Fast Helmholtz solvers for seismic problems

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The Helmholtz equation appears in a wide range of science and engineering disciplines in which wave propagation is modeled. Solving this equation numerically is notoriously difficult. A landmark contribution in reducing the computational complexity is the publication of the Complex Shifted Laplace preconditioner by the TU Delft 10 years ago. This preconditioner is successfully used by scientists and practitioners. Also Shell has implemented this in their software to analyze seismic data.

Seismic data represent one of the main sources of information on the Earth's interior. To interpret seismic data, Shell generally rely on the solutions of the elasto-dynamic wave equation. This notably allows us to synthetize a seismic experiment that consists of the recording of the wave fields due to an excitation at specific receiver locations. Because a seismic data set consists of thousands to hundreds of thousands of shot gathers, thousands to millions of solutions of the wave equation are required during an imaging algorithm. To simulate a modern active seismic experiment, we need to solve the wave equation in a domain of about twenty to thirty kilometers in each lateral direction and about ten kilometers in depth. Depending on the frequency content of the source excitation, this leads to a discretized domain of hundred to thousand points in each direction when we use a classic finite-difference scheme. Consequently the seismic simulation has a significant computational cost. Depending on the applications, we are therefore looking for the most efficient numerical approach. The time-domain wave equation is relatively easy to use since standard time-marching approaches, leading to explicit schemes, are efficient. In fact, when we need to simulate the complete band- limited time response, the complexity of the time-marching algorithm is optimal. However, certain imaging algorithms may only require the responses at discrete frequencies. In these cases, we may wonder if solving the frequency domain wave equation, i.e. the Helmholtz equation in the acoustic case, would be more efficient.

In recent work the Scientific Computing group at TU Delft has developed an algorithmic extension that further reduces the computational burden. It is based on the Complex Shifted Laplace preconditioner enhanced with the multi-level Krylov approach. In this method, a Flexible Krylov method is used. The preconditioner is a combination of the complex shifted Laplace preconditioner and a coarse-grid acceleration. Various ways are known to use this combination. Rigorous Fourier Analysis is used to investigate the spectrum of a two-grid variant of these methods. It appears that most of the eigenvalues are now clustered around one. Application of the method to industrial problems show a decrease of the needed wall-clock time with a factor 10 in comparison with the complex shifted Laplace preconditioner. In this MSc thesis, we will focus on the following topics:

- The Helmholtz solver used by Shell will be extended by the multi-level Krylov approach based on a matrix-free implementation. The exact implementation is unclear yet, since a matrix- free deflation operator still need to be designed.

- The efficiency of the enhanced solver will be showed by realistic seismic experiments with grid sizes of typically 400 x 400 x 100 (corresponding to a spatial domain of 40km x 40 km x 10 km).
- The solvers are investigated for anisotropic problems. Benchmark problems can be solved by e.g. Comsol Multiphysics.
- A more-detailed comparison between the Helmholtz solver based on the Complex Shifted Laplace preconditioner with the time-domain solvers used by Shell will be carried out.