Production and Demand Management

C. D'Ambrosio, F. Lacalandra, J. Lellep, K. Vuik, A. Bischi, T. Parriani, E. Martelli, E. de Klerk, A. Marandi, and L. Schewe

1   Optimal Oil Wells Placement

The optimal oil wells placement problem, a crucial problem in reservoir engineering, consists in determining the optimum number, type, design, and location of oil wells to optimize the hydrocarbon production and the drilling costs.

C. D'Ambrosio
CNRS, École Polytechnique, Institut Polytechnique de Paris, Palaiseau, France

F. Lacalandra
QuanTek, Bologna, Italy

J. Lellep
University of Tartu, Tartu, Estonia

K. Vuik
Delft University of Technology, Delft, Netherlands

A. Bischi (✉)
Skolkovo Institute of Science and Technology, Moscow, Russian Federation
e-mail: a.bischi@skoltech.ru

T. Parriani
Optit Srl, Bologna, Italy

E. Martelli
Politecnico di Milano, Milan, Italy

E. de Klerk
Tilburg University, Tilburg, Netherlands

A. Marandi
Eindhoven University of Technology, Eindhoven, Netherlands

L. Schewe
University of Edinburgh, Edinburgh, UK

In industry, the decision to drill a well or not and its location is taken by reservoir engineers trusting their professional expertise. These decisions strongly relate to the understanding of the impact of different influencing engineering and geological parameters. However, such influence is very complex (nonlinear) and changing over time, thus a deep understanding of such phenomena requires more than human experience. Satisfying solutions could be provided by practitioners, but optimization methods can lead to improved configurations.

From a mathematical modeling viewpoint, the number of water injector and producer wells and the number of branches could be represented by integer variables. In addition, continuous variables as wells and branches design in the reservoir, the length of the branches, etc can be considered. The functions to optimize are generally computed from the outputs of a reservoir fluid flow simulator, costly in computational time: the outputs to optimize are the quantities of produced oil and water, and the quantities of injected water, needed for the production).

The two most widely considered objective functions are:

- maximize the quantity of produced oil;
- maximize the revenue of a wells configuration with Net Present Value (NPV) function. This function combines oil revenue, water management (water injection and separation), and drilling costs.

In both cases, given a wells location, the objective function value is provided by a numerical simulator. As we do not have access an analytic formula of the objective function, the problem is modeled as a Black-Box optimization problem. Hence, we have no information about the continuity, differentiability, or convexity of the objective function.

Constraints are generally physical ones, ensuring the practical realizability of the solution and the correct behavior of the simulator. A useful constraint is also the water cut constraint that consists in applying some reactive control on each producer to avoid producing much water which impacts negatively on the NPV. Such reactive control shuts off producers when the water cut, i.e., the ratio between the water rate produced and the sum of water and oil rates produced, is higher than a given threshold. It is also possible to add constraints during the production, e.g., produce a minimal quantity of oil for instance.

Thus, the oil well placement problem can be modeled as a Black-Box MINLP problem, a very challenging problem both from a theoretical and a computational viewpoint. Note also that, as no convexity assumption holds, one should perform some kind of global search to avoid being trapped in local minima.

In practice, nowadays well placement optimization is an iterative procedure that can be divided into the following procedures:

- Using engineering judgment, guess initial well(s) location.
- Use an optimization algorithm based on user-defined decision variables to suggest possible improved well location(s).
- Apply a reservoir response model to report to the optimization algorithm the performance of the proposed well locations.

- Include the effect of uncertainty in reservoir properties, economic factors, etc, which can be an optional step.
- Calculate the objective function (e.g. quantity of produced oil or NPV).
- Repeat steps 2–5 until stopping criteria (set by user) are met.

The approaches to problems 1–5, may differ in the optimization algorithm, reservoir response modeling technique, and available decision variables and constraints.

