Efficient p-multigrid solvers for Isogeometric Analysis

Isogeometric Analysis [1] can be considered as an extension of the Finite Element Method (FEM) using high-order splines to discretize the weak formulation. As a result, complex geometries with curved boundaries can be represented more accurately. Finding a solution to the resulting linear system of equation remains, however, a challenging task.

Multigrid methods are considered among the most efficient solution techniques, especially for elliptic PDE's (i.e. Poisson type equations). However, the application of (geometric) multigrid solvers to solve linear systems of equations arising in Isogeometric Analysis leads to convergence rates that highly depend on the approximation order p of the basis functions. That is, the higher p is, the more iterations are needed and, hence, the more costly the solution process becomes.

Recently, an alternative solution strategy has been proposed, in which a multigrid hierarchy is constructed based on different approximation orders (i.e. p-multigrid). Combined with a smoother that is based on an ILUT factorization, the resulting p-multigrid method [2] has shown to be very efficient independent of the choice of h and p. Currently, an existing p-multigrid code is available, written in the C++ library G+Smo [3].

Research description

The goal of this master project to apply the multigrid solver within a high performance computing (HPC) framework such that it can be used to solve large linear systems of equations. Access to parallel computer systems (e.g. INSY) is provided. Within this master project the following tasks are foreseen:

- The student is expected to preform a (short) literature study to get familiar with Isogeometric Analysis, p-multigrid methods and the G+Smo library.
- Starting from the existing p-multigrid code, the student has to extend this code to make it suitable for parallel computing. In particular, attention has to be paid in parallelizing the ILUT smoother, or, alternatively, block ILUT smoothers.
- The resulting method is then applied on large scale to solve Poisson type problems and performance (i.e. scalability, timings) should be analyzed.

The student is expected to have basis knowledge of the Finite Element Method (FEM) and an interest in (parallel) programming.

References

1. T.J.R. Hughes, J.A. Cottrell and Y. Bazilevs, *Isogeometric Analysis: CAD, Finite Elements,NURBS, Exact Geometry and Mesh Refinement*, CMAME, 194, 4135 - 4195, 2005
2. R.Tielen, M. Möller, D. Göddeke and C. Vuik, *Efficient p-Multigrid Methods for Isogeometric Analysis*, arXiv: 1901.01685
3. G+Smo (Geometry plus Simulation modules), *http://github.com/gismo*