

Shock detection for discontinuous Galerkin solutions using multiwavelets

Thea Vuik*, Jennifer Ryan

*M.J.Vuik@tudelft.nl

Introduction

Climate models are used for various purposes, such as computing weather forecasts, predicting hurricanes, and to look at climate changes, ocean circulations, etc. It is very important to find accurate results. In order to create such simulations, models based on nonlinear PDE's are constructed and solved via numerical methods, such as the discontinuous Galerkin method (DG).

Aim

In practical applications, initial conditions may contain discontinuities, or the solution of a nonlinear equation may develop a discontinuity at a certain time. An example is given in Figure 1.

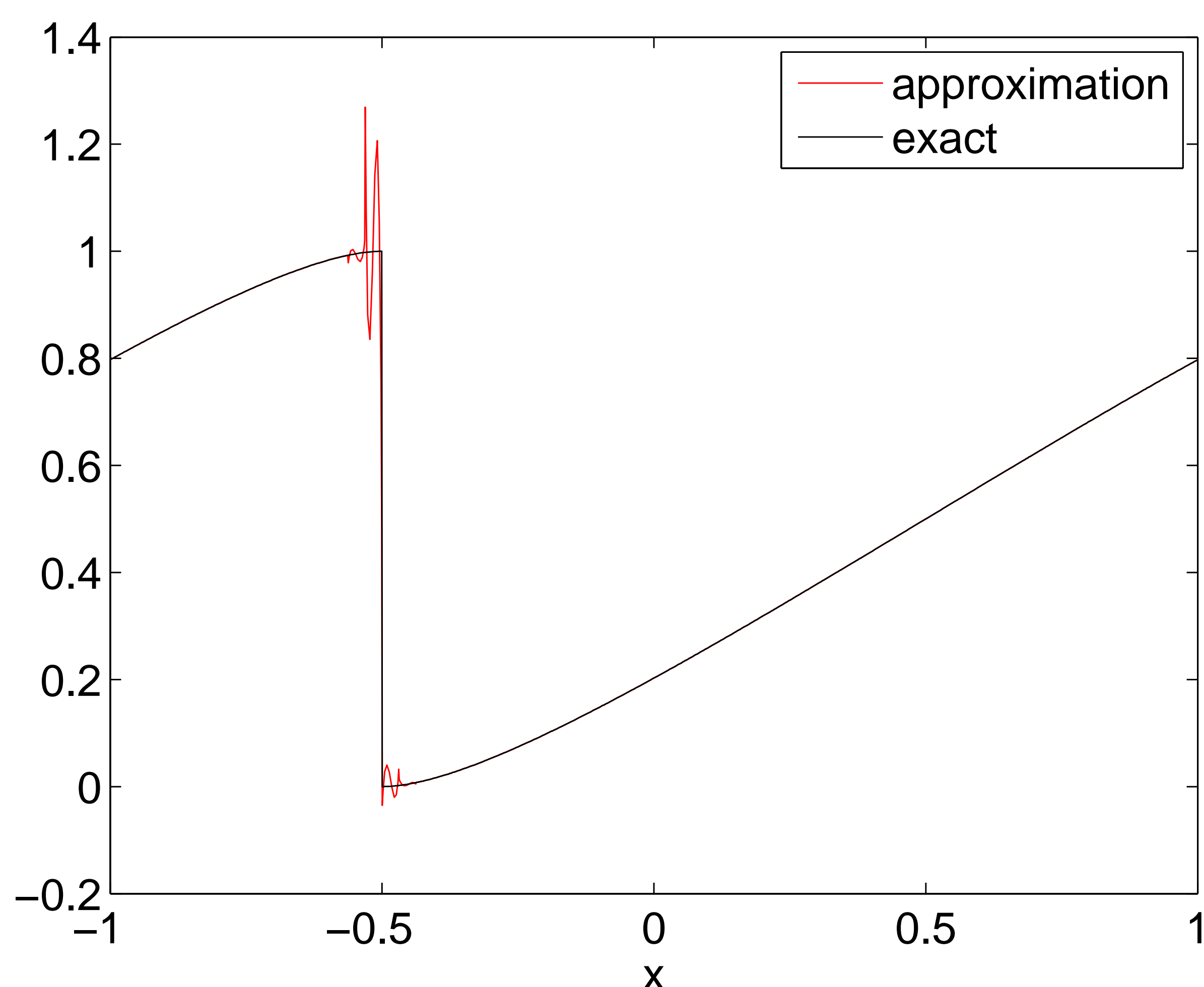


Figure 1: Solution of inviscid Burgers' equation, $T = 1$, 64 elements

Many numerical methods apply a limiter near the discontinuities, to reduce the spurious oscillations that develop in the discontinuous regions. Currently, one such limiter is the moment limiter for DG. In 1D, the results are fine, but in 2D, the moment limiter has problems with shock detection.