2 Optimization of the Gas-Lift Process

In the gas industry the key problem is the optimal gaslift with minimum energy consumption. The mathematical complexity of this optimization problem is connected with the matter that the corresponding control problem is of non-regular structure, boundary conditions of this problem include the control parameters. The gaslift method is of special importance at the initial period after the flowing of the oil fields [13, 14, 303]. The motion in the gaslift process is known to obey the hyperbolic nonlinear partial differential equations. Therefore, at gaslift operation of the borehole cavity the problem of optimization with boundary control is of special interest. However, with the original formulation of the problem of optimal control one encounters certain difficulties. The averagings of the hyperbolic equation describing the time profile of motion by the gaslift method are given here [13, 303]. It rearranges a partial derivative equation in the nonlinear ordinary differential equations. The strategy of constructing the objective quadratic functional with the use of the weight coefficients lies in minimizing the volume of the gas injected in the annular space and maximizing the desired volume of the Gas-Liquid Mixture (GLM) at the end of the lifter. In this case, the aim lies in solving the corresponding optimization problem where the volume of the injected gas which is used as the initial data and plays the role of the control action. The impossibility of using the standard methods to construct the corresponding controllers is a disadvantage of this approach. Yet, since at certain time intervals the boundary control is constant, the numerical data obtained can be readily compared with the production data. Using the method of time averaging, the partial derivative equations of motion of gas and GLM motion proposed in [13] are rearranged in the ordinary differential equations. The problem of optimal boundary control with the quadratic functional is formulated on the basis of the above considerations. The results obtained can be used to control the gaslift borehole cavity at oil extraction. For solution of the considered problem of boundary controls, the gradient method [303] is modified by describing the corresponding Euler-Lagrange equations [61].

3   Total Gas Recovery Maximization

In the short term operation, the most important problem is related to the total gas recovery maximization. In order to withdraw as much natural gas from a reservoir as possible, one option is to use waterflooding. This leads to the problem of finding an optimal water injection rate with respect to different objectives, such as the maximal ultimate recovery, or the total revenues. Indeed there are several objective functions due to different aspects of the problem.

Modeling and algorithmic considerations:

Consider two wells drilled on the surface of the gas reservoir, one for gas recovery and one for water injection. Therefore, let r(t) denote the withdrawal rate of gas which is bounded by the maximum rate of gas extraction rm(t). Through the water injection, well water is injected into the reservoir at the nonnegative rate s(t). This model assumes a constant g which is the ratio of gas entrapped behind the injected water to the volume of water at any time. The model aims at maximizing the ultimate gas recovery and can be posed in a nonlinear form. Some researchers discuss several other objective functions. For example, the objective function to maximize the present worth value of the net revenues for internal rate of return.

The application of concepts from systems and control theory to oil and gas production is the unifying idea behind the current research theme Production Systems and Subsurface Characterisation and Flow.

Past In the previous years, research and development was focused on three main areas:

1. The innovation of concepts for the hydrocarbons production process. This includes the application of smart wells, advanced, geophysical monitoring techniques, downhole treatment, the separation and conversion of substances and the injection of residuals (waste) [318, 432]. Closed-loop 'measurement and control' concepts from system theory will play an important role;
2. The development of an integrated 'real-time' dynamic simulation, inversion and validation environment for reservoir, well and processing facilities [233]. This environment will be used to test and evaluate newly developed technology from our groups and other sources. This environment is used as a learning environment and for work process analysis and optimisation;
3. Laboratory of innovation. The analysis and testing of methods, techniques and work processes to accelerate the process of innovation in the energy and production sector.

Present Currently, the application of concepts from systems and control theory to oil and gas production is the unifying idea behind the research themes production systems and subsurface characterisation and flow. By means of modelling, monitoring and control, the production systems theme aims at stabilising and optimising production in order to achieve production targets, which are being expected from an operator through long term contracts [145, 227].

Future: Smart Wells and Smart Fields Smart well technology involves down-hole measurement and control of well bore and reservoir flow. Drilling and completion techniques have advanced significantly over the last years and allow for the drilling of complex multi-lateral and extended reach wells, and the installation of down-hole inflow control valves, measurement devices for flow, pressure and temperature, and processing facilities such as hydro-cyclones in the well bore. Smart fields technology, also referred to as 'e-field' or 'digital oilfield' technology involves the use of reservoir and production system models in a closed-loop fashion [146]. The measurements may originate from sensors in smart wells, but could also involve simple surface measurements from conventional wells, or originate from other sources such as time-lapse seismics. Research in smart fields is now focused on the development of concepts and algorithms to improve hydrocarbon production though the use of systems and control theory. Future research will address the reservoir management aspects on time scales from months to many years, and in particular the development of techniques for closed-loop reservoir management. We are also developing methods to speed up the modelling and simulation part an order of magnitude [206]. For this reason we combine fast and robust iterative methods for large linear systems with Model Order Reduction insights originating from Optimal Control research. This combination has already led to very good results [102]. Various groups from the Delft University of Technology, Padua University and EPFL Lausanne collaborate in order to develop a new generation of simulators.

