Multiwavelets and outlier detection for troubled-cell indication

In honour of Prof. Chi-Wang Shu on his 60th birthday

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Summary

Solutions of nonlinear hyperbolic PDEs usually develop discontinuities. To avoid spurious oscillations we apply a limiter. Since limiters tend to act on smooth extrema as well, the use of a good troubled-cell indicator is necessary. In this poster, the use of multiwavelets and outlier detection for troubled-cell indication is presented.

DG and multiwavelets

There is an exact relation between the DG coefficients and the multiwavelet coefficients.

$$u_h(x) = \sum_{j=0}^{2^n-1}\sum_{\ell=0}^{k} u_j^{(\ell)}\phi_\ell(\xi_j) = \underbrace{\sum_{\ell=0}^{k} s_{\ell 0}^0 \phi_\ell(x)}_{\substack{\text{global average}\\ \in V_0^{k+1}}} + \underbrace{\sum_{m=0}^{n-1}\sum_{j=0}^{2^m-1}\sum_{\ell=0}^{k} d_{\ell j}^m \psi_{\ell j}^m(x)}_{\substack{\text{finer details}\\ \in W_m^{k+1}}}.$$

Multiwavelet troubled-cell indicator

Multiwavelet coefficients decay in smooth regions. Elements I_j and I_{j+1} are detected as troubled if

$$|\tilde{d}_{kj}^{n-1}| > C \cdot \max\{|\tilde{d}_{kj}^{n-1}|, j = 0, \ldots, 2^n-1\}, \quad C \in [0,1].$$

Parameter C determines the strictness of the indicator. We must choose an appropriate value of C for each test problem.

Outlier detection

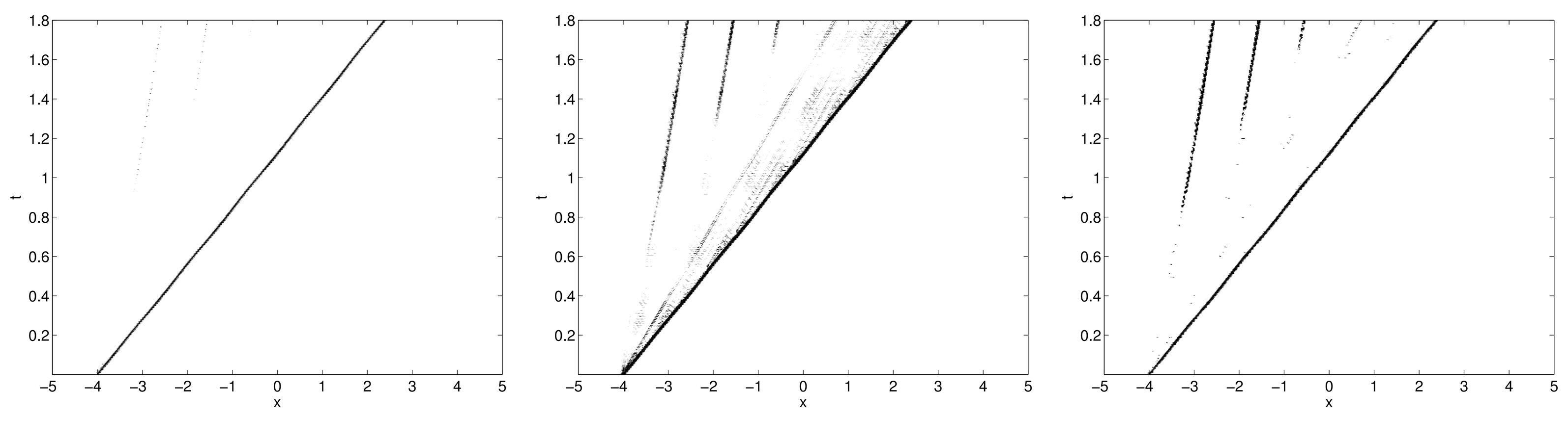
To eliminate parameter C, we apply outlier detection to the multiwavelet coefficients:

Send in a suitable troubled-cell indication vector $\mathbf{D}$.
Split this vector into local vectors, $\mathbf{d}$.
for all local vectors **do**
Sort $\mathbf{d}$ to obtain $\mathbf{d}^s$.
Compute the quartiles Q_1 and Q_3.
Determine outliers:

$$d_j \notin [Q_1 - 3(Q_3 - Q_1), Q_3 + 3(Q_3 - Q_1)].$$

end for
Check detected outliers in left and right half of local region with neighboring regions.

Investigate results for Euler equations: Shu-Osher problem.



(a) $C = 0.1$ (b) $C = 0.01$ (c) Outlier detection

Figure 1: Time-history plot of detected troubled cells, Shu-Osher problem, $k = 2$, 512 elements.

Parameters no longer problem dependent!

Structured triangular meshes

It is difficult to write multiwavelets on triangles as a tensor products. Therefore, we use a bivariate form that allows appropriate subdivisions.

