Parameter estimation and uncertainty for simulating waterflooding in reservoir simulation

Supervisors: Arnold Heemink (TUD) and Johan Romate (Shell/TUD)

Problem background
In petroleum reservoir engineering waterflooding is a technique to enhance the oil recovery from a reservoir. In waterflooding water is injected in one or more places (injection wells) in a reservoir under high enough pressure for the oil in the reservoir to be pushed by the injected water towards the producing wells of the reservoir.
Here, we consider waterflooding in one space dimension. On one end water is injected and on the other end oil and water are produced. We assume oil and water to be incompressible. The two-phase flow model of incompressible fluid flow through a porous medium in one space dimension we consider is given by the transport equations for the phase masses for oil and water (w=water, o=oil),

$$\frac{\partial}{\partial t}(\phi\rho_o S_o) + \frac{\partial}{\partial x}(\phi\rho_o v_o) = 0,$$

$$\frac{\partial}{\partial t}(\phi\rho_w S_w) + \frac{\partial}{\partial x}(\phi\rho_w v_w) = 0,$$

and Darcy's equations,

$$q_o = -\frac{kk_{ro}}{\mu_o}\frac{dp_o}{dx} = -\lambda_o\frac{dp_o}{dx},$$

$$q_w = -\frac{kk_{rw}}{\mu_w}\frac{dp_w}{dx} = -\lambda_w\frac{dp_w}{dx}.$$

These equations are written in terms of the fluid phase pressures (p_o and p_w) and the fluid phase volume fractions (the saturations S_o and S_w), which need to be solved from these equations given appropriate data. The actual velocities v_i are related to the Darcy (or superficial) velocities q_i by $q_i = \phi v_i$. Additionally, we require that the saturations add to one:

$$S_o + S_w = 1.$$

Due to surface tension the oil and water pressures are not equal. The difference between the two pressures is the capillary pressure, p_c:

$$p_c = p_o - p_w.$$

Furthermore, the reservoir is taken horizontal: the effect of gravity is neglected. The densities ρ_o and ρ_w are constant, as are the porosity ϕ, the absolute permeability k and the viscosities μ_o and μ_w.
The relative permeabilities (or "relperms") k_{ro} and k_{rw} and the capillary pressure p_c are modelled as functions of the water saturation. These functions vary from (oil) field to field and so are not known exactly in advance. In practice, these functions of the saturations are determined in an experimental setup: a piece of rock ("core") is taken from the reservoir at hand and in a laboratory the oil is pushed out from the core by injected water in a controlled experiment. This core flood experiment is done such that a one-dimensional description can be used to model the two-phase flow through the rock. In the one-dimensional model some functional model is assumed for the relative permeabilities and the capillary pressure and the experiment is simulated with those coefficients. By comparing the outcome of the one-dimensional model with the

experiment it is possible to improve the model for the coefficients until a best fit is achieved. The estimated parameters are then used in the reservoir simulation model for that reservoir.

Assignment

This assignment is about the development of a numerical model that should allow users to do easier parameter estimation for relperm and capillary pressure models based on core flood modelling and experiments, assess the parameters' sensitivities and uncertainties as well as their covariances. It is also desirable to understand the inter-parametric dependency, since relperms and capillary pressures are not unrelated.

The numerical method used to do this parameter estimation will be the Markov-chain Monte Carlo (MCMC) method. This method allows for uncertainties in the measurements and will predict among other things the uncertainties in the obtained parameters. The parameter estimation problem is formulated as a constrained optimization problem where the objective function reflects the mismatch between the measurements and the model outcome, and hence has to be minimized.

The parameters to be estimated are the parameters defining the relperm and capillary pressure functions that need to be constructed. These functions can be parameterized in different ways with varying number of parameters. For instance, they can be given as parameterized correlation functions or as tables for interpolating polynomials. One of the problems to be investigated is the selection of the parameterization and the range of the parameters. It is also not clear how to set the necessary constraints on the parameters of the table such that monotonicity of the curve is guaranteed whenever the data are monotonous, despite the uncertainty in the data. Hence, can we have "monotonicity under uncertainty"?

A desirable feature of the tool to be developed is the presentation and visualization of the uncertainties in the data and model results. The suggested approach may allow sensitivity analysis for the unknown parameters. The method also provides the ranges of possible solution parameters using the MCMC method, so the issue then is how to present these in a convenient way as part of the analysis.

The data for the parameter estimation can come from various experiments, and hence may have quite different uncertainties. This should be taken care of. Furthermore, there may be data outliers. What to do with outliers: is there a systematic way to handle them? One way of handling them may be through the objective function.

The assignment thus is about answering the following questions:

1. Given some parameterizations of relperm and capillary pressure curves, what can we say about the influence of the parameterization choice on the uncertainty and quality of the solution, and consequently, what is the most suitable parameterization?

2. How should the parameter estimation problem be formulated, taking into account the different uncertainties of the data, handling of outliers, the monotonicity constraints, etc.?

This needs to be demonstrated with implementation in a software program. This program also needs to be such that uncertainties and sensitivities in the solutions and in the data can be made visible in an easy way.

Literature

[1] K. Aziz; A. Settari: *Petroleum Reservoir Simulation* Applied Science Publishers, London, 1979.

[2] D. Loeve, F. Wilschut, R.H. Hanea, J.G. Maas, P.M.E. van Hooff, P.J. van den Hoek, S.G. Douma, J.F.M. Van Doren: Simultaneous determination of relative permeability and capillary pressure curves by assisted history matching several SCAL experiments.
Paper presented at the International Symposium of the Society of Core Analysts, held in Austin, Texas, USA 18-21 September, 2011.