the use of SIMPLE-type preconditioners in maritime CFD applications

Christiaan Klaij

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Overview

Problem description: maritime applications require large, unstructured grids

- matrix-free approach for coupled Navier-Stokes system
- only compact stencil for velocity and pressure sub-systems

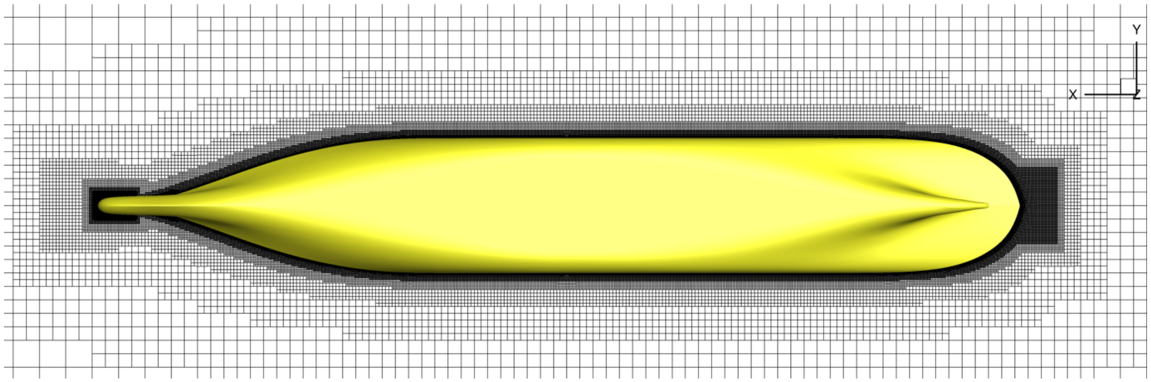
Proposed solution: solve coupled system with Krylov subspace method and SIMPLE-type preconditioner

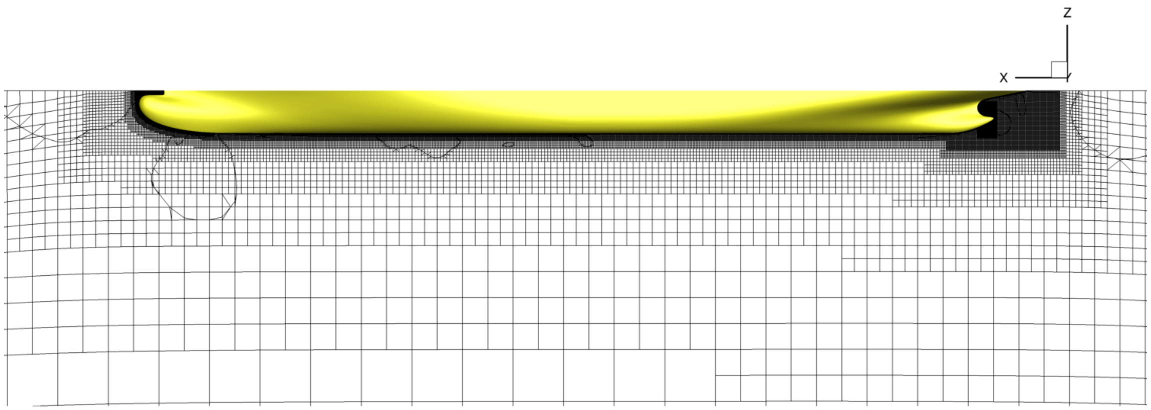
- coupled matrix not needed to build preconditioner
- special treatment of stabilization

Evaluation: SIMPLE as solver versus SIMPLE as preconditioner

- reduction in number of non-linear iterations and wall-clock time?

Container vessel (unstructured grid)





RaNS equations

k-ω turbulence model

$y^+ \approx 1$

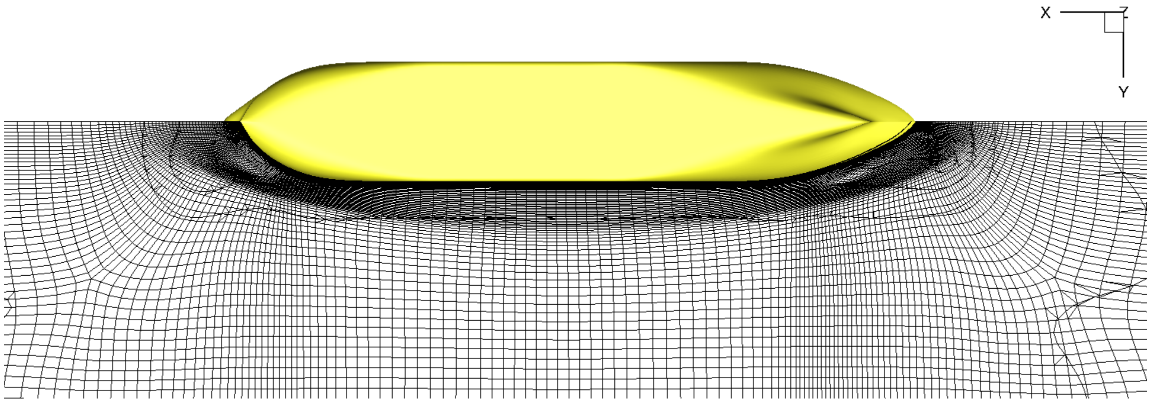
Model-scale:

