Msc thesis proposal:

Variance Reduction by the COS Method for PFE (Potential Future Exposure)

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**Background**

Counterparty credit risk (CCR) is the risk that a party to an over-the-Counter derivatives contract (such as interest rate swap or currency swap) may fail to perform on its contractual obligations, causing losses to the other party.

A common measure for the quantification of the CCR is the so-called Potential Future Exposure (PFE), which is defined as the 97.5% quantile of the future exposure distribution of a netting set or of a higher-level portfolio at a future time point, as illustrated by Figure 1.



Figure 1 Illustration of the future exposure distribution

The netting set is a portfolio of trades, whereby the mark-to-market (MTM) prices of the trades are allowed to be netted based on a bilateral netting agreement between the bank and its counterparty.

The future exposure, i.e. the total positive MtM price, of a netting set is a random variable, since the driving risk factors of the trades (such as interest rates, FX rates, credit spreads, etc) at a future time point are not yet known today.

PFE is usually adopted by banks to define the trading limit to control CCR. The standard numerical method, if not the only method, in industry for PFE calculations is the Monte Carlo simulation.

In this MC framework, the risk factors are first simulated according to pre-selected stochastic models, then the simulated scenarios are fed to the pricing functions of the trades to yield their MtM prices at a future time point, and at last we aggregate them up to yield the future exposure distribution of the whole portfolio and return the 97.5%-th quantile as the PFE value.

**Purpose of this thesis research**

Because PFE is a quantile in the tail of the exposure distribution, the MC simulation method requires a large number of simulations is needed to achieve an acceptable level of accuracy.

In practice, in general it is not possible to afford the number of simulations required by convergence. Thus, the MC estimator of the PFE in practice is a stochastic variable with non-vanishing variance.

The purpose of this thesis research is to utilize the Fourier cosine (COS) method to construct a variance reduction technique of the MC estimator of the PFE.

The COS method is an efficient semi-analytical method to solve the inverse Fourier transform. The key insight is the Fourier coefficients can be directly sampled from the the Fourier transform.

A previous thesis at FF Quant has successfully developed the COS method on calculating PFEs for portfolios involving a modest number of risk factors [2].

In the Monte Carlo framework, the Fourier transform of the probability density of a random variable, i.e., the characteristics function, can be estimated using the simulated sample. Using the simulated characteristics function of the credit exposure or another highly correlated variable would allow to construct to an efficient control variable to reduce the variance of the MC estimator of the PFE.

**Contact**

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**Reference**

1. A novel option pricing method based on Fourier-cosine series expansions. F. Fang and C. W. Oosterlee. SIAM J. Sci. Comput.,31(2):826-848, 2008
2. [[2311.12575] Fast calculation of Counterparty Credit exposures and associated sensitivities using fourier series expansion (arxiv.org)](https://arxiv.org/abs/2311.12575)
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