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On a comparison of Newton Raphson solvers for power flow problems

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Abstract

A general framework is given for applying the Newton-Raphson method to solve power flow problems, using power and current-mismatch functions in polar, Cartesian coordinates and complex form. These two mismatch functions and three coordinates, result in six versions of the Newton-Raphson method for the solution of power flow problems. We present a theoretical framework to compare these variants for PQ-buses and PV-buses. Furthermore, the convergence behavior is investigated by numerical experiments. This enables us to compare new versions with existing versions of the Newton power flow methods. We conclude that the polar current-mismatch and Cartesian current-mismatch versions that are developed in this paper, performed the best result for both distribution and transmission networks.

1 Introduction

A power flow computation that determines the steady state behavior of the network is one of the most important tools for grid operators. The solution of a power flow computation can be used to assess whether the power system can function properly for the given generation and consumption. Therefore, power flow computations are performed in power system operation, control and planning.

The power flow or load flow problem is the problem of computing the voltage *magnitude* $|V_i|$ and *angle* δ_i in each bus of a power system where the power generation and consumption are given. Over the years, a lot of power flow solution techniques [1–15] have been developed on transmission networks. Gauss-Seidel (G-S), Newton power flow (N-R) and Fast Decoupled Load Flow (FDLF) based algorithms are the most widely used methods for the solution of transmission power flow problems. In practice, the Newton power flow method is preferred in terms of quadratic convergence and improved robustness [16]. Furthermore, many new methods [17–28] have been developed for distribution power flow problems and generally they are divided into two main categories such as modification of conventional power flow solution methods (G-S, N-R, FDLF) and Backward-Forward Sweep (BFS)-based algorithms. Several reviews on distribution power flow solution methods can be found in [29–32].

In this paper, we focus on the Newton based power flow methods for balanced transmission and distribution networks. Depending on problem formulations (power or current mismatch) and coordinates (polar, Cartesian and complex form), the Newton-Raphson method can be applied in six different ways as a solution method for power flow problems. These six versions of the Newton power flow method are considered as the fundamental Newton power flow methods from which the further modified versions [8–15] are derived. Table 1 shows the previously published papers that considered each variation of the Newton power flow method.

The most widely used version is the Newton power flow method using the power-mismatch and polar coordinates which is introduced in [2]. In this method, the reactive power mismatch ΔQ and the voltage magnitude correction ΔV for each PV bus are eliminated from the Jacobian matrix equations and therefore the order of the equation is $(2N - N_g - 2)$.

Mismatch formulation	Coordinates		
	Polar	Cartesian	Complex form
Power	[2]	[33]	[34]
Current	[35]	[35, 36]	

Table 1: Known versions of the Newton power flow method

In the version using the power-mismatch and Cartesian (rectangular) coordinates introduced in [33], the reactive power mismatch ΔQ is not eliminated from the Jacobian matrix equations for each PV bus but replaced by a voltage-magnitude-squared mismatch equation

$$\Delta|V|^2 = (|V|^{sp})^2 - (V^r)^2 + (V^m)^2. \tag{1}$$

Therefore, the order of the Jacobian matrix equation is $(2N-2)$ and it is concluded that the method is slightly less reliable and less rapid in convergence than the polar version in [33].

Although it is mentioned in [2, 16] that the complex power flow formulation does not mathematically lead to an analytic function of the complex voltage because of conjugate terms, the paper [34] investigated the version of Newton power flow method using the power-mismatch in complex form. In paper [34], the Jacobian matrix equations are developed in complex form for a PQ bus whereas two separate equations are created for a PV bus. The correction values of complex voltage for the PQ and PV buses are computed separately using different tolerances at each iteration. However, it is preferred to calculate correction values for both the PQ and PV buses using common Jacobian matrix equations and the same tolerance.

The version using the current-mismatch and a mix of Cartesian and polar coordinates is discussed in [35]. In this method, each PQ bus is represented by two equations that are constructed from the real and imaginary parts of the complex current-mismatch function. A PV bus is represented by a single active power mismatch ΔP and the voltage-magnitude-squared mismatch equation (1). The order of the Jacobian matrix equation is $(2N-2)$ and it is concluded in [16] that these versions perform less satisfactorily than the power-mismatch versions.

The version using the current-mismatch and Cartesian coordinates is considered again in [36]. This method introduces a new dependent variable ΔQ for each PV bus and an additional equation relating the corrections in polar and Cartesian coordinates:

$$\Delta|V| = \frac{V^r}{|V|}\Delta V^r + \frac{V^m}{|V|}\Delta V^m \tag{2}$$

$$\Delta\delta = \frac{V^r}{|V|^2}\Delta V^m - \frac{V^m}{|V|^2}\Delta V^r. \tag{3}$$

Using equations (2) and (3), this method makes the Jacobian matrix equation square in order to have a unique solution. In this method, the real ΔI^r and imaginary ΔI^m current-mismatch functions are expressed in terms of the real ΔP and reactive ΔQ power-mismatch functions. Then the reactive power-mismatch ΔQ is considered as a dependent variable for each PV bus and computed at each Newton iteration. Minor attempts were made to speed up the solution method using a partly constant approximation of the Jacobian during the iterations, but the results were not encouraging [36].

We did not find any discussion covering the Newton power flow method using the current-mismatch in complex form.

All variations of Newton power flow method are developed by different researchers in different ways. This paper aims to discuss all six versions of the Newton power flow method using a common framework and to introduce new developments to improve the performance of other versions besides the most used version using the power-mismatch and polar coordinates [2]. Major improvements were done by us in Cartesian power-mismatch, polar current-mismatch and Cartesian current-mismatch versions. In versions using Cartesian coordinates, equations (2) and (3) are used for PV buses instead of the voltage-magnitude-squared mismatch equation (1). In case of versions using the current-mismatch regardless of the choice of the coordinates, the reactive power Q is considered as a dependent variable for each PV bus. Thus, we compute the correction ΔQ at each iteration and update Q using the computed corrections. In case of the Cartesian power-mismatch, the order of the system is decreased to $(2N - N_g - 2)$ whereas [33] uses a system with the order $(2N - 2)$. The complex current-mismatch and complex power-mismatch versions are developed only for PQ buses.

This paper is structured as follows. In section 2, a mathematical model of a power system is introduced in general. Section 3 mathematically describes the power flow problem. The Newton-Raphson method and its all six versions for the solution of power flow problems are explained in section 4. The numerical result of the solution techniques on balanced distribution and transmission networks, is presented in section 5. Finally, the conclusion is given in section 6.

