Modelling water flow in a ditches network of a Dutch polder

Presentation literature review
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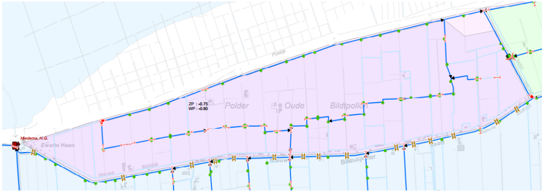
Introduction

Framework

- Acacia Water; enabling access to clean and safe water
- Project Spaarwater
  - Mitigation measures for salinity, such as
    - Water harvesting on private farms
    - Underground freshwater storage
    - Drip irrigation
    - Drainage optimization
  - Electrical conductivity (EC) meter to measure water salinity



Problem statement

- Implementing mitigation measures & EC measurements: combine these to **predict** water salinity?
- Using the EC measurements
  - Interpolating the data somehow?
  - What governs the water flow in a ditches network?
  - What are the underlying mathematical principles?
- Final goal: to develop a visual tool showing the effects of mitigation measures on water flow and salinity

Literature study so far

Main research question:

- How do we model the water flow using a **fast** mathematical algorithm?

Research subquestion:

- How do we incorporate water salinity and geographical features in the model

9 months total

- 3 months: literature study
- 6 months: implementation mathematical model

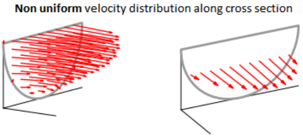
Theory for open channel water flow

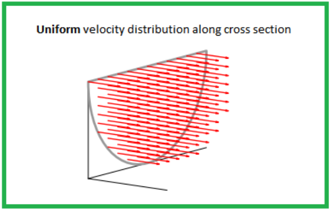
Definitions and prior conditions

1. Uniform velocity of flow

- Only considering velocities u_x in the x direction
- $v(t,x)$ is the *average velocity*, over a given cross section $A(t,x)$

$$v(t,x) = \frac{1}{A(t,x)} \int\int_{(y,z)\in A} u_x(t,x,y,z)$$





Definitions and prior conditions

2. Discharge

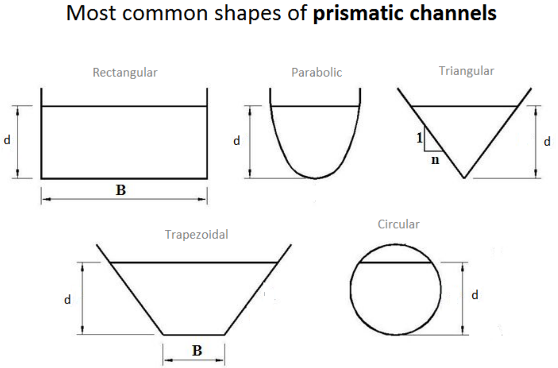
- Volumetric flow rate through a given cross sectional area $A(t,x)$

$$Q(t,x) = A(t,x)v(t,x)$$

- Again, considering one dimensional flow in the x direction

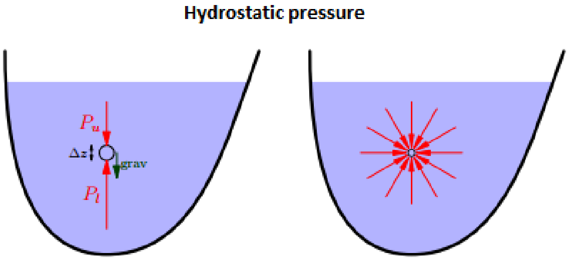
Definitions and prior conditions

3. Prismatic channel



4. Hydrostatic pressure

- Pressure is assumed to behave similarly to in stagnant water, hence
  - No water flow due to vertical pressure differences
  - Pressure increases linearly with depth



The shallow water equations: Saint Venant equations

The mathematical equations describing one dimensional open channel flow are given by

$$\begin{cases} \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = Q_l \\ \frac{\partial Q}{\partial t} + \frac{\partial Qv}{\partial x} = gA(S_0 - S_f - S_p) \end{cases} \tag{1}$$

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Saint Venant equations: the continuity equation

The **continuity equation** is the first equation of the Saint Venant equations

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = Q_l \tag{2}$$

- Where Q is the discharge, A is the cross sectional area and Q_l is the lateral inflow
- Describes the conservation of volume of flow: flow in equals flow out

The shallow water equations: Saint Venant equations

The **equation of motion** is the second equation of the Saint Venant equations

$$\frac{\partial Q}{\partial t} + \frac{\partial Qv}{\partial x} = gA(S_0 - S_f - S_p) \quad (3)$$

- Where S_0 is the channel bed slope, S_f is the empirically derived friction slope and $S_p = \frac{\partial d}{\partial x}$ is the change in water depth along the x direction
- Describes the conservation of momentum
- (3) describes very detailed, local motion of flow but can be simplified to more global flow motion

