Open-source CFD software: FEATFLOW

The FEA(S)T groups

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Lake Tahoe, January 7, 2009
Overview

1. FeatFlow
   - FeatFlow 1.x
   - FeatFlow 2.0
2. Preprocessing
   - DeViSoR Grid3D
3. Example application
   - Poisson equation
4. Postprocessing
   - General Mesh Viewer
5. UnConventional HPC
   - FEAST
FeatFlow 1.x

Finite Element Analysis Toolbox + Flow solver

High performance unstructured finite element package for the numerical solution of the incompressible Navier-Stokes equations

- based on the finite element packages Feat2D and Feat3D
- written in Fortran 77 (and some C routines) by Stefan Turek
- designed for education, scientific research and industrial applications
- full source-code and user manuals are available online
- many extensions are not included in the official release 😞

Visit the FeatFlow homepage

http://www.featflow.de
http://www.featflow.de/album

Theoretical background
Discretization techniques

Incompressible Navier-Stokes equations

\[ u_t - \nu \Delta u + u \cdot \nabla u + \nabla p = f, \quad \nabla \cdot u = 0, \quad \text{in } \Omega \times (0, T] \]

- Spatial discretization techniques
  - nonconforming rotated multilinear finite elements for \( u \)
  - piecewise constant pressure approximation for \( p \)
  - Samarskij upwind or streamline diffusion stabilization

- Temporal discretization techniques
  - implicit one-step-\( \theta \)-scheme (Backward Euler, Crank-Nicolson)
  - implicit fractional-step-\( \theta \) scheme (second-order accurate)
  - adaptive time-stepping based on local discretization error
Solution techniques

Discretized incompressible Navier-Stokes equations

Given $u^n$, $g$ and $k$, solve for $u = u^{n+1}$ and $p = p^{n+1}$

$$[M + \theta k N(u)]u + kBp = g, \quad B^T u = 0$$

where $g = [M - \theta_1 k N(u^n)]u^n + \theta_2 kf^{n+1} + \theta_3 kf^n$

- Nonlinear/linear solution strategies
  - coupled fixed point defect correction method CC2D/CC3D
  - nonlinear discrete projection scheme PP2D/PP3D
  - linear multigrid techniques with adaptive step-length control
  - ILU/SOR or Vanka-like block Gauß-Seidel smoother/solver
# FeatFlow 2.0

**Finite Element Analysis Toolbox + Flow solver**

The modern successor of FeatFlow 1.x for the numerical solution of **flow problems** by the finite element method

- modular object-oriented design by Michael Köster et al.
- written in Fortran 95 (kernel + applications)
- external libraries in F77/C (BLAS, UMFPACK, LAPACK)
- designed for education of students and scientific research
- detailed in-place documentation of the source-code

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**Official release not yet available; get the ALPHA snapshot**

[http://www.featflow.de/download/Featflow2_2.0ALPHA.tar.gz](http://www.featflow.de/download/Featflow2_2.0ALPHA.tar.gz)
Prerequisites

Unix/Linux and Mac OS X
- compatible C and F95 compiler
  - GCC and G95 version 0.91
  - Intel® C++/Fortran Compilers for Linux
  - Sun Studio C, C++ and Fortran Compilers
- GNU make utility
  - Makefiles are provided for all applications

Windows XP, Vista
- Microsoft® VisualStudio 2003, 2005 or 2008
  - Project files are provided for all applications
- Intel® C++/Fortran Compilers for Windows
- Cygwin™ environment
  - General Mesh Viewer (GMV)
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Linux is used for this workshop
Getting **FEATFLOW** 2.0

- Unpack the downloaded archive file
  
  $ tar xvzf Featflow2_2.0ALPHA.tar.gz

- Change into the base directory
  
  $ cd Featflow2 ; ls

  applications     Globals.power     object
  bin              Globals.sparc      readme.txt
  codefragments    Globals.x86       Rules_apps_f90.mk
  feat2win.txt     Globals.x86_64    Rules_apps.mk
  Globals.alpha    kernel           Rules_libs.mk
  Globals.ia64     libraries        VERSIONS
 Globals.mac       Makefile
  Globals.mk       matlab
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  Globals.ia64    libraries     VERSIONS
  Globals.mac     Makefile
  Globals.mk      matlab
Top-level build options

