ATHENS Course Introduction to Finite Elements

Computer Assignment Day 4

Galerkin Finite Element Solution of a Diffusion Equation in Two Dimensions

1 Introduction

In this assignment we consider on the domain $(x, y) \in \Omega = [-0.4, 0.4] \times [-0.4, 0.4]$ for times $t > 0$ the diffusion equation

$$E(x,y)\frac{\partial u}{\partial t} = \nabla \cdot (D(x,y)\nabla u) + S(x,y,t). \tag{1}$$

The conductivity E and the diffusivity D are constant in time t but depend on the location (x, y). The source term S depends on time t and location (x, y). On the outer boundary $\partial\Omega$ of the domain Ω we have the following Dirichlet boundary condition

$$u = 0, \tag{2}$$

and we have on Ω the initial condition

$$u(x, y, 0) = 0. \tag{3}$$

Figure 1 shows the domain on which we consider Equation (1) with boundary condition (2). Using the numbering as in this figure, we have the following definitions of the conductivity E and diffusivity D:

$$E(x,y) = \begin{cases} E_{\text{met}} & \text{if } (x,y) \in \Omega_2, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

$$D(x,y) = \begin{cases} D_{\text{met}} & \text{if } (x,y) \in \Omega_2, \\ D_{\text{air}} & \text{otherwise.} \end{cases} \tag{5}$$

2 The stationary problem

Consider the stationary version of Equation (1)

$$-\nabla \cdot (D(x,y)\nabla u) = S(x,y), \tag{6}$$

with the source term S given by

$$S(x,y) = \begin{cases} -A & \text{if } (x,y) \in \Omega_4, \\ A & \text{if } (x,y) \in \Omega_5, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Download the code for this problem from `http://ta.twi.tudelft.nl/nw/users/domenico/intro_fem/intro_fem.html` and open with Matlab the file `main.m`.

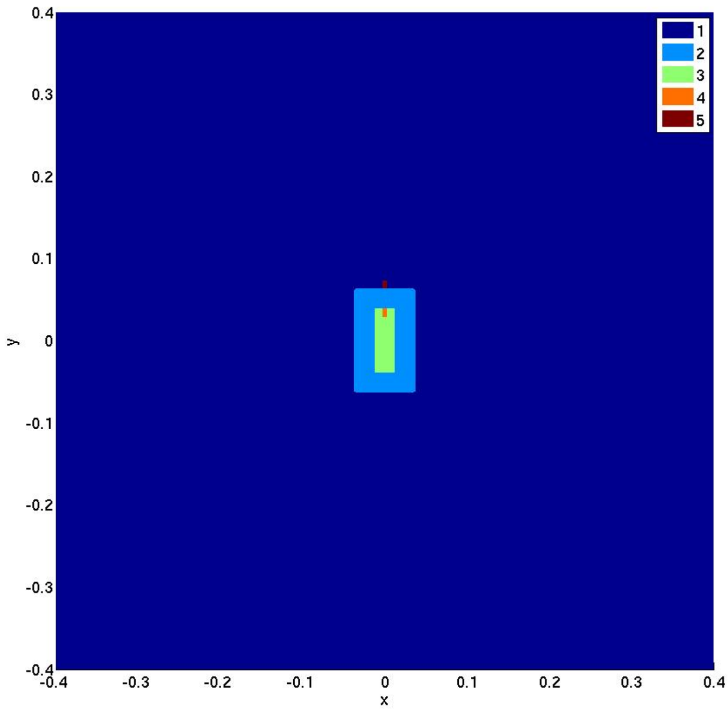

Figure 1: The domain Ω that we consider in this assignment.

Assignment 1

Use the `help` command of Matlab to see which file has what functionality. Will the functions `Diffusion` and `Source` do as prescribed as in Definitions (5) and (7)?

Assignment 2

Give a weak formulation of problem (6), in which the order of the spatial derivatives is minimized. Give furthermore the element matrix and element vector.

Assignment 3

Write the Matlab routine `StiffnessElementMatrix.m`. Hint: See the help already provided in the template.

Assignment 4

Write the Matlab routine `ElementVector.m`. Hint: See the help already provided in the template.