$\mathrm{Re} = 1.3 \cdot 10^7$

13.3m cells

max aspect ratio $1 : 1600$

Tanker (block-structured grid)

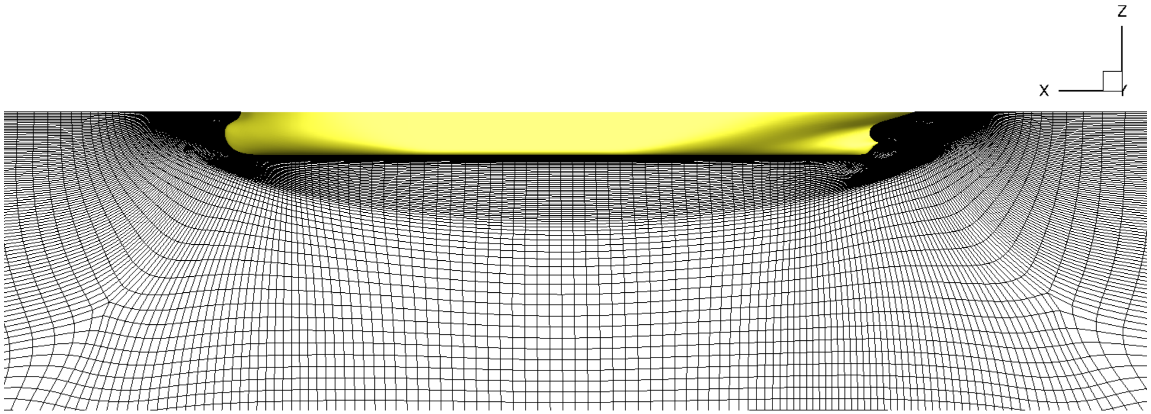


Model-scale:

