



Multi Grid Presentation

Jos van Kan

`j.vankan@math.tudelft.nl`

TU Delft

Two Grid Algorithm

- 1: Presets: $\mathbf{u}_h^0, \mathbf{r}_h^0 = \mathbf{f}_h - A\mathbf{u}_h^0$
- 2: $\mathbf{u}_h^{\text{prs}} = S(\mathbf{u}_h^0, \mathbf{b}, A, n_0)$ {Presmoothing}
- 3: $\mathbf{r}_H = R_{Hh}\mathbf{r}_h$

Two Grid Algorithm

- 1: Presets: $\mathbf{u}_h^0, \mathbf{r}_h^0 = \mathbf{f}_h - A\mathbf{u}_h^0$
- 2: $\mathbf{u}_h^{\text{prs}} = S(\mathbf{u}_h^0, \mathbf{b}, A, n_0)$ {Presmoothing}
- 3: $\mathbf{r}_H = R_{Hh}\mathbf{r}_h$
- 4: Solve $A_H\mathbf{c}_H = \mathbf{r}_H$
- 5: $\mathbf{u}_h^{\text{cgc}} = \mathbf{u}_h^p + P_{hH}\mathbf{c}_H$ {Coarse Grid Correction}

Two Grid Algorithm

- 1: Presets: $\mathbf{u}_h^0, \mathbf{r}_h^0 = \mathbf{f}_h - A\mathbf{u}_h^0$
- 2: $\mathbf{u}_h^{\text{prs}} = S(\mathbf{u}_h^0, \mathbf{b}, A, n_0)$ {Presmoothing}
- 3: $\mathbf{r}_H = R_{Hh}\mathbf{r}_h$
- 4: Solve $A_H\mathbf{c}_H = \mathbf{r}_H$
- 5: $\mathbf{u}_h^{\text{cgc}} = \mathbf{u}_h^p + P_{hH}\mathbf{c}_H$ {Coarse Grid Correction}
- 6: $\mathbf{u}_h^{\text{pos}} = S(\mathbf{u}_h^{\text{cgc}}, \mathbf{b}, A, n_1)$ {Postsmoothing}

Multi Grid Algorithm

MGRecursive ($A_\ell, \mathbf{r}_\ell, \mathbf{c}_\ell, \ell$)

if $\ell < p - 1$ **then**

$\mathbf{c}_\ell = S(\mathbf{0}, \mathbf{r}_\ell, A_\ell, n_0)$ {Presmoothing}

else

Solve $A_{p-1}\mathbf{c}_{p-1} = \mathbf{r}_{p-1}$ {Direct solution on coarsest level}

end if

Multi Grid Algorithm

MGRecursive ($A_\ell, \mathbf{r}_\ell, \mathbf{c}_\ell, \ell$)

if $\ell < p - 1$ **then**

$\mathbf{c}_\ell = S(\mathbf{0}, \mathbf{r}_\ell, A_\ell, n_0)$ {Presmoothing}

$\mathbf{r}_{\ell+1} = R_\ell(\mathbf{r}_\ell - A_\ell \mathbf{c}_\ell)$ {Calculate coarse grid residual}

$A_{\ell+1} = R_\ell A_\ell P_{\ell+1}$ {Calculate coarse grid matrix}

else

Solve $A_{p-1} \mathbf{c}_{p-1} = \mathbf{r}_{p-1}$ {Direct solution on coarsest level}

end if

Multi Grid Algorithm

MGRRecursive ($A_\ell, \mathbf{r}_\ell, \mathbf{c}_\ell, \ell$)

if $\ell < p - 1$ **then**

$\mathbf{c}_\ell = S(\mathbf{0}, \mathbf{r}_\ell, A_\ell, n_0)$ {Presmoothing}

$\mathbf{r}_{\ell+1} = R_\ell(\mathbf{r}_\ell - A_\ell \mathbf{c}_\ell)$ {Calculate coarse grid residual}

$A_{\ell+1} = R_\ell A_\ell P_{\ell+1}$ {Calculate coarse grid matrix}

call MGRRecursive ($A_{\ell+1}, \mathbf{r}_{\ell+1}, \mathbf{c}_{\ell+1}, \ell + 1$)

$\mathbf{c}_\ell = \mathbf{c}_\ell + P_{\ell+1} \mathbf{c}_{\ell+1}$ {Coarse grid correction}

else

Solve $A_{p-1} \mathbf{c}_{p-1} = \mathbf{r}_{p-1}$ {Direct solution on coarsest level}

end if

Multi Grid Algorithm

MGRRecursive ($A_\ell, \mathbf{r}_\ell, \mathbf{c}_\ell, \ell$)

if $\ell < p - 1$ **then**

$\mathbf{c}_\ell = S(\mathbf{0}, \mathbf{r}_\ell, A_\ell, n_0)$ {Presmoothing}

$\mathbf{r}_{\ell+1} = R_\ell(\mathbf{r}_\ell - A_\ell \mathbf{c}_\ell)$ {Calculate coarse grid residual}

$A_{\ell+1} = R_\ell A_\ell P_{\ell+1}$ {Calculate coarse grid matrix}

call MGRRecursive ($A_{\ell+1}, \mathbf{r}_{\ell+1}, \mathbf{c}_{\ell+1}, \ell + 1$)

$\mathbf{c}_\ell = \mathbf{c}_\ell + P_{\ell+1} \mathbf{c}_{\ell+1}$ {Coarse grid correction}

$\mathbf{c}_\ell = S(\mathbf{c}_\ell, \mathbf{r}_\ell, A_\ell, n_1)$ {Postsmoothing}

else

Solve $A_{p-1} \mathbf{c}_{p-1} = \mathbf{r}_{p-1}$ {Direct solution on coarsest level}

end if