2 Power system model

Power systems are modeled as a network of buses (nodes) and branches (transmission lines) whereas a network bus represents a system component such as a generator, load and transmission substation etc. There are three types of network buses such as a slack bus, a generator bus (PV-bus) and a load bus (PQ-bus). Each bus in the power network is fully described by the following four electrical quantities:

$|V_i|$: the voltage magnitude
δ_i : the voltage phase angle
P_i : the active power
Q_i : the reactive power

Depending on the type of the bus, two of the four electrical quantities are specified as shown in Table 2:

Bus type	Number of buses	Known	Unknown
slack node or swing bus	1	$\lvert V_i\rvert, \delta_i$	P_i, Q_i
generator node or PV-bus	N_g	$P_i, \lvert V_i\rvert$	Q_i, δ_i
load node or PQ-bus	$N - N_g - 1$	P_i, Q_i	$\lvert V_i\rvert, \delta_i$

Table 2: Network bus type

Here, i is the index of the bus, N_g is the number of generator buses and N is the total number of buses in the network. For more details on the power system model we refer to [37].

3 Power flow problem

The power flow, or load flow, problem is the problem of computing the voltage magnitude $|V_i|$ and angle δ_i in each bus of a power system where the power generation and consumption are given. According to Kirchoff's Current Law (KCL), the relation between the current I injected at the network buses and the bus voltages V, is described by the admittance matrix $\mathbf{Y}$:

$$I = \mathbf{Y}V \quad \leftrightarrow \quad \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdot & \cdot & Y_{1N} \\ Y_{21} & Y_{22} & \cdot & \cdot & Y_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ Y_{N1} & Y_{N2} & \cdot & \cdot & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_N \end{bmatrix} \tag{4}$$

where I_i is the injected current and V_i is the voltage in phasor domain at bus i and Y_{ij} is the element of the admittance matrix. The injected current at bus i can be computed from equation (4) as follows:

$$I_i = \sum_{k=1}^{N} Y_{ik} V_k. \tag{5}$$

The mathematical equations for the power flow problem are given by:

$$S_i = V_i I_i^* \tag{6}$$

$$= V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* \tag{7}$$

where S_i is the injected complex power at bus i and I_i^* is the complex conjugate of the injected current I_i. Mathematically, the power flow problem comes down to solving a nonlinear system of equations.

4 Newton power flow solution methods

The Newton based power flow methods use the Newton-Raphson (NR) method that is applied to solve a nonlinear system of equations $F(\vec{x}) = 0$. In the NR method, the linearized problem is constructed as the Jacobian matrix equation

$$-J(\vec{x})\Delta\vec{x} = F(\vec{x}) \tag{8}$$

where $J(\vec{x})$ is the square Jacobian matrix and $\Delta\vec{x}$ is the correction vector. The Jacobian matrix is obtained by $J_{ik} = \frac{\partial F_i(\vec{x})}{\partial x_k}$ and is highly sparse in power flow applications [2, 16]. The iteration process of the Newton based power flow method is shown in Algorithm 1. Traditionally, a direct solver is used to solve the Jacobian matrix equation. Convergence of the method is mostly measured in the residual norm $||F(\vec{x}^h)||$ or relative residual norm $||\frac{F(\vec{x}^h)}{F(\vec{x}^0)}||$ of the mismatch function $F(\vec{x}^h)$ at each iteration. The Newton power flow method has a quadratic convergence when iterates are close enough to the solution. The Newton power flow methods formulate $F(\vec{x})$ as power or current mismatch functions and designate the unknown bus voltages as the problem variables $\vec{x}$ using three different coordinates such as polar, Cartesian and complex form. The list of problem variables defined in different coordinates is shown in Table 3.

Algorithm 1 Newton's power flow method

```
h := 0
given initial iterate x⃗^0
while not converged do
    solve the correction −J(x⃗^h)Δx⃗^h = F(x⃗^h)
    update iterate x⃗^{h+1} := x⃗^h + Δx⃗^h
    h := h + 1
end while
```

Coordinates	Vector $\vec{x}$
Polar ($V_i = \|V_i\|e^{\imath\delta_i}$)	$\left[\delta_1, \cdots, \delta_N, \|V_1\|, \cdots, \|V_N\|\right]^T$
Cartesian ($V_i = V_i^r + \imath V_i^m$)	$\left[V_1^m, \cdots, V_N^m, V_1^r, \cdots, V_N^r\right]^T$
Complex form (V_i)	$\left[V_1, \cdots, V_N\right]^T$

Table 3: Variable $\vec{x}$ in different coordinates

4.1 Power-mismatch formulation:

The power flow problem (7) is formulated as the power-mismatch function $F(\vec{x})$ as follows:

$$\begin{aligned} F_i(\vec{x}) = \Delta S_i(\vec{x}) &= S_i^{sp} - S_i(\vec{x}) \\ &= S_i^{sp} - V_i \sum_{k=1}^{N} Y_{ik}^* V_k^* \qquad \forall i \in 1...N \end{aligned} \tag{9}$$

where $S_i^{sp} = P_i^{sp} + \imath Q_i^{sp}$ is the specified complex power injection at bus i. In general, specified active power P_i^{sp} and reactive power Q_i^{sp} injections at bus i are given by following equations:

$$P_i^{sp} = P_i^G - P_i^L \tag{10}$$

$$Q_i^{sp} = Q_i^G - Q_i^L \tag{11}$$

where P_i^G and Q_i^G are specified active and reactive power generation whereas P_i^L and Q_i^L are specified active and reactive power loads respectively. Here, P_i^L and Q_i^L are modeled as a constant power load.

The complex power-mismatch function (9) is separated into real equations and variables using polar and Cartesian coordinates. Table 4 shows power-mismatch functions in different coordinates. An application of first order Taylor approximation to the power-mismatch functions results in a linear system of equations 8 that is solved by all Newton iteration. Table 5 displays the equations that compute the elements of the Jacobian matrix in different coordinates which are the partial derivatives of the power-mismatch function.