$ make [ALT=xxx] [ID=yyy] <target>

- Some values for target
  - `help` - print additional help and further option
  - `id` - print out settings for current ID
  - `all` - compile all libraries and application modules
  - `apps` - compile all application modules
  - `libs` - compile all libraries
Top-level build options

$ make [ALT=xxx] [ID=yyy] <target>

- Some values for target
  - help: print additional help and further option
  - id: print out settings for current ID
  - all: compile all libraries and application modules
  - apps: compile all application modules
  - libs: compile all libraries

- Some available make modifiers
  - ALT=xxx: specify alternative ID-xxx to use
  - ID=yyy: override the autodetected architecture ID by yyy
Building **FEATFLOW** 2.0, cont’d.

### Understanding the ALT/ID concept

```latex
$ \textit{make id} \quad \text{run on } \textit{x86\_64} \text{ GNU/Linux}
```

**Machine-ID (Barracuda)**: pc64-core2-linux

**Compilers to be used:**
- C compiler: `/usr/bin/gcc`
- C++ compiler: `/usr/bin/g++`
- Fortran compiler: `/usr/local/g95/32bit_integers/0.91/bin/g95`
- F-Library archiver: `/usr/bin/ar`
- C-Library archiver: `/usr/bin/ar`

**Flags to be used:**
- `OPTFLAGS = -O3 -m64 -ffast-math -fexpensive-optimizations -fprefetch-loop-arrays -mmmx -msse -msse2 -msse3`
- `OPTFLAGSC =`
- `OPTFLAGSCPP =`
- `OPTFLAGSF =`
- `OPTFLAGSDEBUG = -g`
- `OPTFLAGSCEDEBUG =`
- `OPTFLAGSCPPDEBUG =`
- `OPTFLAGSFDEBUG = -O0 -g -Wall -fbounds-check -ftrace=full`
- `FCFLAGS = -pipe -fmod= -march=nocona`
- `CCFLAGS = -pipe -march=nocona`
- `CPPFLAGS = -pipe -march=nocona`
- `BUILDLIB = feat3d feat2d sysutils umfpack2 amd umfpack4 minisplib lapack blas`
- `BLASLIB = (standard BLAS, included in installation package)`
- `LAPACKLIB = (standard LAPACK, included in installation package)`
- `LDLIBS =`
- `LDFLAGS =`
Building FEATFLOW 2.0, cont’d.

Understanding the ALT/ID concept

$ make ID=pc-core2-linux id          run on x86_64 GNU/Linux

Machine-ID (Barracuda) : pc-core2-linux

Compilers to be used:
- C compiler: /usr/bin/gcc
- C++ compiler: /usr/bin/g++
- Fortran compiler: /usr/local/g95/32bit_integers/0.91/bin/g95
- F-Library archiver: /usr/bin/ar
- C-Library archiver: /usr/bin/ar

Flags to be used:
- OPTFLAGS = -O3 -m32 -ffast-math -fexpensive-optimizations -fprefetch-loop-arrays -march -msse -msse2 -msse3
- OPTFLAGSC =
- OPTFLAGSCPP =
- OPTFLAGSF =
- OPTFLAGSDEBUG = -g
- OPTFLAGSCDEBUG =
- OPTFLAGSCPPDEBUG =
- OPTFLAGSFDEBUG = -O0 -g -Wall -fbounds-check -ftrace=full
- FCFLAGS = -pipe -fmod= -march=nocona
- CCFLAGS = -pipe -march=nocona
- CPPFLAGS = -pipe -march=nocona
- BUILDLIB = feat3d feat2d sysutils umfpack2 amd umfpack4 minisplib lapack blas
- BLASLIB = (standard BLAS, included in installation package)
- LAPACKLIB = (standard LAPACK, included in installation package)
- LDLIBS =
- LDFLAGS =
Building **FEATFLOW 2.0**, cont’d.

