi \). Equation (2) of Campman et al. (2003) is now obtained by noting that vertical velocity, \( v_3^1 = i \omega u_3^1 \). Finally, because — by assumption — the scattering takes place immediately under the receivers, we substitute the field measured at the receivers for the field at the scattering depth on the right-hand-side of the representation. Note that the domain of integration is now \( \Sigma \), the surface occupied by the receivers and \( \mathbf{x}_j = (x, y) \). We allow the impedance distribution to depend on frequency. Because we lump all scattering at one depth (at the surface), in this way, we convert depth dependence into frequency dependence, much like is the case in a dispersion relation. In contrast to Born-type imaging methods—often used in seismics—, our approximation does include multiple scattering.

**Inverse scattering and modeling with the approximate equation and sensitivity**

We formulate inversion in terms of minimizing a misfit between the data and the data from our forward model Eq. (4) (Campman et al., 2003):

\[
F = \frac{||d^1 - K\sigma||^2}{||d^1||^2} + \lambda ||\sigma||^2, \quad (5)
\]

where \( d^1 \) is the scattered energy from one reference event used to find the impedance function and \( \lambda \) is a parameter determining the weight given to minimization of the norm of the model relative
to minimization of the residual norm. The operator \( K \) is defined through:

\[
\{K \sigma\}(\mathbf{x}_j, z_0, \omega, \mathbf{x}^S) = \int_{\Sigma_j \in \Sigma} u^G_3(\mathbf{x}_j - \mathbf{x}_{j'}, \Delta z, \omega) \sigma(\mathbf{x}_{j'}, z_1, \omega) d(x'_{j'}, z_1, \omega, \mathbf{x}^S)d\mathbf{x}_j'.
\] (6)

Here, \( d \) is the reference event measured at the geophones \((z_1 \approx z_0)\). This equation holds for all angles of incidence. From the minimization problem (5), we estimate the surface impedance function by iteratively updating \( \sigma \) using a conjugate gradient method. Campman et al. (2003), predict and subtract near-surface scattered energy from controlled measurements of scattered ultrasonic waves near the surface of a laboratory model, giving a proof of principle of the method. Here, we apply the same method on simple synthetic models to qualitatively evaluate the sensitivity of the approximate scattering equation with respect to two major assumptions in the derivation.

The first assumption is that we neglect the horizontal components of the elastic wave field. The second is that we assume that scattering takes place immediately under the receivers.

First, we consider a shallow scatterer immediately under the surface of a homogeneous elastic half space. The reference (P-wave) event is shown in Fig. 1(a). From this event we estimate an impedance function using Eq. (5). Then we predict the scattered field on different events with increasing angle of incidence up-to about 30 degrees. In Fig. 1(b), the event with an angle of about 30 degrees is shown. Predicting and subtracting the scattered energy from this event, yields the event shown in Fig 1(c). The continuity of the target event is restored, implying that we accurately predicted the scattered energy. It is interesting to see that also the continuity of the S-wave is restored, using the same impedance function — derived from a P-wave.

Next we consider the same model, but with the scatterer about a half wave length (of the scattered Rayleigh wave) from the surface. We can still find an impedance function that accurately fits the reference event. However, when we predict the scattered energy of target events with increasing angles of incidence, the predicted scattered field does not match the true scattered field anymore at
about 10 degrees. In Fig 2(a), the true scattered field for a target event with angle of incidence of 10 degrees is shown. In Fig. 2(b), the predicted scattered field for the same event is shown. The main energy in the scattered field is still reasonably well predicted, but it is clear that at this point our assumption of a shallow scatterer does not hold anymore.

Finally, we look at a result of predicting and subtracting scattered energy from target events. Consider the total field of the event in Fig 1(b), for example. Averaging the data from 60 to 100 m in space produces the wavelet shown in Fig. 3 with the solid line (this is the type of averaging that occurs in a field array). On the other hand, averaging the corrected data from Fig 1, in the same interval, produces the dotted line, which almost coincides with the wavelet after averaging the true incident field (not shown here). The same wavelets are shown in Fig. 3, but now in the frequency domain. This picture shows that the prediction-and-subtraction algorithm restores the high-frequency content in the wavelet.

Discussion and conclusions
From the elastic wave equation for particle displacement, we derive a scalar equation for the vertical component of the velocity. This formula captures the essence of the physics of near-surface scattering. We test the sensitivity of the approximate equation with respect to two major assumptions: 1) the scattering takes place immediately under the receivers and 2) for upcoming waves we can neglect the horizontal components of the velocity field. Our study shows that the equation is accurate, when the scatterers are not deeper than half a wave length of the Rayleigh wave.

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References