4   Optimal Scheduling of Energy Hubs and CCHP Systems

The future development of electric and thermal energy generation, transport and distribution relies on the exploitation of both conventional and renewable energy sources via a wide variety of energy conversion technologies; on the top of that electric and thermal energy storage could be utilized in order to match the demand with response exploiting more effectively the possible synergies between the installed units.

In this context Combined Heat and Power (hereafter CHP) power plants and engines are particularly attractive due to the higher efficiency when compared to conventional units generating only one energy commodity. CHP units can be classified into two main categories:

- one-degree-of-freedom units feature a single independent operating variable, the load (defined as the current fuel input rate divided by the maximum one), which controls the two energy outputs (e.g., electric and thermal power). As a result, for a certain power plant or engine load, it is not possible to vary the share of the two energy outputs according to customer needs. Examples of one-degree-of-freedom CHP units are internal combustion engines and gas turbines with waste heat boiler, backpressure steam cycles, and combined cycles with back-pressure steam turbine.

- Two-degree-of-freedom units feature two independent operating variables, the load and another one (such as a steam extraction valve) adjusting the share of the two energy outputs. Although these systems are more complex and typically more costly, the second control variable increase the operational flexibility of the unit. Examples are steam cycles with extraction condensing steam turbine (a steam extraction valve controls the steam bled from the turbine and used to provide heat to the customer).

It is worth noting that also more sophisticated units featuring three independent variables exist (e.g. CHP natural gas combined cycle with post firing and extraction-condensing steam turbine). Moreover, looking at the energy outputs, some units can be configured so as to cogenerate cooling power in addition to electricity and heat. Such units are called Combined, Cooling, Heat and Power (CCHP). Examples are units made by an internal combustion engine, a waste heat boiler and an absorption chiller (converting heat into chilling power).

Systems featuring several CCHP or CHP units may be integrated with other units such as boilers, heat pumps, and energy storage systems within so-called Energy Hubs. The sizes may range from few hundreds of kW for buildings to hundreds of MW for industrial users and or district heating networks.

Three main types of challenging optimization problems arise when dealing with such integrated systems:

- short-term scheduling, also called unit commitment,
- long-term operation planning,
- design or retrofit of the energy hub.

The short-term unit commitment problem can be stated as follows:
    Given:

- the considered time horizon (e.g., 1 day, 2 days, 1 week) and an appropriate discretization into time periods (e.g., 1 h, 15 min),
- forecast of electricity demand profile,
- forecast of heating and cooling demand profile,
- forecast of ambient temperature,
- forecast of time-dependent price of electricity (sold and purchased),
- performance maps of the installed units,
- operational limitations (start-up rate, ramp-up, etc.) of units,
- efficiency and Maximum capacity of storage systems;

    optimize the following independent variables:

- on/off of units,
- load of units,
- share among heat and power (only for two-degree-of-freedom units),
- energy storage level (hence charge/discharge rate) in each time period (for each energy storage system);

so as to minimize the operating costs (fuel + operation and maintenance + electricity purchase) minus the revenues from electricity sale for the given time horizon while fulfilling the following constraints:

- energy balance constraints for each time interval, e.g. electric energy, thermal energy, etc.,
- start-up constraints for each time unit, for each unit,
- ramp-up constraints for each time unit, for each unit,
- performance maps relating the independent control variables of the units with their energy outputs (e.g. output thermal power as a function of the load),
- a number of case-specific side constraints, e.g. maximum number of daily turns-on/off, for each unit; precedence constraints between units; minimum time unit permanence in on/off states, for each unit etc.

All constraints, except the performance maps of the units, can be easily formulated as linear equalities or inequalities. Performance maps of units are generally nonlinear and often not convex functions yielding to a nonconvex Mixed Integer NonLinear Program.

Due to the large number of variables, both integer and continuous, commercially available global MINLP solvers are not capable of finding the global optimum within reasonable time limits [404]. Besides genetic algorithms [236] or Tabu search [291] from late nineties or other solutions going from Lagrangian relaxation [57] to heuristic algorithms based on engineering practice for simple problems [46], the most effective approaches are based on the linearization of performance maps so as to obtain a Mixed Integer Linear Program (MILP) [307]. This allows to use efficient MILP solvers, such as Cplex [214] and Gurobi [184], and have better guarantees on the quality of the returned solution [404]. The performance maps of the machines can be linearized using either the convex hull representation [254] or classic piecewise linear approximations [89] of 1D [456] and 2D functions [46]; the latter kept into account also daily storage facing an large increase of computational effort, ranging from two to three orders of magnitude.