### Understanding the ALT/ID concept

```bash
$ make ALT=ifc id
```

run on x86_64 GNU/Linux

---

Machine-ID (Barracuda) : pc64-core2-linux-ifc

Compilers to be used:
- C compiler: /usr/local/intel/cce/10.1.021/bin/icc
- C++ compiler: /usr/local/intel/cce/10.1.021/bin/icpc
- Fortran compiler: /usr/local/intel/fce/10.1.021/bin/ifort
- F-Library archiver: /usr/local/intel/fce/10.1.021/bin/xiar
- C-Library archiver: /usr/local/intel/fce/10.1.021/bin/xiar

Flags to be used:
- OPTFLAGS = -O3 -ipo -xT
- OPTFLAGSC =
- OPTFLAGSCPP =
- OPTFLAGSF =
- OPTFLAGSDEBUG = -g
- OPTFLAGSCDEBUG = -traceback
- OPTFLAGSCPPDEBUG = -traceback
- OPTFLAGSFDEBUG = -warn all -check all -traceback
- FCFLAGS = -cm -fpe0 -vec-report0 -module
- CCFLAGS = -vec-report0
- CPPFLAGS = -vec-report0
- BUILDLIB = feat3d feat2d sysutils umfpack2 amd umfpack4 minisplib lapack blas
- BLASLIB = (standard BLAS, included in installation package)
- LAPACKLIB = (standard LAPACK, included in installation package)
- LDLIBS =
- LDFLAGS = -lsvm1

---

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Understanding the ALT/ID concept

$ make [ALT=xxx] [ID=yyy] <target>

- compiler settings are defined in the global configuration files `Global.[alpha,ia64,mac,power,sparc,x86,x86_64]`
- new compilers and/or architectures can be easily included
- special purpose settings, e.g. `pc64-opteron-linux-ifclarge`
Understanding the ALT/ID concept

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Building the Poisson example application

$ cd applications/poisson
$ make

... after some time ...

Done, poisson-pc64-core2-linux is ready.
Understanding the ALT/ID concept

$ make [ALT=xxx] [ID=yyy] <target>

- compiler settings are defined in the global configuration files Global.[alpha,ia64,mac,power,sparc,x86,x86_64]
- new compilers and/or architectures can be easily included
- special purpose settings, e.g. pc64-opteron-linux-ifclarge

Building the Poisson example application

$ cd applications/poisson
$ make debug turn on debugging facilities of the compiler
... after some time ...
Done, poisson-pc64-core2-linux is ready.
Preprocessing: *Grid generation*
1. Construct initial coarse grid in the external preprocessing step
2. Generate hierarchy of regularly refined meshes in the application
Geometric multigrid approach

1. Construct initial coarse grid in the external preprocessing step
2. Generate hierarchy of regularly refined meshes in the application

Two-level ordering strategy
- adopt all coordinates from coarser grid levels unchanged
- introduce new coordinates at edge/face/cell midpoints
Coarse grid description

- File format is described in the Feat2D/Feat3D manuals
- Supported element types: triangles, quads (2D) and hexahedra (3D)
Coarse grid description

- File format is described in the `FEAT2D/FEAT3D` manuals
- Supported element types: triangles, quads (2D) and hexahedra (3D)
- Domain triangulation is specified in TRI file (2D/3D)
  - coordinate values of vertices in the interior
  - parameter values of vertices at the boundary
  - elements in terms of their corner nodes
  - first two lines are treated as header/comments!
Coarse grid description

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- Domain triangulation is specified in TRI file (2D/3D)
  - coordinate values of vertices in the interior
  - parameter values of vertices at the boundary
  - elements in terms of their corner nodes
  - first two lines are treated as header/comments!
- Boundary parametrization is specified in PRM file (only 2D)
  - each boundary component is described by \( p \in [0, p_{\text{max}}] \)
  - the ‘interior’ is located left to the boundary
    \( \rightarrow \) do not mix up (counter-)clockwise orientation
  - supported boundary types: lines, (arcs of) circles
DeViSoR Grid3D

Coarse grid generator for FeatFlow and Feast

- written in Java + OpenGL and published under the GPL
- available at http://www.feast.uni-dortmund.de
- send requests, bug reports to devisor@featflow.de
- on-line help system and self-contained tutorial included

- Unpack the downloaded archive file
  $ unzip grid-3.0.21.zip

- Start the application
  $ cd grid-3.0.21
  $ java -jar grid3d.jar
Alternative grid generators

- GiD – the personal pre- and post-processor
  - Evaluation version is available at http://gid.cimne.upc.es
  - Fully automatic structured and unstructured coarse grid generator
  - Supports triangular, quadrilateral, tetrahedral, hexahedral elements
  - Provides an effective easy-to-use and geometric user interface