Coordinates	Power-mismatch function: $F_i(\vec{x}) = \Delta S_i(\vec{x})$
Polar	$\Delta P_i(\vec{x}) = P_i^{sp} - \sum_{k=1}^{N} \lvert V_i\rvert\lvert V_k\rvert(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$
	$\Delta Q_i(\vec{x}) = Q_i^{sp} - \sum_{k=1}^{N} \lvert V_i\rvert\lvert V_k\rvert(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$
Cartesian	$\Delta P_i(\vec{x}) = P_i^{sp} - \sum_{k=1}^{N} \Big(V_i^r(G_{ik}V_k^r - B_{ik}V_k^m) + V_i^m(B_{ik}V_k^r + G_{ik}V_k^m)\Big)$
	$\Delta Q_i(\vec{x}) = Q_i^{sp} - \sum_{k=1}^{N} \Big(V_i^m(G_{ik}V_k^r - B_{ik}V_k^m) - V_i^r(B_{ik}V_k^r + G_{ik}V_k^m)\Big)$
Complex form	$\Delta S_i(\vec{x}) = S_i^{sp} - V_i \sum_{k=1}^{N} Y_{ik}^* V_k^*$

Table 4: Power-mismatch function in different coordinates

Coordinates		$J_{ik} = \frac{\partial F_i(\vec{x})}{\partial x_k}$
Polar	$i \neq k$	$\frac{\partial \Delta P_i(\vec{x})}{\partial \lvert V_k\rvert} = -\lvert V_i\rvert(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$
		$\frac{\partial \Delta Q_i(\vec{x})}{\partial \lvert V_k\rvert} = -\lvert V_i\rvert(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$
		$\frac{\partial \Delta P_i(\vec{x})}{\partial \delta_k} = -\lvert V_i\rvert\lvert V_k\rvert(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$
		$\frac{\partial \Delta Q_i(\vec{x})}{\partial \delta_k} = -\lvert V_i\rvert\lvert V_k\rvert(-G_{ik}\cos\delta_{ik} - B_{ik}\sin\delta_{ik})$
	$i = k$	$\frac{\partial \Delta P_i(\vec{x})}{\partial \lvert V_i\rvert} = -\Big(2\lvert V_i\rvert G_{ii} + \sum_{i\neq k} \lvert V_k\rvert(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})\Big)$
		$\frac{\partial \Delta Q_i(\vec{x})}{\partial \lvert V_i\rvert} = -\Big(-2\lvert V_i\rvert B_{ii} + \sum_{i\neq k} \lvert V_k\rvert(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})\Big)$
		$\frac{\partial \Delta P_i(\vec{x})}{\partial \delta_i} = -\sum_{i\neq k} \lvert V_i\rvert\lvert V_k\rvert(-G_{ik}\sin\delta_{ik} + B_{ik}\cos\delta_{ik})$
		$\frac{\partial \Delta Q_i(\vec{x})}{\partial \delta_i} = -\sum_{i\neq k} \lvert V_i\rvert\lvert V_k\rvert(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$
Cartesian	$i \neq k$	$\frac{\partial \Delta P_i(\vec{x})}{\partial V_k^r} = -\Big(V_i^r G_{ik} + V_i^m B_{ik}\Big)$
		$\frac{\partial \Delta Q_i(\vec{x})}{\partial V_k^r} = -\Big(V_i^m G_{ik} - V_i^r B_{ik}\Big)$
		$\frac{\partial \Delta P_i(\vec{x})}{\partial V_k^m} = -\Big(V_i^m G_{ik} - V_i^r B_{ik}\Big)$
		$\frac{\partial \Delta Q_i(\vec{x})}{\partial V_k^m} = V_i^r G_{ik} + V_i^m B_{ik}$
	$i = k$	$\frac{\partial \Delta P_i(\vec{x})}{\partial V_i^r} = -\Big(V_i^r G_{ii} + V_i^m B_{ii} + \sum_{k=1}^{N}(G_{ik}V_k^r - B_{ik}V_k^m)\Big)$
		$\frac{\partial \Delta Q_i(\vec{x})}{\partial V_i^r} = -\Big(V_i^m G_{ii} - V_i^r B_{ii} + \sum_{k=1}^{N}(B_{ik}V_k^r + G_{ik}V_k^m)\Big)$
		$\frac{\partial \Delta P_i(\vec{x})}{\partial V_i^m} = -\Big(V_i^m G_{iu} - V_i^r B_{ii} + \sum_{k=1}^{N}(B_{ik}V_k^r + G_{ik}V_k^m)\Big)$
		$\frac{\partial \Delta Q_i(\vec{x})}{\partial V_i^m} = V_i^r G_{ii} + V_i^m B_{ii} - \sum_{k=1}^{N}(G_{ik}V_k^r - B_{ik}V_k^m)$
Complex form	$i \neq k$	$\frac{\partial \Delta S_i(\vec{x})}{\partial V_k} = -V_i Y_{ik}^*$
	$i = k$	$\frac{\partial \Delta S_i(\vec{x})}{\partial V_i} = -\Big(V_i Y_{ii}^* + I^*\Big)$

Table 5: The partial derivatives of the power-mismatch function in different coordinates

4.1.1 Polar power-mismatch version (NR-p-pol [2])

The Jacobian matrix equation (8) derived from the power-mismatch function in polar coordinates is given in the partitioned form for convenience of presentation:

$$-\left[\begin{array}{c|c} J^{11} & J^{12} \\ \hline J^{21} & J^{22} \end{array}\right]\left[\begin{array}{c} \Delta\delta \\ \Delta\lvert V\rvert \end{array}\right] = \left[\begin{array}{c} \Delta P \\ \Delta Q \end{array}\right] \tag{12}$$

where sub-matrices are given as $J^{11} = \frac{\partial \Delta P}{\partial \delta}$, $J^{12} = \frac{\partial \Delta P}{\partial \lvert V\rvert}$, $J^{21} = \frac{\partial \Delta Q}{\partial \delta}$ and $J^{22} = \frac{\partial \Delta Q}{\partial \lvert V\rvert}$. The Jacobian matrix equation (12) has to be modified for all PV buses since a voltage magnitude

$|V_j|$ is specified instead of reactive power Q_j in case of a PV bus j. Since Q_j^{sp} is not given, the reactive power mismatch ΔQ_j is not formulated for all PV buses. All partial derivatives of it with respect to voltage magnitude $|V_i|$ and angle δ_i cannot be taken. Similarly, $\Delta|V_j|$ does need to be computed for a PV bus j since $|V_j|$ is now known. Therefore, we eliminate all the $\frac{\partial \Delta P_i}{\partial |V_j|}$, $\frac{\partial \Delta Q_i}{\partial |V_j|}$, $\frac{\partial \Delta Q_j}{\partial \delta_i}$ and $\frac{\partial \Delta Q_j}{\partial |V_i|}$ from the Jacobian matrix $J(\vec{x})$, $\Delta|V_j|$ from the correction vector $\Delta\vec{x}$ and ΔQ_j from the power mismatch vector $F(\vec{x})$ for each PV bus j. The order of the resulting Jacobian matrix equation is $(2N - N_g - 2)$.