The so described problem assumes that forecasts of energy demands and prices are accurate and their uncertainty is limited. If data uncertainty needs to be considered, the short-term scheduling problem can be extended and reformulated either as a two-stage stochastic program [15, 66] or a robust optimization problem with recourse [313, 467].

As an additional challenge, when determining the optimal scheduling of CHP units, it is necessary to take into account of the European Union regulation for high efficiency CHP units [104]. If a CHP unit achieves throughout the whole year a primary energy saving index above a threshold value, incentives are granted. Being a yearly-basis constraint, it poses the need of considering the whole operating year as time horizon when determining the optimal scheduling of CHP units. The same requirement concerns energy hubs featuring seasonal storage systems [161] capable of efficiently storing energy for several months. Since tackling the scheduling problem for the whole year as a single MILP is impracticable, metaheuristics based on time decomposition to reach near optimal solutions in a reasonable amount of

time have been proposed. Bischi et al. [47] proposed a rolling horizon algorithm in which the time horizon is partitioned into weeks. The extension of the MILP model from 1 day to 7 days may imply an increase of computational time from few sec for a single day to tens of minutes for the week (with MILP gap below 0.1%) but it allows to better manage the thermal storage system accounting for the weekly periodicity of the users' demand. Within the rolling-horizon algorithm, the weekly MILP subproblems are solved in sequence from the current week till the end of the year. The yearly-basis constraints related to the CHP incentives are included in each weekly MILP subproblem by estimating the energy consumption and production of the future weeks of the year with the corresponding typical operating weeks (previously determined and optimized). If the yearly basis CHP incentive constraints are not met, the rolling horizon algorithm is repeated considering a higher (less optimistic) energy consumption for the future weeks. Thanks to the decomposition of the operating year into weekly subproblems, the computational time required to optimize the whole year of operation with a tight relative MILP gap (0.1%) ranges from 1 day to 3 days, making the algorithm an effective scheduling and control tool for energy hubs featuring CHP units.

Finally it is worth pointing out that, due to growing industrial interest in the optimal operation of complex energy systems for providing cooling, heating and power (e.g., energy service companies, multi-utilities managing district heating networks as well as power plant operators), several tools are already available on the market [42].

5   The Pooling problem

The pooling problem arises in the chemical process and petroleum industries. It is a generalization of a minimum cost network flow problem where products possess different specifications (e.g. sulphur concentration). In a pooling problem, flow streams from different sources are mixed in intermediate tanks (pools) and blended again in the terminal points. At the pools and terminals, the quality of a mixture is given as the volume (weight) average of the qualities of the flow streams that go into them.

There are three types of tanks: inputs or sources, which are the tanks to store the raw materials, pools, to blend incoming flow streams and make new compositions, and outputs or terminals, to store the final products. According to the links among different tanks, pooling problems can be classified into three classes:

- Standard pooling problem: in this class there is no flow stream among the pools. It means that the flow streams are in the form of input-output, input-pool and pool-output.
- Generalized pooling problem: here, flow streams between the pools are allowed.
- Extended pooling problem: here, the problem is to maximize the profit (minimize the cost) on a standard pooling problem network while complying with con-

straints on nonlinearly blending fuel qualities such as those in the Environmental Protection Agency (EPA) Title 40 Code of Federal Regulations Part 80.45.

There are many equivalent mathematical formulations for the pooling problem, such as P-, Q-, PQ- and HYB- formulations, and all of them may be formulated as nonconvex (bilinear) problems, and consequently the problem can possibly have many local optima. More information about different formulations may be found in [183].

Despite the strong NP-hardness of a pooling problem in general, proved in [11], and even for problems with a unique pool, proved in [12], or with single-flow restriction, proved in [190], there are classes of pooling problems for which algorithms with polynomial running time exist; see e.g. [28, 53, 189, 191]. Furthermore, much progress in solving small to moderate size instances to global optimality has been made since 1978, when Haverly in [193] described the P-formulation and solved small standard pooling problems using recursive linear programming. A common approach is to construct good lower and upper bounds for use in a branch-and-bound framework; see e.g. [147]. To have tighter lower bounds, different methods have been proposed in the literature including Lagrangian approaches [4], (piecewise) linear relaxations [100, 101, 305], modification of polynomial optimization hierarchies [292], and convex nonlinear relaxations [274]. The first software that is developed specifically to solve pooling problems is called APOGEE [305], where the authors make use of an iterative piecewise linear relaxation, of which it is proved in [100] that the first iteration may result in a lower bound far from the optimal value.

