Computer Assignment EE4550: Block 3

Finite Element Simulation of the Magnetic Field in a C-Shaped Core

The objective of this assignment is to guide the student through the finite simulation of the magnetic field excited by a multiple turn winding in a C-shaped ferromagnetic core.

Let the computational geometry Ω consist of the following four part: the ferromagnetic core domain Ω_{core}, the left part of the winding $\Omega_{winding-left}$, the right part of the winding $\Omega_{winding-right}$ and the air domain Ω_{air}. We thus have that

$$\Omega = \Omega_{core} \cup \Omega_{winding-left} \cup \Omega_{winding-right} \cup \Omega_{air} \, . \tag{1}$$

Let $u(x, y)$ denote the z-component of the vector magnetic potential $\mathbf{A}$, i.e., $\mathbf{A} = (0, 0, u(x, y))$. This

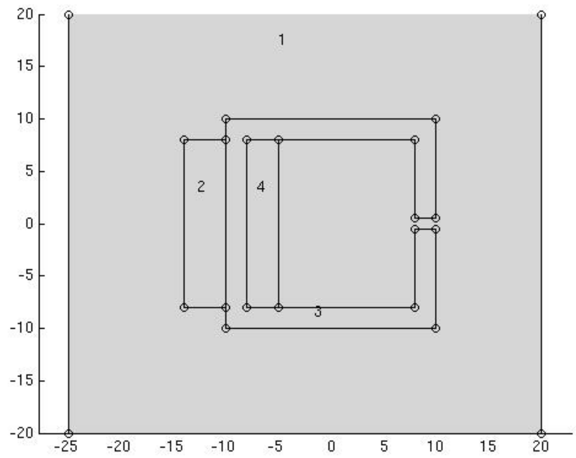


Figure 1: Geometry of the C-shaped core with winding and surrounding air.

implies that the relationship $\mathbf{B} = \nabla \times \mathbf{A} = \text{curl}\,\mathbf{A}$ for the magnetic flux $\mathbf{B} = (B_x, B_y, 0)$ reduces to

$$B_x(x, y) = \frac{\partial u}{\partial y} \text{ and } B_y(x, y) = -\frac{\partial u}{\partial x} \, . \tag{2}$$

Let μ denote the magnetic permeability of the material. Then $\mu = \mu_0 \, \mu_r$, where μ_0 and μ_r are the magnetic permeability of vacuum the relative permeability of the material, respectively. The values of μ_0 and μ_r are given by $\mu_0 = 4\pi 10^{-7} \text{H m}^{-1}$

$$\mu_r = \begin{cases} 1000 \text{ for } (x, y) \in \Omega_{core} \\ 1 \quad\;\;\; \text{for } (x, y) \in \Omega_{air} \cup \Omega_{winding-left} \cup \Omega_{winding-right} \, . \end{cases} \tag{3}$$

The magnetic field $\mathbf{H} = (H_x, H_y, 0)$ is then through the constitutive relation $\mathbf{H} = \frac{1}{\mu}\mathbf{B}$ given by

$$H_x(x, y) = \frac{1}{\mu} B_x(x, y) = \frac{1}{\mu}\frac{\partial u}{\partial y} \text{ and } H_y(x, y) = \frac{1}{\mu} B_y(x, y) = -\frac{1}{\mu}\frac{\partial u}{\partial x} \, . \tag{4}$$

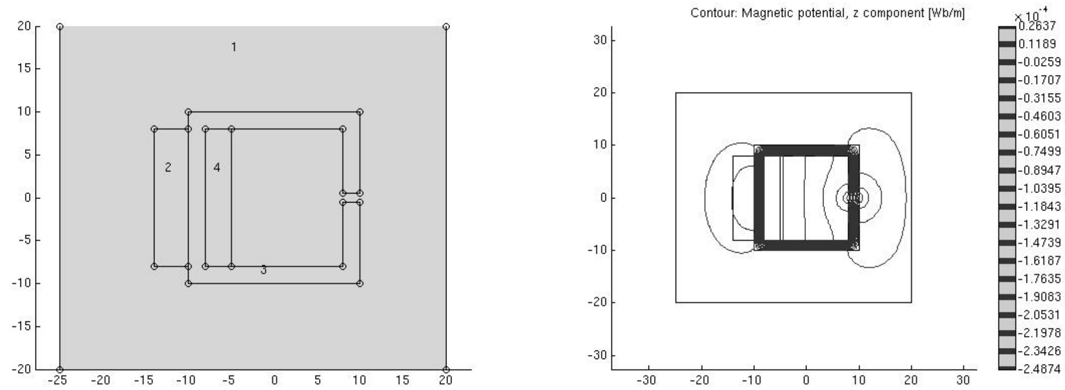
Let $J_{e,z}(x, y)$ denote the applied current density in the z-direction and assume its values to be given by

$$J_{e,z} = \begin{cases} J_0 \text{ for } (x, y) \in \Omega_{winding-left} \\ -J_0 \text{ for } (x, y) \in \Omega_{winding-right} \\ 0 \text{ elsewhere} \, . \end{cases} \tag{5}$$

The magnetic vector potential component $u(x, y)$ is then found by solving the following diffusion equation with discontinuous diffusion coefficient diffusion equation

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial u}{\partial y}\right) = J_{e,z} \tag{6}$$

supplied with the homogeneous boundary condition $u = 0$.



(a) Geometry (b) Computed magnetic field lines

Figure 2: Geometry and computed magnetic field lines in the C-shaped core.

Assignment 1 (Geometry Definition)