4.1.2 Cartesian power-mismatch version (NR-p-car)

The Jacobian matrix equation (8) derived from the power-mismatch function in Cartesian coordinates is given as follows:

$$-\left[\begin{array}{c|c} J^{11} & J^{12} \\ \hline J^{21} & J^{22} \end{array}\right] \left[\begin{array}{c} \Delta V^m \\ \Delta V^r \end{array}\right] = \left[\begin{array}{c} \Delta P \\ \Delta Q \end{array}\right] \tag{13}$$

where sub-matrices are given as $J^{11} = \frac{\partial \Delta P}{\partial V^m}$, $J^{12} = \frac{\partial \Delta P}{\partial V^r}$, $J^{21} = \frac{\partial \Delta Q}{\partial V^m}$ and $J^{22} = \frac{\partial \Delta Q}{\partial V^r}$. The Jacobian matrix equation (13) has to be modified for all PV buses for the same reason as the polar power-mismatch version 4.1.1 had. In this versions, the reactive power-mismatch ΔQ_j cannot be formulated for a PV bus j and therefore all partial derivatives $\frac{\partial \Delta Q_j}{\partial V_k^m}$ and $\frac{\partial \Delta Q_j}{\partial V_k^r}$ cannot be taken.

In paper [33], the reactive power mismatch ΔQ is replaced by a voltage-magnitude-squared mismatch equation (1) for all PV buses and therefore all partial derivatives $\frac{\partial \Delta Q_j}{\partial V_k^m}$ and $\frac{\partial \Delta Q_j}{\partial V_k^r}$ are also replaced by $\frac{\partial \Delta |V_j|^2}{\partial V_k^m}$ and $\frac{\partial \Delta |V_j|^2}{\partial V_k^r}$ respectively. Moreover, the order of the Jacobian matrix equation remains $(2N - 2)$ and it is concluded that the method is slightly less reliable and less rapid in convergence than the polar power-mismatch version 4.1.1 in [33].

In this paper, we develop a new approach that improves the performance of this version. In our approach, the reactive power-mismatch ΔQ_j is removed from the power-mismatch vector $F(\vec{x})$ for all PV buses and therefore all partial derivatives $\frac{\partial \Delta Q_j}{\partial V_k^m}$ and $\frac{\partial \Delta Q_j}{\partial V_k^r}$ are also eliminated from the Jacobian matrix $J(\vec{x})$. As a result of the elimination, the Jacobian matrix becomes a rectangular matrix. In order to make the Jacobian matrix square, we use the equation (2) with $\Delta|V_j| = 0$ since $|V_j|$ is now specified for a PV bus j. This gives us the relation between the corrections ΔV_j^r and ΔV_j^m as:

$$\Delta V_j^r = -\frac{V_j^m}{V_j^r} \Delta V_j^m \tag{14}$$

Using (14), the column of the Jacobian matrix with respect to the derivatives $\frac{\partial \Delta P_i}{\partial V_j^r}$ and $\frac{\partial \Delta Q_i}{\partial V_j^r}$ are added to the column with respect the derivatives $\frac{\partial \Delta P_i}{\partial V_j^m}$ and $\frac{\partial \Delta Q_i}{\partial V_j^m}$ as follows:

$$\frac{\partial \Delta P_i}{\partial V_j^m} \Delta V_j^m = \Big(\frac{\partial \Delta P_i}{\partial V_j^m} - \frac{V_j^m}{V_j^r} \frac{\partial \Delta P_i}{\partial V_j^r}\Big) \Delta V_j^m \tag{15}$$

$$\frac{\partial \Delta Q_i}{\partial V_j^m} \Delta V_j^m = \Big(\frac{\partial \Delta Q_i}{\partial V_j^m} - \frac{V_j^m}{V_j^r} \frac{\partial \Delta Q_i}{\partial V_j^r}\Big) \Delta V_j^m \tag{16}$$

Then the correction ΔV_j^r can be eliminated from the correction vector $\Delta\vec{x}$ for each PV bus j and therefore the order of the Jacobian matrix equation is $(2N - N_g - 2)$.

4.1.3 Complex power-mismatch version (NR-p-com)

The Jacobian matrix equation (8) derived from the power-mismatch function in complex form is given as follows:

$$-\left[\; J \;\right]\left[\; \Delta V \;\right] = \left[\; \Delta S \;\right] \tag{17}$$

where the Jacobian matrix $J = \frac{\partial \Delta S}{\partial V}$ is obtained by taking the partial derivatives of the complex power-mismatch functions with respect to the complex voltage V. The Jacobian matrix equation (17) holds for all PQ buses but not for all PV buses because the complex power mismatch ΔS cannot be formulated for all PV buses. Therefore, this version can be applied to the power flow problem on the network with only a slack bus and PQ buses.

4.2 Current-mismatch formulation:

The current equation (5) and the power flow problem (6) are used to formulate the current-mismatch function $F(\vec{x})$ as follows:

$$\begin{aligned} F_i(\vec{x}) &= \Delta I_i(\vec{x}) = I_i^{sp} - I_i(\vec{x}) \\ &= \left(\frac{S_i^{sp}}{V_i}\right)^* - \sum_{k=1}^{N} Y_{ik} V_k \qquad \forall i \in 1...N \end{aligned} \tag{18}$$

where $I_i^{sp} = \left(\frac{S_i^{sp}}{V_i}\right)^*$ is the specified complex current injection at bus i.

The current-mismatch function (18) can be also expressed in terms of the power-mismatch function (9) as follows:

$$\Delta I_i = \left(\frac{\Delta S_i}{V_i}\right)^* \qquad \text{(complex)} \tag{19}$$

$$= \frac{\cos\delta_i \Delta P_i + \sin\delta_i \Delta Q_i}{|V_i|} + \imath \frac{\sin\delta_i \Delta P_i - \cos\delta_i \Delta Q_i}{|V_i|} \qquad \text{(polar)} \tag{20}$$

$$= \frac{V_i^r \Delta P_i + V_i^m \Delta Q_i}{|V_i|^2} + \imath \frac{V_i^m \Delta P_i - V_i^r \Delta Q_i}{|V_i|^2} \qquad \text{(Cartesian).} \tag{21}$$

The complex current-mismatch function (18) is separated into real equations and variables using polar and Cartesian coordinates. Table 6 shows the current-mismatch functions in different coordinates. An application of a first order Taylor approximation to the current-mismatch function results in a linear system of equations 8 that is solved in every Newton iteration. Table 7 displays the equations to compute the elements of the Jacobian matrix in different coordinates which are the partial derivatives of the current-mismatch function.

