Msc thesis proposal:

Pricing European and Barrier Options under SABR Model via Trigonometric Expansion

Fang Fang[[1]](#footnote-2)

**Background**

In FF Quant Advisory (FFQ) there has been a series of research projects going on, focusing on extending the Fourier Cosine method (COS) to multi-dimensions. This Msc thesis project is part of the series. The subject of this thesis project is about fast and efficient valuation of barrier options under SABR model.

When it comes to pricing barrier options, stochastic volatility models are more suitable than Black-Scholes model that is based on constant volatility assumption, in the sense that the implied volatility surface can be fit very well. SABR model is a stochastic volatility model popular in industry.

In one of the completing Msc thesis projects at FFQ, we have tested the idea of solving the Partial Differential Equation (PDE) for the characteristic function (ch.f) of the survival density function using *trigonometric expansions*, under Heston’s model, which is another popular stochastic volatility model among practitioners. The expansion coefficients are solved via solving the linear system resulted from inserting the expansion into the corresponding PDE.

The trigonometric expansion we referred to here is the resulting expansion from integrating out a Fourier series expansion of the first order derivative of the target function.

In literature there are also increasing number of research papers on the application of deep learning method in derivative pricing, e.g., deep hedging. Though deep learning can induce powerful representations, with multiple layers of neurons, these models are generally intractable or untransparent.

Regarding the machine learning methods, there have been new classes of neuron networks based on Fourier representations. E.g., Fourier neural networks and Fourier neural operators have been proposed to solve partial different equations ([1], [2], [3]).

The valuation problem of options under SABR model is more difficult than under Heston’s model, as there exists no analytical or semi-analytical solution. The Hagan formula is popular in industry for approximation of European option prices, but only within a certain range of the parameters. Existing numerical methods one can rely on are Monte Carlo simulation and finite difference method for directly solving the PDEs. Both are, however, time consuming, and thus, are not suitable for on-the-fly calculations, or for serving to generate reference values for machine learning.

**Challenge**

There are two challenges in this research.

The first one is to construct the cosine or sine expansion structure, according to boundary and initial conditions of the PDE.

The second challenge is to extend the number of dimensions by including the model parameters as variables of the function. When the number of dimensions is high, there is “ the curse of dimensionality”.

To tackle the second challenge, we propose to apply the dimension reduction technique developed in FFQ (such as [4]), the so-called COS-CPD method.

**The goal and content of this thesis**

We aim at achieving the following objectives:

* Take the same route as the currently completing Msc thesis, to construct a trigonometric expansion for the option pricing function under SABR model;
* Apply the COS-CPD method to solve the resulting linear system, the solution of which are the coefficients of the trigonometric expansion;
* Formulating the method in terms of a neural network that takes the Fourier basis functions as the neurons.
* Comparing this method with some existing Fourier network and deep learning methods.

**Contact**

If you are interested to enter the field of quantitative finance, this is a very good starting point. Please feel free to contact me directly if this topic is of your interest, or if you would like to learn more details: fang.fang@ffquant.nl or f.fang@tudelft.nl

**Reference**

1. “Fourier neural networks as function approximators and differential equation solvers”, Marieme Ngom and Oana Marin, Statistical Analysis and Data Mining, June 2021, DOI:10.1002/sam.11531.
2. “Fourier neural operator for parametric partial differential equations”, Zongyi Li, Nikola Kovachki, Kamyar Azizzadenesheli, Burigede Liu, Kaushik Bhattacharya, Andrew Stuart, Anima Anandkumar, [**I**CLR 2021 Conference paper](https://ffquantnl.sharepoint.com/sites/FFQuant/Shared%20Documents/00%20Research/2023-12%20Master%20Thesis%20-%20barrier%20option%20pricing%20under%20SABR%20via%20COS-CPD/ICLR%202021%20Conference%C2%A0paper), https:// <https://openreview.net/forum?id=c8P9NQVtmnO>.
3. “On universal approximation and error bounds for Fourier neural operators”, Nikola Kovachki, Samuel Lanthaler, Siddhartha Mishra, Journal of Machine Learning Research, 22 (2021) 1-76.

1. [https://repository.tudelft.nl/islandora/object/uuid:f6ffbcd6-df14-425b-84bb-63e4e105205c](https://repository.tudelft.nl/islandora/object/uuid%3Af6ffbcd6-df14-425b-84bb-63e4e105205c) Dimension reduction techniques for multi-dimensional numerical integrations based on Fourier-cosine series expansion, Zhimin Cheng, 2022-10-21.

**About FF Quant Advisory B.V.**

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We provide quantitative consulting services to banks, insurance companies and other financial institutions. Our expertise include the development, validation and audit of regulatory and non-regulatory risk models and of pricing models for financial instruments.

We are also specialized in researching, developing and testing quantitative toolkits. Other services include, but are not limited to, backtesting of trading strategies, applying machine learning techniques to replace traditional quantitative models, etc.

1. Part-time Assistant professor at the Applied Mathematics Department of TU Delft; Director of FF Quant Advisory B.V. https://fsquaredquant.nl/ [↑](#footnote-ref-2)