Non-symmetric benchmark problem

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This problem concerns a pressure calculation for the incompressible Navier-Stokes equations. For the discretization a finite volume technique is used combined with boundary fitted coordinates. This results in a structured matrix with at most 9 non-zero elements per row. The matrix looks like a discretization of a Laplace equation, however it is weakly non-symmetric due to the treatment of the boundary conditions. The physical domain and a coarse finite volume grid are given in Figure 1. Homogeneous Neumann boundary conditions are posed

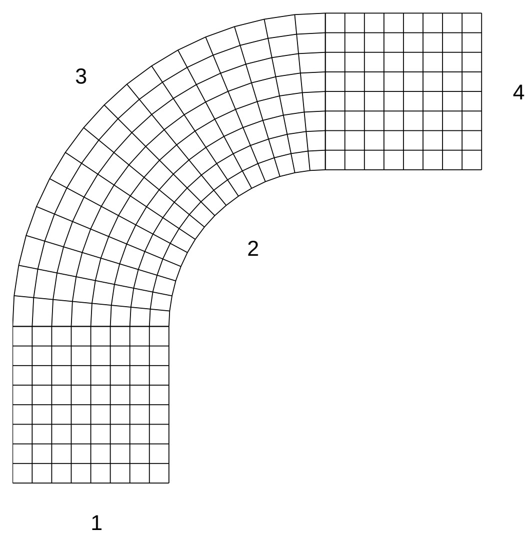

Figure 1: The domain and grid for the non-symmetric problem

on Boundary 1, 2, and 3, whereas on Boundary 4 a Dirichlet condition is used. The problem is solved on an $M \times 4M$-grid with $M = 16, 32, 64, 128$. We refer to [3, 4, 2, 1] for further information.

Description of the data structure

The matrix, solution and right-hand-side vector are stored in the files matrix*.dat. In order to read the routine a Fortran file read.f is provided. The problem is discretized with nx volumes in the x-direction and ny volumes in the y-direction. In the problems ny = 4nx. This means that n1 = nx + 1 unknowns are used in the x-direction and n2 = ny+1 in the y-direction. The dimension of the matrix is n = n1*n2. In order to facilitate virtual points the elements of some rows are all equal to zero. The nonzero diagonals are stored in matrix(1:n,9). The following relation is valid:

$$matrix(i,1) = A_{i,i}$$

$$matrix(i,2) = A_{i,i-n1-1}$$

$$matrix(i,3) = A_{i,i-n1}$$

$$matrix(i,4) = A_{i,i-n1+1}$$

$$matrix(i,5) = A_{i,i-1}$$

$$matrix(i,6) = A_{i,i+1}$$

$$matrix(i,7) = A_{i,i+n1-1}$$

$$matrix(i,8) = A_{i,i+n1}$$

$$matrix(i,9) = A_{i,i+n1+1}$$

Finally a Fortran file matvec.f is given to see how a matrix vector product can be implemented.

References

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