Coordinates	Current-mismatch function: $F_i(\vec{x}) = \Delta I_i(\vec{x})$
Polar	$\Delta I_i^r(\vec{x}) = \frac{P_i^{sp}\cos\delta_i + Q_i^{sp}\sin\delta_i}{\lvert V_i\rvert} - \sum_{k=1}^{N} \lvert V_k\rvert(G_{ik}\cos\delta_k - B_{ik}\sin\delta_k)$ $\Delta I_i^m(\vec{x}) = \frac{P_i^{sp}\sin\delta_i - Q_i^{sp}\cos\delta_i}{\lvert V_i\rvert} - \sum_{k=1}^{N} \lvert V_k\rvert(G_{ik}\sin\delta_k + B_{ik}\cos\delta_k)$
Cartesian	$\Delta I_i^r(\vec{x}) = \frac{P_i^{sp}V_i^r + Q_i^{sp}V_i^m}{(V_i^r)^2+(V_i^m)^2} - \sum_{k=1}^{N}(G_{ik}V_k^r - B_{ik}V_k^m)$ $\Delta I_i^m(\vec{x}) = \frac{P_i^{sp}V_i^m - Q_i^{sp}V_i^r}{(V_i^r)^2+(V_i^m)^2} - \sum_{k=1}^{N}(G_{ik}V_k^m + B_{ik}V_k^r)$
Complex form	$\Delta I_i(\vec{x}) = \left(\frac{S_i^{sp}}{V_i}\right)^* - \sum_{k=1}^{N} Y_{ik}V_k$

Table 6: Current-mismatch function in different coordinates

Coordinates		$J_{ik} = \frac{\partial F_i(\vec{x})}{\partial x_k}$
Polar	$i \neq k$	$\frac{\partial \Delta I_i^r(\vec{x})}{\partial\lvert V_k\rvert} = -(G_{ik}\cos\delta_k - B_{ik}\sin\delta_k)$ $\frac{\partial \Delta I_i^m(\vec{x})}{\partial\lvert V_k\rvert} = -(G_{ik}\sin\delta_k + B_{ik}\cos\delta_k)$ $\frac{\partial \Delta I_i^r(\vec{x})}{\partial\delta_k} = \lvert V_k\rvert(G_{ik}\sin\delta_k + B_{ik}\cos\delta_k)$ $\frac{\partial \Delta I_i^m(\vec{x})}{\partial\delta_k} = -\lvert V_k\rvert(G_{ik}\cos\delta_k - B_{ik}\sin\delta_k)$
	$i = k$	$\frac{\partial \Delta I_i^r(\vec{x})}{\partial\lvert V_i\rvert} = -(G_{ii}\cos\delta_i - B_{ii}\sin\delta_i) - \frac{P_i^{sp}\cos\delta_i + Q_i^{sp}\sin\delta_i}{\lvert V_i\rvert^2}$ $\frac{\partial \Delta I_i^m(\vec{x})}{\partial\lvert V_i\rvert} = -(G_{ii}\sin\delta_i + B_{ii}\cos\delta_i) - \frac{P_i^{sp}\sin\delta_i - Q_i^{sp}\cos\delta_i}{\lvert V_i\rvert^2}$ $\frac{\partial \Delta I_i^r(\vec{x})}{\partial\delta_i} = \lvert V_i\rvert(G_{ii}\sin\delta_i + B_{ii}\cos\delta_i) - \frac{P_i^{sp}\sin\delta_i - Q_i^{sp}\cos\delta_i}{\lvert V_i\rvert}$ $\frac{\partial \Delta I_i^m(\vec{x})}{\partial\delta_i} = -\lvert V_i\rvert(G_{ii}\cos\delta_i - B_{ii}\sin\delta_i) + \frac{P_i^{sp}\cos\delta_i + Q_i^{sp}\sin\delta_i}{\lvert V_i\rvert}$
Cartesian	$i \neq k$	$\frac{\partial \Delta I_i^r(\vec{x})}{\partial V_k^r} = -G_{ik}$ $\frac{\partial \Delta I_i^m(\vec{x})}{\partial V_k^r} = B_{ik}$ $\frac{\partial \Delta I_i^r(\vec{x})}{\partial V_k^m} = B_{ik}$ $\frac{\partial \Delta I_i^m(\vec{x})}{\partial V_k^m} = -G_{ik}$
	$i = k$	$\frac{\partial \Delta I_i^r(\vec{x})}{\partial V_i^r} = -G_{ii} - \frac{P_i^{sp}((V_i^r)^2-(V_i^m)^2)+2V_i^rV_i^mQ_i^{sp}}{\lvert V_i\rvert^4}$ $\frac{\partial \Delta I_i^m(\vec{x})}{\partial V_i^r} = -B_{ii} + \frac{Q_i^{sp}((V_i^r)^2-(V_i^m)^2)-2V_i^rV_i^mP_i^{sp}}{\lvert V_i\rvert^4}$ $\frac{\partial \Delta I_i^r(\vec{x})}{\partial V_i^m} = B_{ii} + \frac{Q_i^{sp}((V_i^r)^2-(V_i^m)^2)-2V_i^rV_i^mP_i^{sp}}{\lvert V_i\rvert^4}$ $\frac{\partial \Delta I_i^m(\vec{x})}{\partial V_i^m} = -G_{ii} + \frac{P_i^{sp}((V_i^r)^2-(V_i^m)^2)+2V_i^rV_i^mQ_i^{sp}}{\lvert V_i\rvert^4}$
Complex form	$i \neq k$	$\frac{\partial \Delta I_i(\vec{x})}{\partial V_k} = -Y_{ik}$
	$i = k$	$\frac{\partial \Delta I_i(\vec{x})}{\partial V_i} = -\left(\frac{S_i^{sp}}{V_i^2} + Y_{ii}\right)$

Table 7: The partial derivatives of the current-mismatch function in different coordinates

