MSc project proposal
Model-Order Reduction of Immersed Finite Element Systems

Supervisors

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Project description

Computational fluid flow simulations are utilized in many applications in science and engineering. For instance, in automotive or aviation (*e.g.*, for passenger cars or unmanned aerial vehicles), engineers need to assess aerodynamic properties of designs for a variety of operating scenarios. The ability to perform fast, even real-time, simulations is critical in such cases, as multiple configurations and flow conditions need to be simulated for tasks such as predicting performance, optimization, uncertainty quantification, and formulation of optimal control strategies. At the same time, it is important that these simulations are accurate and account for the complex geometries that the fluid is flowing around. The need for fast and accurate simulations forms two challenges:

- Mesh generation: Generation of a boundary-fitted mesh for a domain around a complex geometry is a computationally expensive and error-prone process requiring significant manual intervention. The process is recurrent when multiple configurations need to be analyzed.
- Computational expenses: Transient simulations of unsteady flows are computationally expensive. This is exacerbated in case multiple flow conditions (*e.g.*, velocities and directions) must be assessed.

Both challenges can be tackled by combining immersed spline-based finite elements (Figure 1) with reduced-order modeling (ROM, Figure2).

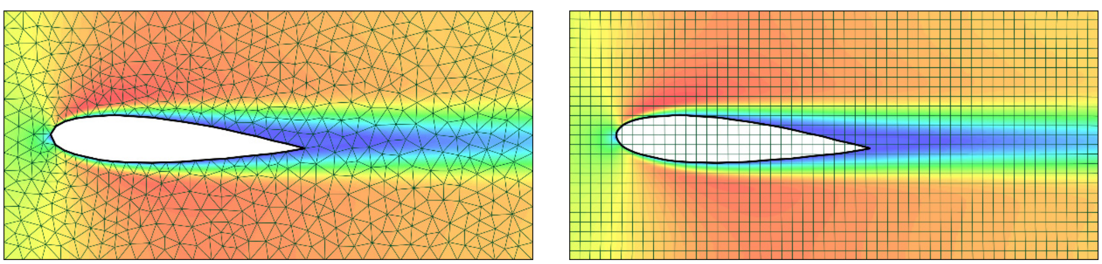

Figure 1: The left figure shows a traditional unstructured mesh, which matches the boundary of the object around which the flow is simulated. The generation of such a mesh can be a laborious procedure, which can be recurrent when multiple configurations must be assessed. The right figure shows an immersed mesh. In this mesh, the background is discretized independently from the geometry around which the flow is computed. Because the generation of the mesh is trivial, immersed finite element methods provide a robust framework for fully automated analyses of complex geometries and enable the application of highly-efficient spline-based (isogeometric) function spaces. The application of immersed methods does require tailored quadrature/integration rules on elements that intersect boundaries of the geometry and a special treatment to resolve ill-conditioning problems in the resulting systems. [1-3]

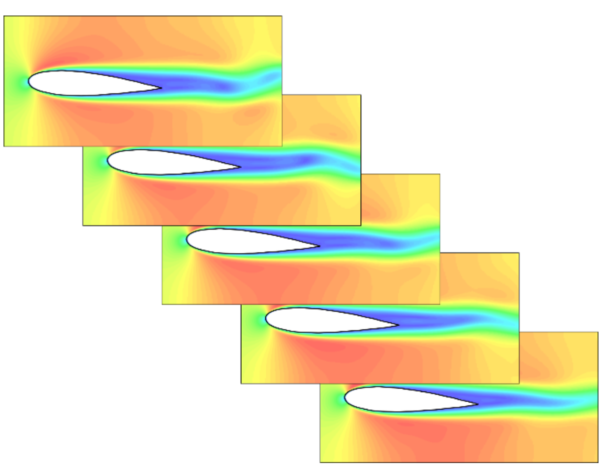

Figure 2: The figure illustrates snapshots of a flow problem which can be used in ROM. In finite elements, every basis function entails a degree of freedom, resulting in very large systems and a large computational cost for transient problems that require large numbers of timesteps. With ROM, the number of degrees of freedom is reduced, and the solution is approximated by a linear combination of a finite number of *modes*, which are often derived from snapshots of a preliminary simulation. Since the number of modes is much smaller than the number of basis functions, this reduces the cost of the simulation. Nonlinear problems generally require quadrature at each timestep, such that besides a reduction of the basis, also a reduction of the quadrature routine is required. [4]

Combining both techniques is not trivial, and in this MSc project we aim to answer the following research questions:

- The first step in ROM is a reduction of the basis. These reduction techniques are generally rooted in linear algebra (e.g., Krylov-based, modal, or POD/SVD techniques). Immersed and spline-based (isogeometric) systems can be strongly ill-conditioned, however [2]. Therefore, we need to assess the suitability of existing basis reduction techniques for these systems.
- For nonlinear and unsteady problems, reduced quadrature is needed for inexpensive computations of system matrices at each timestep or in each nonlinear iteration. Such quadrature techniques are referred to as hyperreduction, with DEIM being the most prominent method [5,6]. While smooth splines already require significantly less integration points than standard finite elements, immersed methods require specialized quadrature for cut elements and for weak enforcement of boundary conditions [2]. An additional challenge is assuring well-posed systems in combination with the reduced basis of the immersed formulation. Therefore, the second goal is the assessment of existing reduced-quadrature techniques for immersed isogeometric systems.
- (Optional) If time allows, we want to apply the developed technique to a problem of computational aeroacoustics, which studies the sound generated by airflow. In computational aeroacoustics, very small scales need to be resolved by the simulation to capture separate vortices, which play an important role in the production of aerodynamic sound. We are mainly interested to investigate the number of snapshots and the size of the reduced space that is required to capture aeroacoustic effects.

Planning

We will start with a literature study of immersed methods and ROM. There has already been one study carried out in this direction by the university of Lausanne, providing valuable insights for this project [7]. For computations, we will make use of the open-source framework Nutils [8], in which all operations required for immersed spline-based finite elements are available. For large simulations we can make use of the DelftBlue supercomputer and the HPC12 cluster of the Flow Physics and Technology department.

References

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