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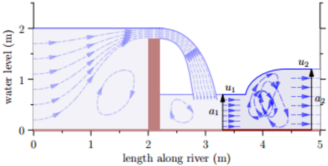
Modelling different types of flow

The full equation of motion describes detailed and local motion of flow

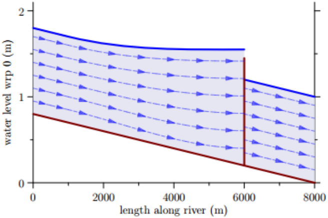
$$\frac{\partial Q}{\partial t} + \frac{\partial Qv}{\partial x} = gA(S_0 - S_f - S_p)$$

- Simplifying this equation yields simpler, global behaviour of flow



Full equation of motion

$$\frac{\partial Q}{\partial t} + \frac{\partial Qv}{\partial x} = gA(S_0 - S_f - S_p)$$



Simplified equation of motion

$$S_0 - S_f - S_p = 0$$

Modelling different types of flow

	I	II	III	IV
Term				

Equation of motion

$$\frac{1}{g}\frac{\partial v}{\partial t} + \frac{v}{g}\frac{\partial v}{\partial x} + \frac{\partial y}{\partial x} + (S_f - S_o) = 0.$$

Model	*Terms*
1. Kinematic wave	IV
2. Diffusion wave	III + IV
3. Steady dynamic wave	II + III + IV
4. Dynamic wave	I + II + III + IV
5. Gravity	I + II + III

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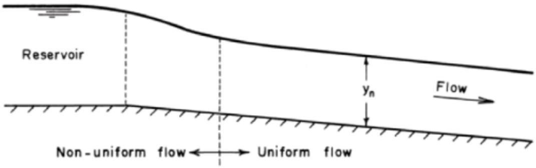
Types of flow: uniform vs nonuniform

The main distinction between types of flow is **uniform** versus **nonuniform** motion of flow

- Uniform flow describes an equilibrium situation in which there is no change in flow velocity along the x direction $\frac{\partial Q}{\partial x} = 0$
- Nonuniform flow describes a situation in which the flow velocity is changing along the x direction $\frac{\partial Q}{\partial x} \neq 0$



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Models for open channel flow

Flow routing; lumped vs distributed

A common method for modelling water flow and discharge, classified as either **lumped** or **distributed** routing

- Lumped routing uses the storage equation $\frac{\mathrm{d}ST}{\mathrm{d}t} = I(t) - O(t)$
  - Applying the conservation of volume of flow
  - Stationary system, made instationary by updating the stationary system for new timesteps
  - First the dicharges are calculated everywhere, then water depths are updated
- Distributed routing is done through solving partial differential equations, e.g. a version of the Saint Venant equations
  - Instationary approach
  - Discharge and water depth are updated simultaneously

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Lumped routing

Distributed routing

Model name	Model assumptions	Model equations	$\frac{\partial Q}{\partial t}$	$\frac{\partial Q}{\partial x}$	Computational time
Flow conservation	Uniform flow, equilibrium discharge per control volume, pseudo-stationary approach	$\begin{cases} \frac{dST}{dt} = I(t) - O(t) \\ S_0 = S_f \end{cases}$ where the second equation gives $Q_{eq} = \frac{1}{n} S_0^{1/2} A R^{2/3}$	×	✓	Of the order $5 n_x n_t$; so the number of spatial segments plus the number of time steps considered
Flow conservation, backwater effects	Gradually varied flow, backwater effects incorporated, pseudo-stationary approach	$\begin{cases} \frac{dST}{dt} = I(t) - O(t) \\ Q = \frac{1}{n} (\frac{\partial d}{\partial x})^{1/2} A R^{2/3} \end{cases}$	×	✓	Of the order $16 n_x n_t$
Steady dynamic wave	Gradually varied flow, subcritical flow so mild slopes, pseudo-stationary approach	$\begin{cases} \frac{\partial Q}{\partial x} = Q_l \\ \frac{\partial Qv}{\partial x} = gA(S_0 - S_f - S_p) \end{cases}$	×	✓	?
Kinematic wave	Distributed routing approach, uniform flow assumed per discretization segment, waves propagate downstream only at equal celerity and do not attenuate	$\begin{cases} \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = Q_l \\ S_0 = S_f \end{cases}$	✓	✓	Of the order $16 n_x n_t$; so the number of spatial segments times the number of time steps considered
Diffusive wave	Distributed routing approach, gradually varied flow with backwater effects, waves propagate downstream only at equal celerity and attenuate as they move downstream	$\begin{cases} \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = Q_l \\ \frac{\partial y}{\partial x} + S_f - S_0 = 0 \end{cases}$	✓	✓	Of the order *constant* × $n_x n_t$, but a little more complicated than the kinematic wave
Dynamic wave	Distributed routing, full dynamic waves which propagate both upstream and downstream, waves strongly attenuate	$\begin{cases} \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = Q_l \\ \frac{\partial Q}{\partial t} + \frac{\partial Qv}{\partial x} = gA(S_0 - S_f - S_p) \end{cases}$	✓	✓	Of the order *constant* × $n_x n_t$, but more complicated than kinematic and diffusive wave