$\mathrm{Re} = 4.6 \cdot 10^6$

2.0m cells

max aspect ratio $1 : 7000$

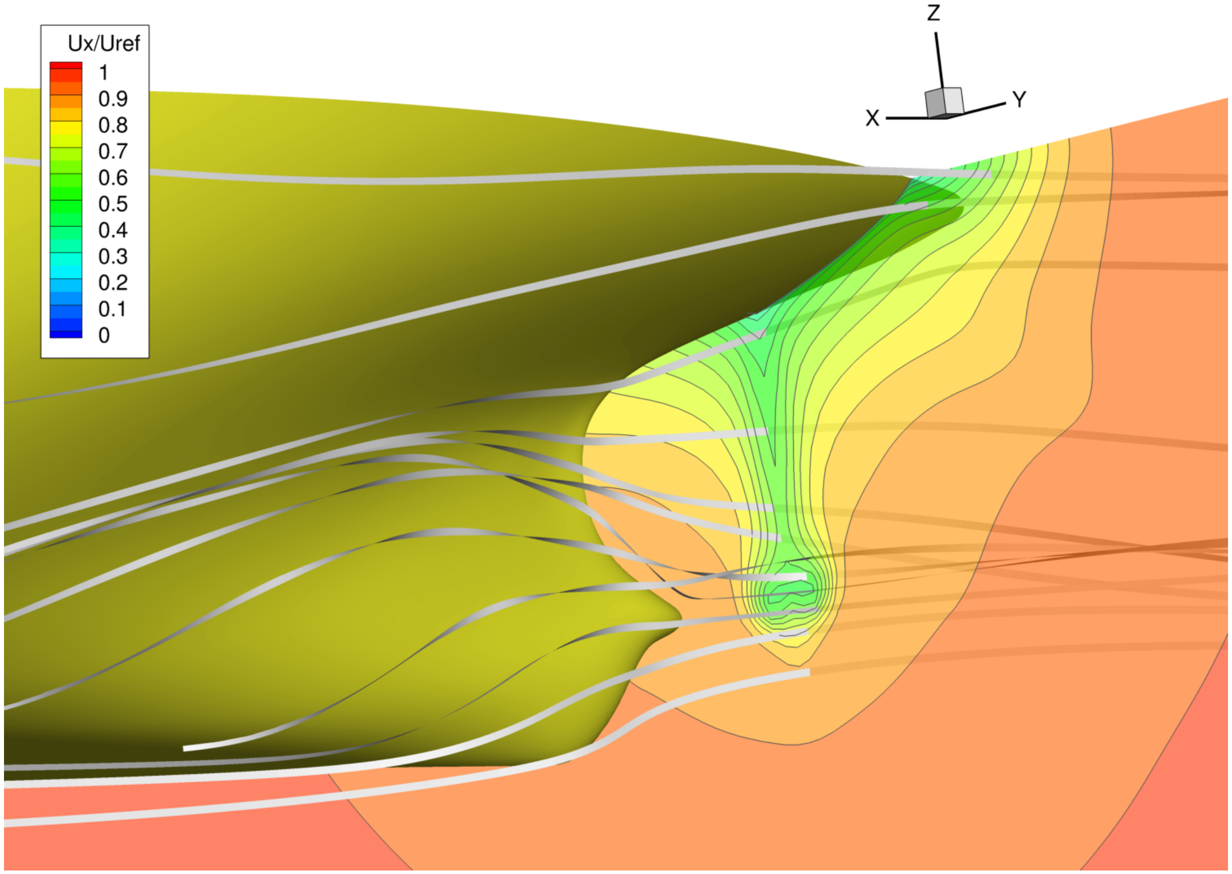


Full-scale:

$\mathrm{Re} = 2.0 \cdot 10^9$

2.7m cells

max aspect ratio $1 : 930\,000$



streamlines around the stern and the axial velocity field in the wake.

Discretization

Co-located, cell-centered finite volume discretization of the steady Navier-Stokes equations with Picard linearization leads to linear system:

$$\begin{bmatrix} Q_1 & 0 & 0 & G_1 \\ 0 & Q_2 & 0 & G_2 \\ 0 & 0 & Q_3 & G_3 \\ D_1 & D_2 & D_3 & C \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ p \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ g \end{bmatrix} \qquad \text{for brevity: } \begin{bmatrix} Q & G \\ D & C \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$$

with $Q_1 = Q_2 = Q_3$.

$\Rightarrow$ Solve system with FGMRES and SIMPLE-type preconditioner

MARIN

SIMPLE-method

Given u^k and p^k:

1. solve $Qu^* = f - Gp^k$
2. solve $(C - DQ^{-1}G)p' = g - Du^* - Cp^k$
3. compute $u' = -Q^{-1}Gp'$
4. update $u^{k+1} = u^* + u'$ and $p^{k+1} = p^k + p'$

with approximation $Q^{-1} \approx \mathrm{diag}(Q)^{-1}$.

$\Rightarrow$ “Matrix-free”: only assembly and storage of Q and $(C - DQ^{-1}G)$. For D, G and C the action suffices.

SIMPLER: additional pressure prediction

Given u^k and p^k, start with a pressure prediction:

1. solve
   $$(C - D\,\mathrm{diag}(Q)^{-1}G)p^* = g - Du^k - D\,\mathrm{diag}(Q)^{-1}(f - Qu^k)$$
2. continue with SIMPLE using p^* instead of p^k

Constraints

Compact stencils are preferred on unstructured grids:

- neighbors of cell readily available; neighbors of neighbors not

Also preferred because of MPI parallel computation:

- domain decomposition, communication

Compact stencil?

✓ Matrix $Q_1 (= Q_2 = Q_3)$

✗ Stabilization matrix C

$\Rightarrow$ modify SIMPLE(R) such that C is not required in l.h.s.

Treatment of stabilization matrix

- In SIMPLE, neglect C in l.h.s. of pressure correction equation

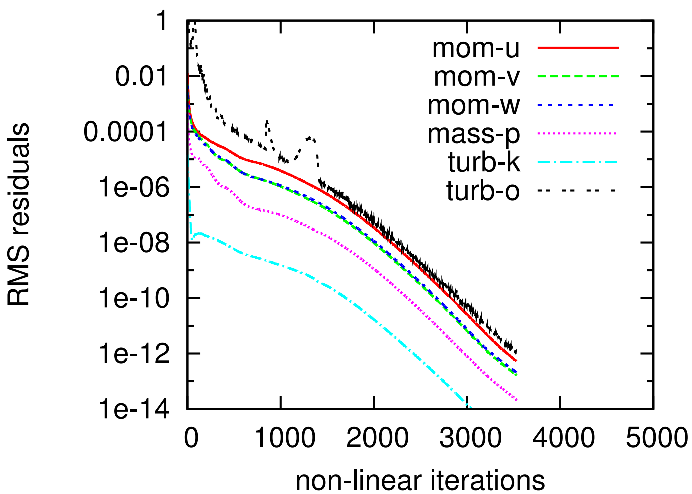
$$(C - D\text{diag}(Q)^{-1}G)p' = g - Du^* - Cp^k$$
$$\Downarrow$$
$$-D\text{diag}(Q)^{-1}Gp' = g - Du^* - Cp^k$$