	Coefficients		
Run	D_{met}	D_{air}	A
1	10^{-3}	1	7.5
2	10^{-3}	1	0.75
3	10^{-3}	1	0.075

Assignment 5

Look at the file `main.m`. Run the code several times with the coefficients as in Table 2 and look at the figures that are created. Use the option `'fine'` with the function `LoadMesh`. What happens when you change the value of the amplitude A?

Assignment 6

Look again at the file `main.m`. Run the code several times with the coefficients as in Run 1 from Table 2 and look at the figures that are created. Run the code with the option `'coarse'`, `'medium'` and `'fine'` with the function `LoadMesh`. What happens when you change the coarseness of the mesh? Can you see in the first figure why this is so? Hint: Use the `zoom` tool.

3 The zero conductivity problem

Consider the zero conductivity version of Equation (1)

$$-\nabla \cdot (D(x,y)\nabla u) = S(x,y,t), \tag{8}$$

with the source term S given by

$$S(x,y,t) = \begin{cases} -A\sin(\omega t) & \text{if } (x,y) \in \Omega_4, \\ A\sin(\omega t) & \text{if } (x,y) \in \Omega_5, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Download the code for this problem from `http://ta.twi.tudelft.nl/nw/users/domenico/intro_fem/intro_fem.html` and open with Matlab the file `main.m`.

Assignment 7

Give a weak formulation of problem (8), in which the order of the spatial derivatives is minimized. Give furthermore the element matrix and element vector.

Assignment 8

Write the Matlab routine `StiffnessElementMatrix.m`. Hint: See the help already provided in the template.

Assignment 9

Write the Matlab routine `ElementVector.m`. Hint: See the help already provided in the template.

Assignment 10

In `main.m` set the values of the diffusivity D and the amplitude A to the values from Run 1 in Table 2. Run three simulations with the value `Period` set to 2π, π and $1/25\pi$. Why don't you see anything happening in the solution with the last simulation?

Assignment 11

In `main.m` set the values of the diffusivity D and the amplitude A to the values from Run 1 in Table 2. Set the value `Period` to π. Run three simulations with the value `T` set to 4, 10 and 100 and on the medium mesh. Why don't you see anything happening in the solution with the first simulation? Which simulation took more time to run?

4 The non-zero conductivity problem

Consider the non-zero conductivity version of Equation (1)

$$E(x,y)\frac{\partial u}{\partial t} = \nabla \cdot (D(x,y)\nabla u) + S(x,y,t). \qquad (10)$$

Download the code for this problem from `http://ta.twi.tudelft.nl/nw/users/domenico/intro_fem/intro_fem.html` and open with Matlab the file `main.m`.

Assignment 12

Give a weak formulation of problem (1), in which the order of the spatial derivatives is minimized. Give furthermore the element matrices and element vector.

Assignment 13

Write the Matlab routine `MassElementMatrix.m`. Hint: See the help already provided in the template.

Assignment 14

Write the Matlab routine `StiffnessElementMatrix.m`. Hint: See the help already provided in the template.

Assignment 15

Write the Matlab routine `ElementVector.m`. Hint: See the help already provided in the template.

Assignment 16

In `main.m` set the values of the diffusivity D and the amplitude A to the values from Run 1 in Table 2. Set the value `Period` to π and use the medium mesh. Run a simulation with 25 time steps with the Euler Forward method. Can you think of a reason why you get these weird solutions? Hint: Look at the matrix M with the command `spy`.

Assignment 17

In `main.m` set the values of the diffusivity D and the amplitude A to the values from Run 1 in Table 2. Set the value `Period` to π and use all three meshes. Run a simulation with 25 time steps with the Euler Backward method. Do you get correct results now?

Assignment 18

In `main.m` set the values of the diffusivity D and the amplitude A to the values from Run 1 in Table 2. Set the value `Period` to π and use all three meshes. Run a simulation with 25 time steps with the Crank-Nicholson method. Do you get correct results now?

Assignment 19

Play around and try various settings of the parameters. Use the Crank-Nicholson method and the fine mesh. Changing which parameter has the most effect on the solution?