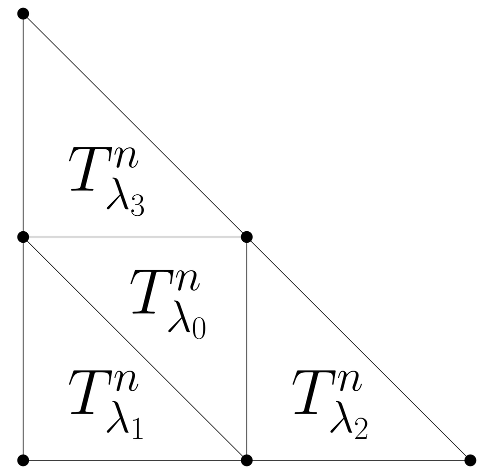
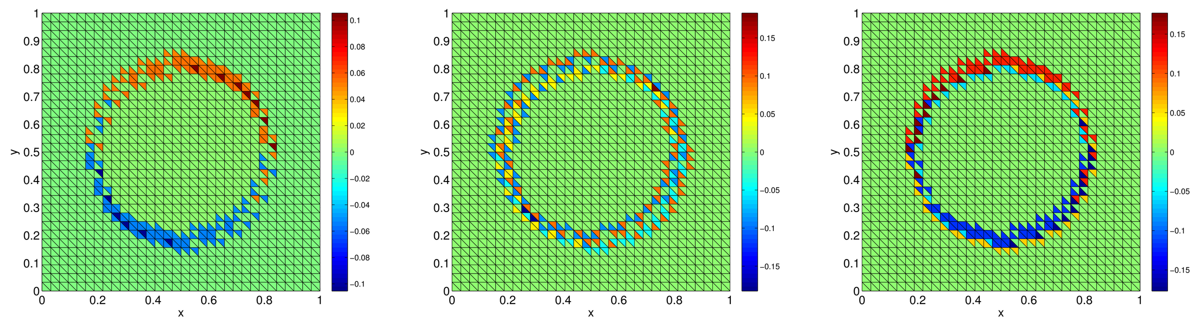
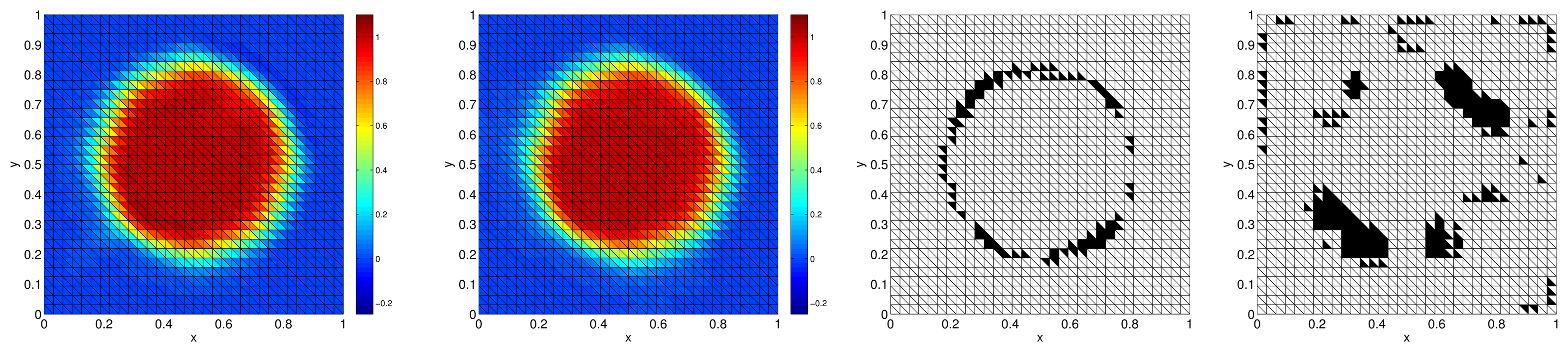


Figure 2: Midpoint subdivision of triangle T_λ^{n-1}.

Detection: use $\tilde{d}_{\ell\lambda}^{m,n-1}$, $m = 1,2,3$, $\ell = 1, \ldots, N_k$. Investigate results for advection equation: circle wave.



(a) $\tilde{d}_{1\lambda}^{1,n-1}$ (b) $\tilde{d}_{2\lambda}^{1,n-1}$ (c) $\tilde{d}_{3\lambda}^{1,n-1}$



(d) $C = 0.9$ (e) Outlier (f) $C = 0.9$ (g) Outlier

Figure 3: Multiwavelet coefficients of initial condition, final-time approximations and corresponding detected troubled cells, $T = \sqrt{2}$, structured triangular mesh based on 32×32 rectangles, $k = 1$.

References

[1] M.J. Vuik. *The use of multiwavelets and outlier detection for troubled-cell indication in discontinuous Galerkin methods.* PhD thesis, TU Delft, 2017.

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