4.2.1 Polar current-mismatch version (NR-c-pol)

The Jacobian matrix equation (8) derived from the current-mismatch function in polar coordinates is given as follows:

$$-\left[\begin{array}{c|c} J^{11} & J^{12} \\ \hline J^{21} & J^{22} \end{array}\right]\left[\begin{array}{c} \Delta\delta \\ \Delta\lvert V\rvert \end{array}\right] = \left[\begin{array}{c} \Delta I^r \\ \Delta I^m \end{array}\right] \tag{22}$$

where sub-matrices are given as $J^{11} = \frac{\partial \Delta I^r}{\partial\delta}$, $J^{12} = \frac{\partial \Delta I^r}{\partial\lvert V\rvert}$, $J^{21} = \frac{\partial \Delta I^m}{\partial\delta}$ and $J^{22} = \frac{\partial \Delta I^m}{\partial\lvert V\rvert}$. Same as the polar power-mismatch version 4.1.1, $\Delta\lvert V_j\rvert$ needs to be computed for a PV bus j since $\lvert V_j\rvert$

is now known. Therefore, we eliminate all the $\frac{\partial \Delta I_i^r}{\partial |V_j|}$ and $\frac{\partial \Delta I_i^m}{\partial |V_j|}$ from the Jacobian matrix $J(\vec{x})$ and $\Delta |V_j|$ from the correction vector $\Delta \vec{x}$ for each PV bus j. As a result of the elimination, the Jacobian matrix becomes a rectangular matrix.

In paper [35], each PQ bus is represented by the real ΔI^r and imaginary ΔI^m current-mismatch functions. A PV bus is represented by the active power-mismatch ΔP and the voltage-magnitude-squared mismatch equation (1). Thus, the order of the Jacobian matrix equation is $(2N-2)$ and it is concluded in [16] that these versions perform less satisfactorily than the power-mismatch versions.

In our approach, the reactive power Q_j is chosen as a dependent variable as $|V|$ and δ for each PV bus j because we use the current-mismatch formulation directly. Since Q_j is an unknown variable, all the first order partial derivatives $\frac{\partial \Delta I_i^r}{\partial Q_j}$ and $\frac{\partial \Delta I_i^m}{\partial Q_j}$ has to be computed as shown in Table 8:

$J_{ij} = \frac{\partial F_i}{\partial x_j}$	$i \neq j$	$\frac{\partial \Delta I_i^r(\vec{x})}{\partial Q_j} = 0$ $\frac{\partial \Delta I_i^m(\vec{x})}{\partial Q_j} = 0$
	$i = j$	$\frac{\partial \Delta I_j^r(\vec{x})}{\partial Q_j} = \frac{\sin \delta_j}{\|V_j\|^{sp}}$ $\frac{\partial \Delta I_j^r(\vec{x})}{\partial Q_j} = -\frac{\cos \delta_j}{\|V_j\|^{sp}}$

Table 8: The partial derivatives of the current-mismatch function in polar coordinates with respect to the reactive power Q

Now we add the derivatives $\frac{\partial \Delta I_i^r}{\partial Q_j}$ and $\frac{\partial \Delta I_i^m}{\partial Q_j}$ into the Jacobian matrix $J(\vec{x})$ becoming a square matrix again and the correction ΔQ_j into the correction vector $\Delta \vec{x}$ for each PV bus j. The initial reactive power Q_j^0 at a PV bus j is computed as follows:

$$Q_j^0 = \sum_{k=1}^{N} |V_j||V_k|(G_{jk} \sin \delta_{jk} - B_{jk} \cos \delta_{jk}). \tag{23}$$

In each Newton iteration, the correction ΔQ_j is computed and the reactive power Q_j is updated using the computed correction.

4.2.2 Cartesian current-mismatch version (NR-c-car)

The Jacobian matrix equation (8) derived from the current-mismatch function in Cartesian coordinates is given as follows:

$$-\left[\begin{array}{c|c} J^{11} & J^{12} \\ \hline J^{21} & J^{22} \end{array}\right] \left[\begin{array}{c} \Delta V^m \\ \Delta V^r \end{array}\right] = \left[\begin{array}{c} \Delta I^r \\ \Delta I^m \end{array}\right] \tag{24}$$

where sub-matrices are given as $J^{11} = \frac{\partial \Delta I^r}{\partial V^m}$, $J^{12} = \frac{\partial \Delta I^r}{\partial V^r}$, $J^{21} = \frac{\partial \Delta I^m}{\partial V^m}$ and $J^{22} = \frac{\partial \Delta I^m}{\partial V^r}$.

In paper [36], the real ΔI^r and imaginary ΔI^m current-mismatch functions are expressed in terms of the real ΔP and reactive ΔQ power-mismatch functions. Then the reactive power-mismatch ΔQ is considered as a dependent variable for each PV bus and computed at each Newton iteration. Minor attempts were made to speed up the solution method using a partly constant approximation of the Jacobian during the iterations, but the results were not encouraging [36].

In our approach, the reactive power Q_j is chosen as a dependent variable for each PV bus j as polar current-mismatch version 4.2.1. Since Q_j is an unknown variable, all the first order partial derivatives $\frac{\partial \Delta I_i^r}{\partial Q_j}$ and $\frac{\partial \Delta I_i^m}{\partial Q_j}$ has to be computed as shown in Table 9:

$J_{ij} = \frac{\partial F_i}{\partial x_j}$	$i \neq j$	$\frac{\partial \Delta I_i^r(\vec{x})}{\partial Q_j} = 0$ $\frac{\partial \Delta I_i^m(\vec{x})}{\partial Q_j} = 0$
	$i = j$	$\frac{\partial \Delta I_j^r(\vec{x})}{\partial Q_j} = \frac{V_j^m}{(V_j^r)^2+(V_j^m)^2}$ $\frac{\partial \Delta I_j^r(\vec{x})}{\partial Q_j} = \frac{-V_j^r}{(V_j^r)^2+(V_j^m)^2}$

Table 9: The partial derivatives of the current-mismatch function in Cartesian coordinates with respect to the reactive power Q

