An implicit numerical model for a transport equations with given flow field

Supervisor: Johan Romate (Shell/TUD)

Problem background
In reservoir simulation equations for conservation of mass, momentum and energy of fluids moving through a porous medium are solved. Typically, there are multiple fluid phases, for instance gas, oil, and water. The complexity of the equations and the various scales of the phenomena involved have led to a variety of numerical methods used, but most software systems currently in use have at least the possibility to solve the equations implicitly, for enhanced stability properties.
In addition to the flow solver extra transport equations may be solved. These transport equations are used for monitoring purposes, or for adding extra physics, components, or properties. Given the velocity field $\mathbf{u}$ of a fluid in a porous medium, an example transport equation for a species or tracer c is

$$c_t + \nabla \cdot (c\mathbf{u}) = \nabla \cdot (\mathbf{D}_{diff} \nabla c) + s(c). \tag{1}$$

Such additional transport equations use the flow field solved by the flow solver, but have their own dynamics otherwise (in the form of source terms, diffusion terms, etc.). It may be desirable to also solve these transport equations implicitly in some form. The most straightforward method is to take everything in the equations implicitly, i.e. at the time level to be solved, but that is most likely not the most efficient way.

Assignment
In this assignment the goal is to develop a good and efficient numerical model for these transport equations, which can be used in combination with an implicit flow solver. This may be a fully implicit method or a partially implicit method. A fully explicit method is normally not desirable due to the severe time step limitations in explicit discretizations of the diffusion term. To combine this with e.g. an explicit high-resolution method for the advection term a fractional step method may be investigated.
The model needs to be developed first in the 1D case, and if possible extended to the 2D case. A further extension is to multiple transport equations simultaneously.

The assignment roughly consists of the following parts:

1. Literature study.
2. Development of an efficient numerical model.
3. Implementation and demonstration of the model.
4. Writing the thesis.