- In SIMPLER, do *not* involve the mass equation when deriving the pressure prediction p^*

$$(C - D\,\text{diag}(Q)^{-1}G)p^* = g - Du^k - D\,\text{diag}(Q)^{-1}(f - Qu^k)$$
$$\Downarrow$$
$$-D\,\text{diag}(Q)^{-1}Gp^* = -D\,\text{diag}(Q)^{-1}(f - Qu^k)$$

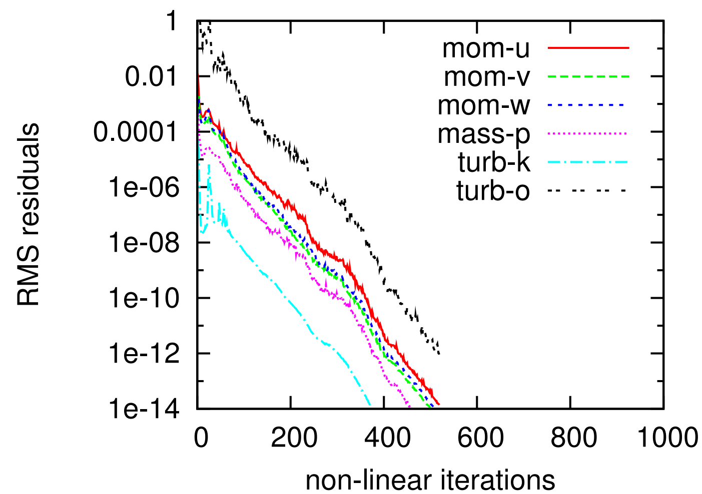
Example of iterative convergence

SIMPLE



$$\omega_u = 0.2 \quad \omega_p = 0.1$$

KRYLOV-SIMPLER



$$\omega_u = 0.8 \quad \omega_p = 0.3$$

Container vessel

Tables show number of non-linear iterations and wall clock time needed to converge to machine precision, starting from uniform flow.

Model-scale $\mathrm{Re} = 1.3 \cdot 10^7$, max cell aspect ratio $1 : 1600$

grid	CPU cores	SIMPLE		KRYLOV-SIMPLER	
		# its	Wall clock	# its	Wall clock
13.3m	128	3187	5h 26mn	427	3h 27mn

Tanker

Model-scale $\mathrm{Re} = 4.6 \cdot 10^6$, max cell aspect ratio $1 : 7000$

grid	CPU cores	SIMPLE		KRYLOV-SIMPLER	
		its	Wall clock	its	Wall clock
0.25m	8	1379	25mn	316	29mn
0.5m	16	1690	37mn	271	25mn
1m	32	2442	57mn	303	35mn
2m	64	3534	1h 29mn	519	51mn

Full-scale $\mathrm{Re} = 2.0 \cdot 10^9$, max cell aspect ratio $1 : 930\,000$

grid	CPU cores	SIMPLE		KRYLOV-SIMPLER	
		its	Wall clock	its	Wall clock
2.7m	64	29 578	16h 37mn	1330	3h 05mn

Summary

- Coupled Navier-Stokes system has 10 blocks, we only assemble and store 2, for the others their action suffices.
- The stabilization matrix C has a wide stencil, we changed SIMPLE(R) so that its assembly and storage is not needed.
- For maritime applications, we find that SIMPLE(R) as preconditioner reduces the number of non-linear iterations by 5 to 20 and the CPU time by 2 to 5. Greatest reduction found for most difficult case.

Summary cont'd

C.M. Klaij and C. Vuik, SIMPLE-type preconditioners for cell-centered, colocated finite volume discretization of incompressible Reynolds-averaged Navier-Stokes equations, *Int. J. Numer. Meth. Fluids* (to appear), 2012.

Contains details on:

- academic benchmark cases (backward-facing step, lid-driven cavity, flat plate)
- choice of relaxation parameters
- choice of linear solvers and relative tolerances for sub-systems
- other variants (MSIMPLE and MSIMPLER)
- ...