Now we add the derivatives $\frac{\partial \Delta I_i^r}{\partial Q_j}$ and $\frac{\partial \Delta I_i^m}{\partial Q_j}$ into the Jacobian matrix $J(\vec{x})$ and the correction ΔQ_j into the correction vector $\Delta \vec{x}$ for each PV bus j. After the addition, the Jacobian matrix become a rectangular matrix. In order to make the Jacobian matrix square, we add the column of the Jacobian matrix with respect to the derivatives $\frac{\partial \Delta I_i^r}{\partial V_j^r}$ and $\frac{\partial \Delta I_i^m}{\partial V_j^r}$ to the column with respect the derivatives $\frac{\partial \Delta I_i^r}{\partial V_j^m}$ and $\frac{\partial \Delta I_i^m}{\partial V_j^m}$ using (14) as follows:

$$\frac{\partial \Delta I_i^r}{\partial V_j^m} \Delta V_j^m = \Big(\frac{\partial \Delta I_i^r}{\partial V_j^m} - \frac{V_j^m}{V_j^r} \frac{\partial \Delta I_i^r}{\partial V_j^r} \Big) \Delta V_j^m \tag{25}$$

$$\frac{\partial \Delta I_i^m}{\partial V_j^m} \Delta V_j^m = \Big(\frac{\partial \Delta I_i^m}{\partial V_j^m} - \frac{V_j^m}{V_j^r} \frac{\partial \Delta I_i^m}{\partial V_j^r} \Big) \Delta V_j^m \tag{26}$$

Then the correction ΔV_j^r can be eliminated from the correction vector $\Delta \vec{x}$ for each PV bus j. The initial reactive power Q_j^0 at a PV bus j is computed as follows:

$$Q_j^0 = \sum_{k=1}^{N} \Big(V_j^m (G_{jk} V_k^r - B_{jk} V_k^m) - V_j^r (B_{jk} V_k^r + G_{jk} V_k^m) \Big). \tag{27}$$

In each Newton iteration, the correction ΔQ_j is computed and the reactive power Q_j is updated using the computed correction.

4.2.3 Complex current-mismatch version (NR-c-com)

The Jacobian matrix equation (8) derived from the current-mismatch function in complex form is given as follows:

$$-\left[\ J\ \right] \left[\ \Delta V\ \right] = \left[\ \Delta I\ \right] \tag{28}$$

where the Jacobian matrix $J = \frac{\partial \Delta S}{\partial V}$ is obtained by taking the partial derivatives of the complex current-mismatch functions with respect to the complex voltage V. As the complex power-mismatch version 4.1.3, this version is also applicable for the power flow problem on the network with only a slack bus and PQ buses.

The bus voltage correction in different coordinates is given in Table 10

Coordinates	Type of Bus	$\vec{x}^{h+1} := \vec{x}^h + \Delta\vec{x}^h$
Polar	PQ and PV	$V_i^{(h+1)} = \lvert V_i\rvert^{(h+1)} e^{\imath\delta_i^{(h+1)}}$
		$\lvert V\rvert_i^{(h+1)} = \lvert V\rvert_i^{(h)} + \Delta\lvert V_i\rvert^{(h)}$ $\delta_i^{(h+1)} = \delta_i^{(h)} + \Delta\delta_i^{(h)}$
Cartesian	PQ and PV	$V_i^{(h+1)} = (V_i^r)^{(h+1)} + \imath(V_i^m)^{(h+1)}$
		$(V_i^r)^{(h+1)} = (V_i^r)^{(h)} + (\Delta V_i^r)^{(h)}$ $(V_i^m)^{(h+1)} = (V_i^m)^{(h)} + (\Delta V_i^m)^{(h)}$
	PQ	$V_i^{(h+1)} = \lvert V_i\rvert^{(h+1)} e^{\imath\delta_i^{(h+1)}}$
		$\Delta\lvert V_j\rvert = \frac{V_j^r}{\lvert V_j\rvert}\Delta V_j^r + \frac{V_j^m}{\lvert V_j\rvert}\Delta V_j^m$ $\Delta\delta_j = \frac{V_j^r}{\lvert V_j\rvert^2}\Delta V_j^m - \frac{V_j^m}{\lvert V_j\rvert^2}\Delta V_j^r$
	PV	$\Delta\delta_j = \frac{\Delta V_j^m}{V_j^r}$
Complex	PQ (NR-p-com)	$V_i^{(h+1)} = V_i^{(h)} + (\Delta V_i^{(h)})^*$
	PQ (NR-c-com)	$V_i^{(h+1)} = V_i^{(h)} + \Delta V_i^{(h)}$

Table 10: Bus voltage correction in different coordinates

5 Numerical experiment

The newly developed/improved versions of the Newton power flow method (Cartesian power-mismatch, polar current-mismatch, Cartesian current-mismatch and complex current-mismatch) discussed in section 4, are compared to the existing versions of the Newton power flow method (polar power-mismatch [2], Cartesian power-mismatch [33] and Cartesian current-mismatch [36]). Two distribution networks (33-bus [38] and 69-bus [39]) and four transmission networks from matpower [40] (case1354pegase, case2737sop, case9241pegase and case13659pegase) are used to test the convergence ability and scalability of all the versions of the Newton power flow solution method. All methods are implemented in Matlab and the relative convergence tolerance is equal to 10^{-5}. The maximum number of iteration is set to 10. All experiments are performed on an Intel computer with four cores i5-4690 3.5GHz CPU and 64Gb memory, running a Debian 64-bit Linux 8.7 distribution.

5.1 Distribution networks

The convergence result of solution methods for distribution networks (DCase33 and DCase 69) is shown in Table 11.

Methods	Test cases					
	DCase33			DCase69		
	iter	time	$\|\|F(\vec{x})\|\|_\infty$	iter	time	$\|\|F(\vec{x})\|\|_\infty$
NR-p-pol [2]	3	0.0096	7.4675e-09	3	0.0089	1.0426e-08
NR-p-car	3	0.0061	1.0433e-09	3	0.0066	8.1777e-09
NR-p-car [33]	3	0.0066	1.0897e-09	3	0.0069	8.0940e-09
NR-p-com	3	0.0050	3.1339e-06	3	0.0050	2.8385e-06
NR-c-pol	2	0.0081	4.0671e-06	2	0.0090	7.6450e-06
NR-c-car	2	0.0070	4.0041e-06	3	0.0082	1.9483e-11
NR-c-car [36]	2	0.0101	4.0040e-06	3	0.0124	1.9483e-11
NR-c-com	3	0.0054	9.9385e-06	5	0.0068	9.7421e-06

Table 11: Distribution networks: DCase33 and DCase69

From Table 11, we see that the Newton power flow versions using the current-mismatch functions have better convergence than versions using the power-mismatch functions regardless of the choice of the coordinates. Thus, we can conclude that the Newton power flow versions using current-mismatch are more suitable for solving distribution power flow problems than versions using power-mismatch functions. Although the complex power-mismatch and the complex current-mismatch versions have the same number of iterations to converge, these versions have linear convergence whereas other versions have quadratic convergence. Thus, the complex versions are the least preferable of the Newton power flow versions. The polar current-mismatch version that is developed in this paper performed the best for both distribution network cases.