In this poster, we present a new shock detector, based on multiwavelets.

Mathematical details

- We use the inviscid Burgers' equation:

$$u_t + \left(\frac{u^2}{2}\right)_x = 0, u^0(x) = \frac{1}{2} + \frac{1}{2} \sin(\pi x), x \in [-1, 1];$$
- DG approximation: $u_h = \sum_{\ell=0}^k u_j^{(\ell)} \phi_{\ell}(\xi), \xi = \frac{2}{\Delta x}(x - x_j);$
- Time stepping: third order TVD RK;
- Multiwavelets: $u_h = S^0(x) + \sum_{m=0}^{n-1} D^m(x), 2^n$ elements.

Results

Using the multiwavelet decomposition, we can easily detect the shock (Figure 2). Contribution D^5 is maximal in this region.

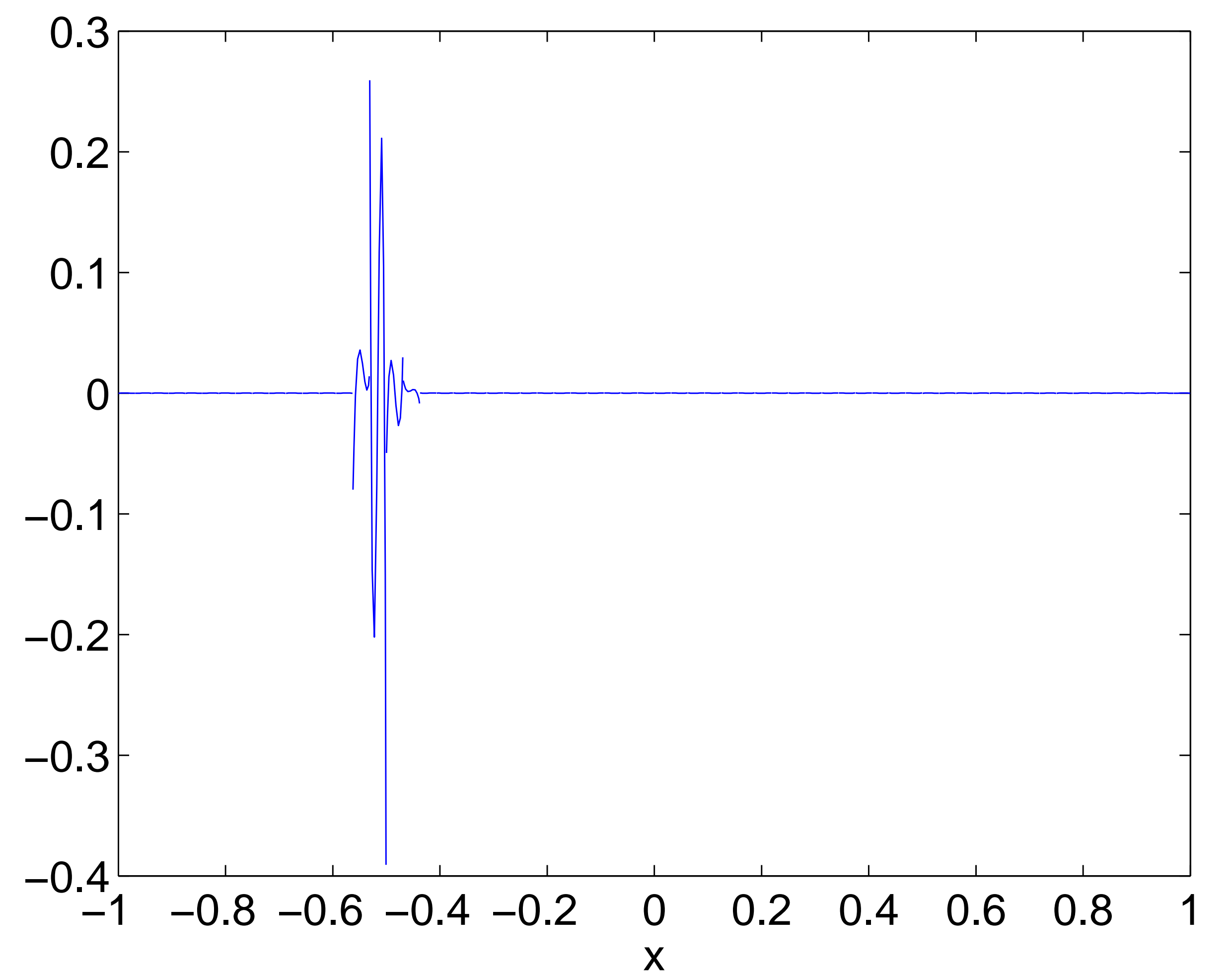


Figure 2: Multiwavelet contribution $D^5(x)$, belongs to Figure 1

Next, we combine the multiwavelet shock detector with the moment limiter, which is applied in the shock region only. Results can be seen in Figure 3.