Due to the high-complexity, different pooling problem instances have been collected in libraries such as [158], which are used as the test bed to assess the performance of newly developed solvers and algorithms for nonlinear optimization problems; see, e.g., [293, 304, 306].

Two interesting generalizations of the pooling problem are:

- more general networks where other types of units than pools are also present, e.g. units that extract pollutants. Mathematically, this generalisation falls within the framework of so-called wastewater management networks; see e.g. [235].
- treating the network topology as a decision variable, as done in [305].

Network and Storage

R. Schwarz, F. Lacalandra, L. Schewe, A. Bettinelli, D. Vigo, A. Bischi, T. Parriani, E. Martelli, K. Vuik, R. Lenz, H. Madsen, I. Blanco, D. Guericke, I. Yüksel-Ergün, and J. Zittel

1 Gas Pipeline Design

Natural gas is considered by many to be the most important energy source for the future. The objectives of energy commodities strategic problems can be mainly related to natural gas and deal with the definition of the "optimal" gas pipelines design which includes a number of related sub problems such as: Gas stations (compression) location and Gas storage locations, as well as compression station

R. Schwarz · R. Lenz · I. Yüksel-Ergün · J. Zittel
Zuse Institute Berlin (ZIB), Berlin, Germany

F. Lacalandra
QuanTek, Bologna, Italy

L. Schewe
University of Edinburgh, Edinburgh, UK

A. Bettinelli · T. Parriani
Optit, Bologna, Italy

D. Vigo
University of Bologna/Optit, Bologna, Italy

A. Bischi
Skolkovo Institute of Science and Technology, Moscow, Russia

E. Martelli
Politecnico di Milano, Milan, Italy

K. Vuik
Delft University of Technology, Delft, Netherlands

H. Madsen (✉) · I. Blanco · D. Guericke
Technical University of Denmark, Lyngby, Denmark
e-mail: hmad@dtu.dk

design and optimal operation such in [255]. Needless to say these problems involve amount of money of the order of magnitude of the tens of billions EUR and often these problems can be a multi-countries problem. From the economic side, the natural gas consumption is expected to continue to grow linearly to approximately 153 trillion cubic feet in 2030, which is an average growth rate of about 1.6% per year. Because of the properties of natural gas, pipelines were the only way to transport it from the production sites to the demanding places, before the concept of Liquefied Natural Gas (LNG). The transportation of natural gas via pipelines remains still very economical.

From an optimization standpoint, the gas pipeline design problems can be divided in the following main sub problems:

1. how to setup the pipeline network, i.e. its topology;
2. how to determine the optimal diameter of the pipelines;
3. how to allocate compressor stations in the pipeline network;

Typically, the mathematical programming formulations of these optimization problems contain many nonlinear/nonconvex and even nonsmooth constraints and objective functions because of the underlying physics of the gas flows that need to be considered. The classic constraints are the so-called Weymouth panhandle equations, which are a potential-type set of constraints and relate the pressure and flow rate through a pipeline.

As in many other situations, problems 1–3 are a single problem but a divide et impera principium is applied. Therefore the problems 1 and 2 are somehow determined via simulations and normally there are—in the first but also in the second problem—many economic drivers, and also political drivers when many countries are involved. From a technical point of view, problem 3, the allocation of the compressor stations, is probably the most challenging. Because of the high setup cost and high maintenance cost, it is desirable to have the best network design with the lowest cost. This problem concerns many variables: the number of compressor stations which is an integer variable, the pipeline length between two compressor stations, and the suction and discharge gas pressures at compressor stations. This problem is computationally very challenging since it includes not only nonlinear functions in both objective and constraints but, in addition, also integer variables.