- GiD2Feat – set of tools to convert GiD meshes to PRM/TRI files
Example application: *Poisson equation*
Possion equation

- Change into the application source directory
  
  ```
  $ cd applications/poisson/src; ls
  ```
  ```
  poissonXd_callback.f90  poisson.f90
  poissonXd_methodYYY.f90
  ```

- Open the application main source file
  
  ```
  $ emacs poisson.f90
  ```
Poisson equation

- Change into the application source directory
  
  $ cd applications/poisson/src; ls

  poissonXd_callback.f90  poisson.f90
  poissonXd_methodYYY.f90

- Open the application main source file
  
  $ emacs poisson.f90

Initialization and finalization

- `system_init()`
  initialize system-wide settings

- `storage_init(999, 100)`
  initialize storage management

- `storage_done()`
  finalize storage management
**Possion equation, cont’d.**

**Sample problem:** \[-\Delta u = f \quad \text{in } \Omega = (0, 1)^2, \quad u = 0 \quad \text{on } \partial \Omega\]

- **right hand side** \[f(x, y) = 32(x(1 - x) + y(1 - y))\]
- **analytical solution** \[u(x, y) = 16x(1 - x)y(1 - y)\]

- Open the demonstration module file
  
  \`
  $ emacs poisson2d_method0_simple.f90
  \`

  - contains the corresponding subroutine
  - includes all required kernel modules
  - provides detailed step-by-step tutorial

- Open the callback function module file
  
  \`
  $ emacs poisson2d_callback.f90
  \`
## Possion equation, cont’d.

### Grid generation
- `boundary_read prm` read boundary parametrization
- `tria_readTriFile2D` read domain triangulation
- `tria_quickRefine2LevelOrdering` perform regular refinement
- `tria_initStandardMeshFromRaw` generate data structures

### Spatial discretization
- `spdiscr_initBlockDiscr2D` prepare block discretization
- `spdiscr_initDiscr_simple` initialize spatial discretization
- `bilf_createMatrixStructure` create scalar matrix structure
- `bilf_buildMatrixScalar` discretize the bilinear form
- `linf_buildVectorScalar` discretize the linear form/r.h.s.
# Possion equation, cont’d.

## Dirichlet boundary conditions
- `boundary_createRegion` specify boundary segment
- `bcasm_newDirichletBConRealBD` calculate discrete b.c.’s
- `matfil_discreteBC` set b.c.’s in system matrix
- `vecfil_discreteBCrhs` set b.c.’s in right hand side
- `vecfil_discreteBCsol` set b.c.’s in solution vector

## Linear BiCGStab solver
- `linsol_initBiCGStab` initialize linear solver
- `linsol_setMatrices` attach system matrix
- `linsol_initStructure, linsol_initData` solve linear system
Solution output

- `ucd_startGMV` start export on GMV format
- `ucd_addVariableVertexBased` add vertex-based solution data
- `ucd_addVariableElementBased` add cell-based solution data
- `ucd_write` write solution data to file

Clean-up and finalization

- `XXX_release, XXX_releaseYYY` free allocated memory
- `XXX_done` stop sub-system

General naming convention of subroutines

- `abbreviatedModulefile_NameOfSubroutine`
Postprocessing: *Visualization of the solution*
General Mesh Viewer

3D scientific visualization tool

- developed at Los Alamos National Lab by Frank Ortega
- available at http://www-xdiv.lanl.gov/XCM/gmv/
- supported OS: UNIX/Linux, Mac OS X, Windows (Cygwin)

- unstructured meshes in 2D/3D
- cutlines, cutplanes, cutspheres
- vertex-based, cell-based data sets
- contour, vector plots

Alternative: importer for ParaView based on development CVS-version
Advanced topics: *Multigrid, Mesh Adaptation, ...*
Sample problem: \(-\Delta u = f\) in \(\Omega = (0, 1)^2\), \(u = 0\) on \(\partial \Omega\)

- right hand side \(f(x, y) = 32(x(1 - x) + y(1 - y))\)
- analytical solution \(u(x, y) = 16x(1 - x)y(1 - y)\)

Open the demonstration module file

\>$\$ emacs poisson2d_method1_mg.f90

- linear geometric multigrid solver
- Jacobi or ILU(0) smoother
- direct coarse grid solver (UMFPACK)
Anisotropic diffusion

