Delft University of Technology

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Applied Finite Elements
2008/2009 Take Home Exams, Second Series

All exercises and references are taken from the book *Numerical Methods in Scientific Computing*.

1. Consider the flat thin plate of Figure 5.5 of the book, clamped in Γ_1.
   Let u, v, t_1, t_2, E, ν, A and B be defined as in Section 5.4.3 of the book.
   Let assume a body force $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ is applied.
   Then the potential energy can be written as:

$$P(\mathbf{u}) = \frac{1}{2}\int\limits_{\Omega} (\sigma_{xx}\varepsilon_x + \sigma_{yy}\varepsilon_y + \gamma_{xy}\tau_{xy})\, d\Omega - \int\limits_{\Omega} \rho(b_1 u + b_2 v)\, d\Omega - \int\limits_{\Gamma_2} (t_1 u + t_2 v)\, d\Gamma\,, \qquad (1)$$

   with ρ the density.

   (a) Derive the Euler-Lagrange equations on Ω and the natural boundary condition on Γ_2.
   Hint: assume $u = \hat{u} + \varepsilon\phi$, $v = \hat{v} + \varepsilon\psi$

   (b) Which smoothness conditions must be satisfied by ϕ and ψ, and which conditions should hold on the boundaries.

   (c) Apply Ritz's method and derive a set of linear equations for the coefficients.
   Hint: use the same type of basis functions for both displacement components.

   (d) The finite element method is used to construct the basis functions.
   The internal element matrix and vector are split into parts referring to the u and v components in the following way:

$$\mathbf{S} = \begin{bmatrix} S_{uu} & S_{uv} \\ S_{vu} & S_{vv} \end{bmatrix}, \qquad \mathbf{f} = \begin{bmatrix} f_u \\ f_v \end{bmatrix}. \qquad (2)$$

   Express the elements of each submatrix and sub vector in terms of the basis functions.

   (e) We restrict ourselves to linear triangles.
   Compute the elements of the submatrices and subvectors.
   The parameters E and ν are constants, but $\mathbf{b}$ and $\mathbf{t}$ depend on space.

   (f) Compute the element matrix and element vector for the boundary element on Γ_2.

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