A comparison of two models

Flow conservation	Uniform flow, equilibrium discharge per control volume, pseudo-stationary approach	$\begin{cases} \frac{dST}{dt} = I(t) - O(t) \\ S_0 = S_f \end{cases}$	×	✓	Of the order $5n_xn_t$; so the number of spatial segments plus the number of time steps considered
Kinematic wave	Distributed routing approach, uniform flow assumed per discretization segment, waves propagate downstream only at equal celerity and do not attenuate	$\begin{cases} \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = Q_l \\ S_0 = S_f \end{cases}$	✓	✓	Of the order $16n_xn_t$; so the number of spatial segments times the number of time steps considered

Note that the full Saint Venant equations are given by

$$\begin{cases} \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = Q_l \\ \frac{\partial Q}{\partial t} + \frac{\partial Qv}{\partial x} = gA(S_0 - S_f - S_p) \end{cases}$$

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Incorporating geographical features

- Waterway bifurcations
  - Confluence conditions; part of research questions
- Weirs
  - Using simple weir formula $Q = C_{weir}\sqrt{g}B(d - d_{weir})^{3/2}$
- Pumping stations, inlets, outles
  - Fixed discharges
- Culverts
  - Adding resistance to channel through shrinking cross sectional areas at the location of the culvert

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Incorporating water salinity

Salt concentrations can be determined through the advection-diffusion equation

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial x} + D\frac{\partial^2 c}{\partial x^2} = f \tag{4}$$

where c is the salt concentration, v the flow velocity, D the diffusion coefficient and f a source term.

There are three options with decreasing complexity from top to bottom, of which the implementation is a point for further research:

- Advection-diffusion equation combined with the Navier Stokes equations with varying densities
- Advection-diffusion equation combined with Saint-Venant equation with constant water density assumed to be $\rho = 1$
- Using a lumped approach; balance equation for salt mass

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Conclusion

Summary

- Main difference between lumped and distributed approach
  - Lumped modelling first calculates spatial variation followed by an update per time step
  - Distributed modelling calculates spatial and time variation simultaneously
- There are many different options of complexity for a water flow model; ranging from a very simple motion of flow to a very detailed, local motion of flow
- More research needs to be done to decide what level of complexity is desired for a specific type of situation

Proposed start modelling equations

The kinematic wave equations

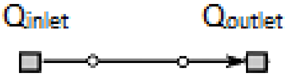
$$\begin{cases} \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = Q_l \\ S_0 = S_f \end{cases}$$

Research questions

1. How do we discretize the kinematic wave equations using the finite volume method?
2. How does the kinematic wave equation compare to the other possible models in terms of its applicability, computational time and accuracy?
3. How do we design the grid such that it represents the geographical structure of a ditches network?
4. How do we decide on the boundary conditions and how do we implement boundary and confluence conditions into the grid, incorporating different flow directions?

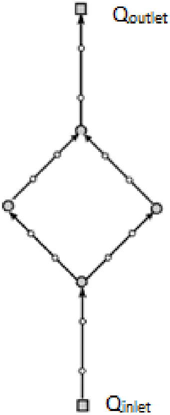
Test model problems

Step 1: Straight ditch with one inlet and one outlet



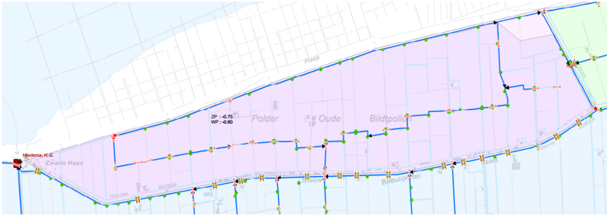
Test model problems

Step 2: Simple network with two waterway bifurcations



Test model problems

Step 3: (Part of) The ditches network of the Oude Bildtpollenpolder



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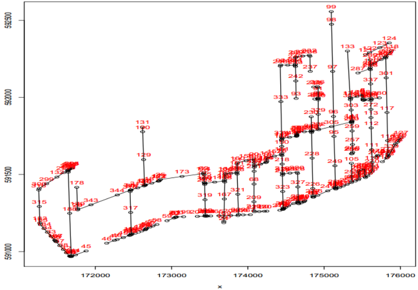

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Questions and discussion