Example: applications/anisotropicdiffusion.f90

\[ D = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \]

\[
\begin{cases}
-\nabla \cdot (D \nabla u) = 0 & \text{in } \Omega \\
u = 0 & \text{on } \Gamma_0 \\
u = 2 & \text{on } \Gamma_1
\end{cases}
\]

- \( k_1 = 100, \quad k_2 = 1, \quad \theta = -\frac{\pi}{6} \)
- linear finite elements, \( h = 1/36 \)
- Galerkin fails: \( u_{\text{min}} = -0.0553 \)
- \( h \)-adaptation: \( u_{\text{min}} = -0.0068 \)
Anisotropic diffusion

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Mesh adaptation

- conformal mesh refinement based on red-green strategy
- vertex-locking algorithm for mesh re-coarsening procedure
- nodal generation function stores birth certificates
  - provides complete characterization of elements
  - youngest node corresponds to refinement level
  - is required for the vertex-locking algorithm
- state function for element characterization
- local two-level ordering strategy

This has been addressed in the talk on Monday.
Mesh adaptation

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Mesh genealogy, revisited

Triangulation $\mathcal{T}_m(\mathcal{E}_m, \mathcal{V}_m)$, $m = 0, 1, 2, \ldots$ consists of

\[ \mathcal{E}_m = \{ \Omega_k : k = 1, \ldots, N_E \} \quad \text{and} \quad \mathcal{V}_m = \{ v_i : i = 1, \ldots, N_V \} \]

- nodal \textbf{generation function} $g : \mathcal{V}_m \to \mathbb{N}_0$ is defined recursively

\[
g(v_i) := \begin{cases} 
0 & \text{if } v_i \in \mathcal{V}_0 \\
\max_{v_j \in \Gamma_{kl}} g(v_j) + 1 & \text{if } v_i \in \Gamma_{kl} := \bar{\Omega}_k \cap \bar{\Omega}_l \\
\max_{v_j \in \partial \Omega_k} g(v_j) + 1 & \text{if } v_i \in \Omega_k \setminus \partial \Omega_k
\end{cases}
\]
Characterization of elements

**Idea I:** State function $s : \mathcal{E}_m \rightarrow \mathbb{Z}$ (in MSB representation)

- Set Bit[0] to 1 for quadrilateral, otherwise set it to zero
- Set Bit[k=1..4] to 1 if both endpoints of edge $k$ have same age
- If no two endpoints have same age, then find local position $k$ of the youngest vertex, set Bit[k] to 1 and negate the state

**Idea II:** Define local ordering strategy within each element *a priori*

<table>
<thead>
<tr>
<th>element characterization</th>
<th>element state</th>
</tr>
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<tbody>
<tr>
<td>triangle/quadrilateral from $\mathcal{T}_0$</td>
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</tr>
<tr>
<td>green quadrilateral</td>
<td>3, 5, 9, 11, 17, 21</td>
</tr>
<tr>
<td>red quadrilateral</td>
<td>7, 13, 19, 25</td>
</tr>
<tr>
<td>inner red triangle</td>
<td>14</td>
</tr>
<tr>
<td>'other' triangle</td>
<td>2, 4, 8</td>
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<td>green triangle</td>
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Characterization of elements

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- If no two endpoints have same age, then find local position $k$ of the youngest vertex, set Bit[k] to 1 and negate the state

Idea II: Define local ordering strategy within each element a priori

<table>
<thead>
<tr>
<th>element characterization</th>
<th>element state</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle/quadrilateral from $\mathcal{T}_0$</td>
<td>0/1</td>
</tr>
<tr>
<td>green quadrilateral</td>
<td>3, 5, 9, 11, 17, 21</td>
</tr>
<tr>
<td>red quadrilateral</td>
<td>7, 13, 19, 25</td>
</tr>
<tr>
<td>inner red triangle</td>
<td>14</td>
</tr>
<tr>
<td>'other’ triangle</td>
<td>2, 4, 8</td>
</tr>
<tr>
<td>green triangle</td>
<td>-8, -4, -2</td>
</tr>
</tbody>
</table>
Characterization of elements