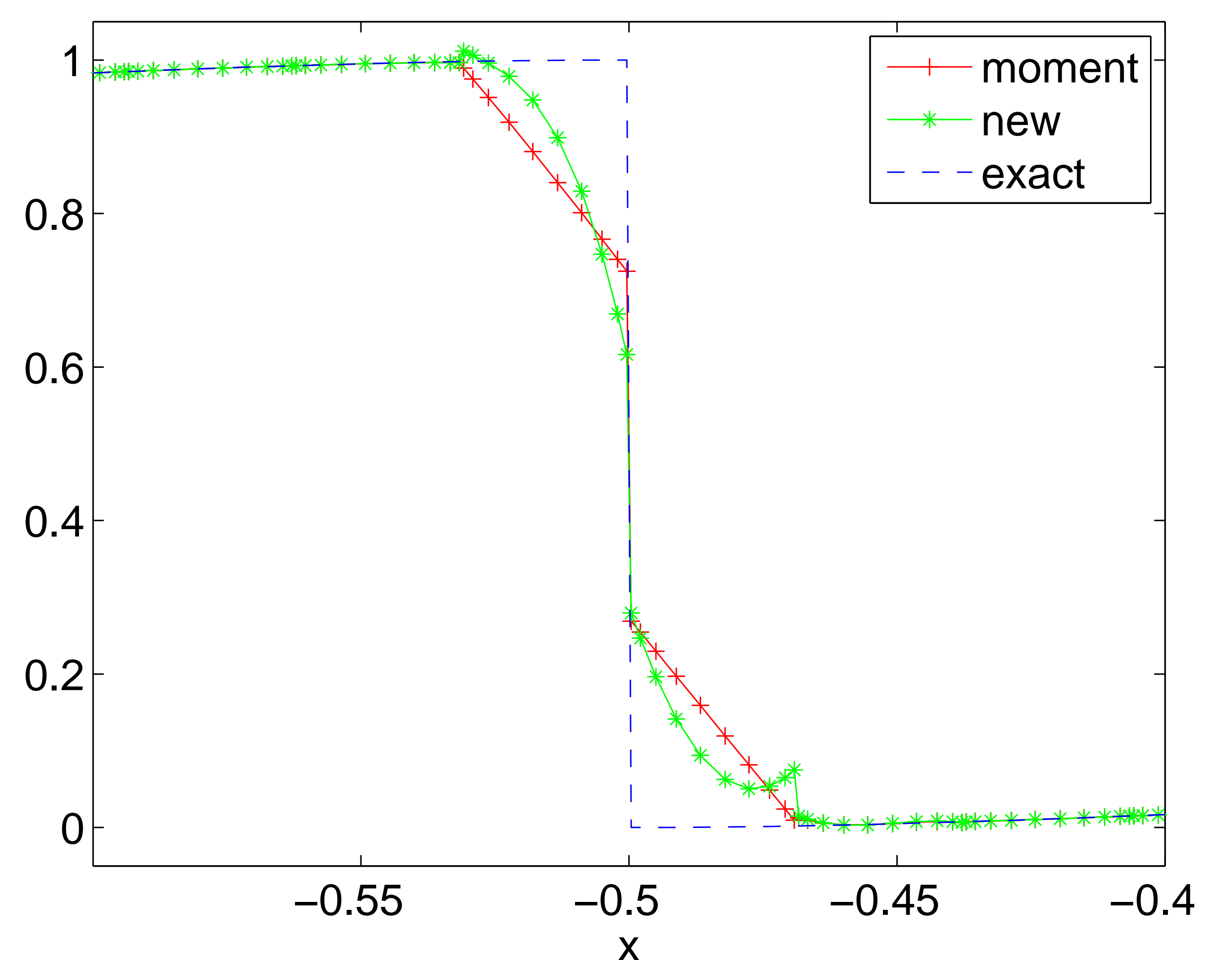


Figure 3: New limiter: moment limiter in shock region

Conclusion

Multiwavelets are a promising shock detector for DG.