5.2 Transmission networks

Since the complex power-mismatch and complex current mismatch versions are developed for only PQ buses, these versions are not applied for transmission networks. The convergence result of solution methods for transmission networks (TCase1354, TCase2737, TCase9241 and TCase13659) is shown in Tables 12 and Table 13.

Methods	Test cases					
	TCase1354			TCase2737		
	iter	time	$\|\|F(\vec{x})\|\|_\infty$	iter	time	$\|\|F(\vec{x})\|\|_\infty$
NR-p-pol [2]	3	0.0284	6.2678e-06	4	0.0640	1.5353e-08
NR-p-car	3	0.0265	1.5795e-06	4	0.0634	2.3500e-06
NR-p-car [33]	3	0.0298	2.2486e-06	5	0.0777	2.8518e-06
NR-c-pol	3	0.0313	8.3005e-10	4	0.0700	6.1735e-07
NR-c-car	3	0.0306	6.1446e-10	4	0.0649	8.6780e-07
NR-c-car [36]	5	0.0507	9.9969e-06	5	0.0838	7.9842e-07

Table 12: Small transmission networks: TCase1354 and TCase2737

For smaller transmission networks TCase1354 and TCase2737, all versions result in the same behavior except the Cartesian current-mismatch version developed in [36] which required more iterations than other versions.

Methods	Test cases					
	TCase9241			TCase13659		
	iter	time	$\|\|F(\vec{x})\|\|_\infty$	iter	time	$\|\|F(\vec{x})\|\|_\infty$
NR-p-pol [2]	6	0.3555	2.1292e-09	5	0.3899	2.2891e-09
NR-p-car	5	0.2908	2.1026e-08	6	0.4689	7.9833e-12
NR-p-car [33]	5	0.3180	2.0742e-06	10	0.8899	1.401e+148
NR-c-pol	3	0.1973	6.4746e-07	4	0.3634	3.4366e-09
NR-c-car	3	0.1993	1.9438e-06	4	0.3619	8.6170e-09
NR-c-car [36]	10	0.6595	0.0023	10	0.9036	1.1482

Table 13: Large transmission networks: TCase9241 and TCase13659

For the large transmission network TCase9241, the version developed in [36] did not converge whereas other versions converged. For this case, the polar current-mismatch and the Cartesian current-mismatch versions developed in this paper converged after only three iterations whereas other versions [2] and [33] and Cartesian power-mismatch needed five to six iterations. For the largest transmission network TCase13659, versions [33] and [36] did not converge whereas other versions developed in this paper and the version given in [2] converged. Moreover, the polar current-mismatch and Cartesian current-mismatch versions converged after four iterations while the most famous version [2] needed five iterations. Thus, we can conclude that the Newton power flow versions using current-mismatch are more preferable for large transmission networks.

6 Conclusion

In this paper, we formulate and analyze the Newton based power flow method used for the solution of non-linear power flow problems. For the various methods we consider two different mismatch functions: the current and the power form and three different coordinate systems: Cartesian, polar and complex formulations. This leads to six different versions of the Newton power flow method. Studying these versions in a common framework enables us to analyse and compare in a unified way. Furthermore, the existing versions of the Newton power flow method [2,33,36] are implemented and compared with the newly developed/improved versions of the Newton power flow method (Cartesian power-mismatch, polar current-mismatch, Cartesian current-mismatch and complex current-mismatch). In case of the polar and Cartesian current-mismatch versions, the reactive power Q is chosen as a dependent variable for each PV bus. Thus, we compute the correction ΔQ at each iteration and update Q using the computed corrections. Equations (2) and (3) are used instead of the voltage-magnitude-squared mismatch equation (1) in versions using Cartesian coordinates. The order of the equation is $(2N - N_g - 2)$ for the versions using the power-mismatch function whereas the order of equation is $(2N - 2)$ for versions using the current-mismatch function. For distribution networks, versions using the current-mismatch function have better convergence than versions using the power-mismatch function regardless of the choice of the coordinates. The polar current-mismatch version that is developed in this paper delivered the best result for both distribution network cases. For smaller transmission networks, all existing and new versions have the same convergence behavior. However, for large transmission networks, versions [33] and [36] did not converge whereas other versions developed in this paper and the version [2] converged. Moreover, the polar current-mismatch and Cartesian current-mismatch versions developed in this paper performed the best results for larger transmission networks.

Therefore, we conclude that the newly developed/improved versions have a better performance than the existing versions of the Newton power flow method for both distribution and transmission networks.

In addition, the Cartesian current-mismatch version has an advantage in the calculation of the Jacobian matrix because its off-diagonal elements are constant and equal to the terms of the nodal admittance matrix. Moreover, depending on the properties of given network, one version can work better than the other versions. Therefore, it is crucial to study which version is more suitable for what kind of power network. In the near future, these newly developed versions will be applied for the solution of unbalanced three-phase distribution networks.

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A Notation

N	:	number of buses in the network
N_g	:	number of generator buses
h	:	iteration counter
$V_k = V_k^r + \imath V_k^m$	:	complex voltage at bus k
$\lvert V_k \rvert, \delta_k$	:	voltage magnitude and angle at bus k
$\vec{x}$	:	vector of unknown variables
$\Delta\vec{x}$	:	correction of unknown variables
$F_k(\vec{x})$	:	complex power or current mismatch function at bus k
$J(\vec{x})$	:	Jacobian matrix of the mismatch function
$\Delta S_k = \Delta P_k + \imath \Delta Q_k$	:	complex power mismatch at bus k
$\Delta I_k = \Delta I_k^r + \imath \Delta I_k^m$	:	complex current mismatch at bus k
$S_k^{sp} = P_k^{sp} + \imath Q_k^{sp}$	:	specified complex power at bus k
$S_k^G = P_k^G + \imath Q_k^G$	:	generated complex power at bus k
$S_k^L = P_k^L + \imath Q_k^L$	:	complex power load at bus k
$Y_{ik} = G_{ik} + \imath B_{ik}$	:	(i, k) the element of nodal admittance matrix

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