**Idea I:** State function \( s : E_m \rightarrow \mathbb{Z} \) (in MSB representation)

- Set Bit[0] to 1 for quadrilateral, otherwise set it to zero
- Set Bit[k=1..4] to 1 if both endpoints of edge \( k \) have same age
- If no two endpoints have same age, then find local position \( k \) of the youngest vertex, set Bit[k] to 1 and negate the state

**Idea II:** Define *local ordering strategy* within each element *a priori*

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![Diagram](image)
Concept of dynamically allocatable arrays in Fortran 9x

- integer, dimension(():), allocatable :: Iarray
- allocate(Iarray(n)) allocate array of size n dynamically
- deallocate(Iarray) deallocate dynamically allocated array
Storage management

Concept of dynamically allocatable arrays in Fortran 9x

- integer, dimension(:), allocatable :: Iarray
- allocate(Iarray(n)) allocate array of size n dynamically
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Concept of dynamically allocatable arrays in FeatFlow

- storage_init, storage_done initialize/finalize storage
- storage_new allocate new memory storage
- storage_realloc re-allocate memory storage
- storage_free deallocate memory storage
- storage_getbase_XXX access memory storage
Storage management, cont’d.

- Supported data: 8/16/32/64 integer, SP/DP real, logical, strings
- Memory storage is accessible via integer handle $i\text{handle}$
- Pointer to the memory is assigned via $\text{storage\_getbase\_XXX}$

**Implementation details**

- memory storage handling is internally mapped to de/allocate
- different storage management can be implemented without changes
- handles can be easily passed to subroutines and functions
- derived types typically consist of a set of handles + scalar data
Storage management, cont’d.

Derived type for triangulation structure

```plaintext
type t_triangulation
    integer :: ndim = 0
    integer :: NVT = 0
    integer :: NMT = 0
    integer :: NAT = 0
    integer :: NEL = 0
    . . .
    integer :: h_DvertexCoords = ST_NOHANDLE
    integer :: h_IverticesAtElement = ST_NOHANDLE
    integer :: h_IneighboursAtElement = ST_NOHANDLE
end type
```
Example: *Using handles*

Create new storage
- \texttt{storage\_new (scall, sname, Isize, ctype, ihandle, \& cinitNewBlock, <rheap>)}
  - \texttt{ctype} ∈ \{ST\_DOUBLE, ST\_SINGLE, ST\_INTx, ST\_CHAR, ST\_LOGICAL\}
  - \texttt{cinitNewBlock} ∈ \{ST\_NEWBLOCK\_ZERO, ST\_NEWBLOCK\_NOINIT\}

Accessing storage, e.g. 2-dimensional double array
- \texttt{real(DP), dimension(:,:), pointer :: p\_Darray}
- \texttt{storage\_getbase\_double2D (ihandle, p\_Darray, <rheap>)}

Releasing storage
- \texttt{storage\_free (ihandle)}
UnConventional High Performance Computing
**Finite Element Analysis & Solution Tools**

High performance finite element package for the efficient simulation of large scale problems on heterogeneous hardware

- written in Fortran 90 and C (MPI communication)
- CFD (Stokes, Navier-Stokes), CSM (Elasticity)
- macro-wise domain decomposition approach
- fast and robust parallel multigrid methods
- unconventional hardware as FEM co-processors

Visit the **FEAST** homepage

http://www.feast.uni-dortmund.de
Scalable Recursive Clustering (ScaRC) solver

Combine domain decomposition and cascaded multigrid methods

Globally unstructured

Locally structured

hide anisotropies to improve robustness
UnConventional HPC

- Graphic processing unit (GPU)
  128 parallel scalar processors
  @ 1.35 GHz, ≈ 350 GFLOP/s
  GDDR3 memory @ 900 MHz

- Cell multi-core processor (PS3)
  7 synergistic processing units
  @ 3.2 GHz, 218 GFLOPS/s
  Memory @ 3.2 GHz
Future vision

Unified **FeaT+FeaST** finite element package

- Decompose the domain into **globally** unstructured macro-cells
- Use generalized tensor product grid or unstructured mesh **locally**
- Reuse FEM co-processors in the **FeaTFlow** package
- Enable special features, (e.g. $h$-adaptation) per macro-cell

On-line resources and additional material

- FeatFlow project:  [http://www.featflow.de](http://www.featflow.de)
- Feast project:  [http://www.feast.uni-dortmund.de](http://www.feast.uni-dortmund.de)