In the case of transmission networks, existing infrastructure is already available, but needs to be expanded to increase the capacity. To this end, new pipelines are often built in parallel to existing ones, effectively increasing the diameter. On the other hand, for the exploitation of new gas fields or off-shore transportation, pipeline systems are designed from scratch with no predetermined topology. Capacity planning and rollout has a time horizon of several years. Accordingly, some optimization models consider multiple stages of network expansion. Many of the planning problems are formulated as mixed-integer non-linear programs (MINLP) with integer variables and nonconvex constraints. To solve these models directly, solvers apply outer approximation and spatial branching. Alternatively, the problem functions can be approximated piecewise linearly, yielding a mixed-integer linear program (MILP) formulation. A survey paper concerned with water networks is

also relevant here [90]. Specialized algorithms make use of the fact that certain subproblems with fixed integer variables have a convex reformulation, which can be solved efficiently and used for pruning [212, 356].

The design of pipeline topologies from scratch is solved with a decomposition, where first a topology is fixed heuristically, and improved by local search. The pipeline diameters are then solved separately [366].

In the case that the network has a tree topology, Dynamic Programming has been applied, both for the choice of suitable pipe diameters [366] as well as compression ratios [58].

Another important aspect is how to treat varying demand scenarios. A finite number of different scenarios can be tackled using decomposition techniques [385]. When the network has a tree topology, also robust variants can be reduced to finitely many scenarios [362].

2 District Heating Network Design

In the current energy market context, District Heating (DH) has an important role, especially in countries with cold climate. DH often leverages on Combined Heat and Power (CHP) units, capable to reduce the consumption of primary energy to fulfill a given electric and thermal request, as well as on existing significant sources of heat generated by industrial processes or waste-to-energy heat generation. Additionally, heating networks will need to increase their flexibility in operation due to an increasing mix of renewable sources, both heat sources or green electricity utilized by heat pumps, distributed generation and smart consumers as well as DH operational temperature reduction and heat storage integration [277, 426].

From a management standpoint, the design of the district heating network is a strategic business issue, since it requires large investments due to the cost of materials and civil works for the realization of the network. Proper strategic design of the network (i.e. definition of the most convenient backbone pipelines to lay down) and tactical targeting of most promising potential customers both aims at maximizing the Net Present Value (NPV) of the investment.

Finding the extension plan for an existing (or eventually empty) DH network that maximizes the NPV at a given time horizon is a challenging optimization problem that can be stated as follows. Given:

- A time horizon (e.g., 15 years)
- A set of power plants, with specific operational limitations (maximum pressure, maximum flow rate,. . .);
- An existing distribution network, with information on the physical properties of the pipes (length, diameter,. . .);
- A set of customers already connected to the network with known heat demand;
- A set of potential new pipes that can be laid down;
- A set of potential new customers that can be reached;

find:

1. The subset of potential new customers that should be reached;
2. Which new pipelines should be installed;
3. The diameter of the new pipes

that maximize the NPV.

Research on modelling approaches for representing the behavior of the thermo-hydraulic network through sets of non-linear equations can be found in the literature (see for example [52] and [343]). Solving systems of non-linear equations is difficult and computationally expensive. For this reason, aggregation techniques of the network elements are often used to model large district heating networks, at the expense of some accuracy [258, 259, 272, 450, 452] and [272].

In [25], an integer-programming model is proposed for the optimal selection of the type of heat exchangers to be installed at the users' premises in order to optimize the return temperature at the plant. The authors achieve good system efficiency at a reasonable cost.

Bordin et al. [54] present a mathematical model to support DH system planning by identifying the most advantageous subset of new users that should be connected to an existing network, while satisfying steady state conditions of the thermo-hydraulic system. Bettinelli et al. [42] extend the model proposed by Bordin et al. [54] with the selection of the diameter for the new pipes and a richer economic model that takes into account

- Production cost and selling revenues;
- Cost for installing and activating new network links;
- Cost for connecting new customers to the network;
- Amortization;
- Taxes;
- Budget constraints.

Moreover, while the investment on the backbone pipelines is done on the first year, new customers are not connected immediately, but following an estimated acquisition curve (e.g., 25% the first year, 15%, the second year, . . .). Hence, the corresponding costs and revenues have to be scaled accordingly.

The thermos-hydraulic model must ensure the proper operation of the extended network. The following constraints are to be imposed:

- Flow conservation at the nodes of the network;
- Minimum and maximum pressures at the nodes;
- Plants operation limit: maximum pressure on the feed line, minimum pressure on the return line, minimum and maximum flow rate;
- Pressure drop along the links;
- Maximum water speed and pressure drop per meter.