# Master of Science Project

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## Computation of Thermo-acoustic Modes in Combustors

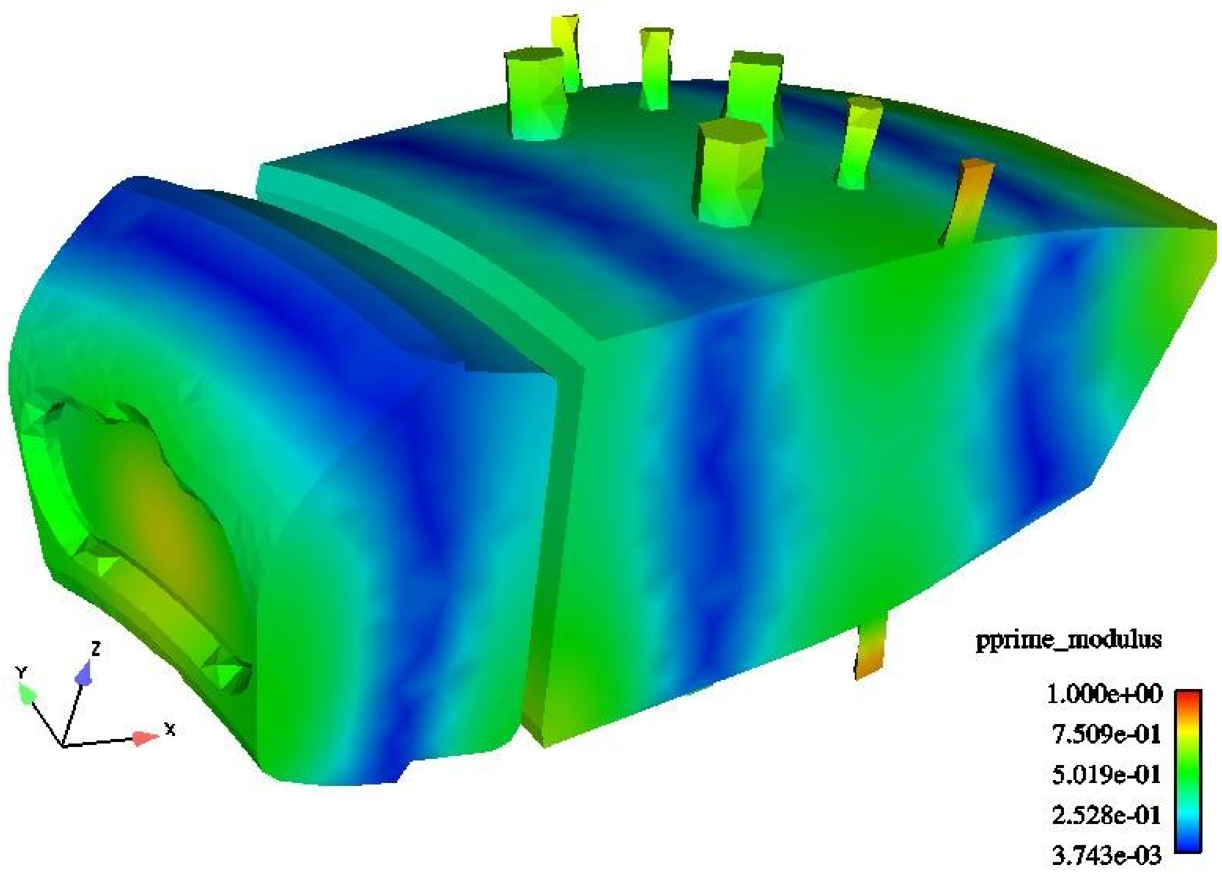
through

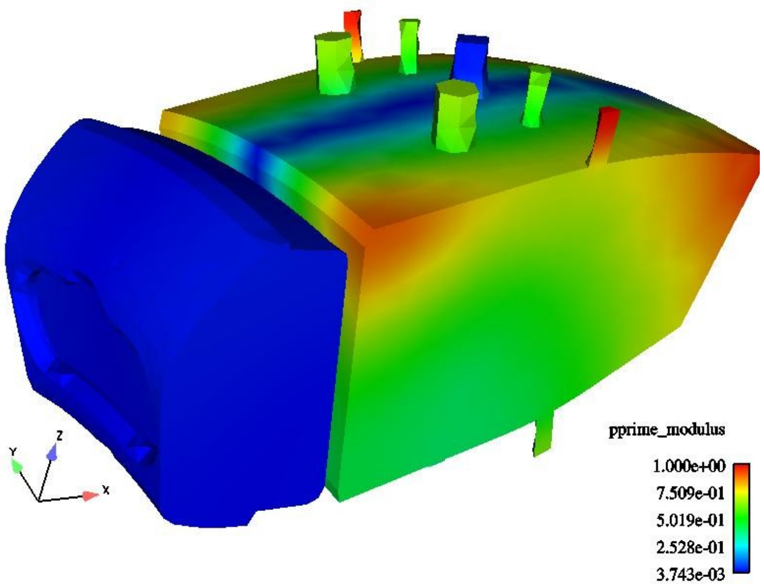
Non-Linear Eigenvalue Problems

using

Arnoldi's method and Jacobi Davidson

# Combustion Oscillations

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7.509e-01
5.019e-01
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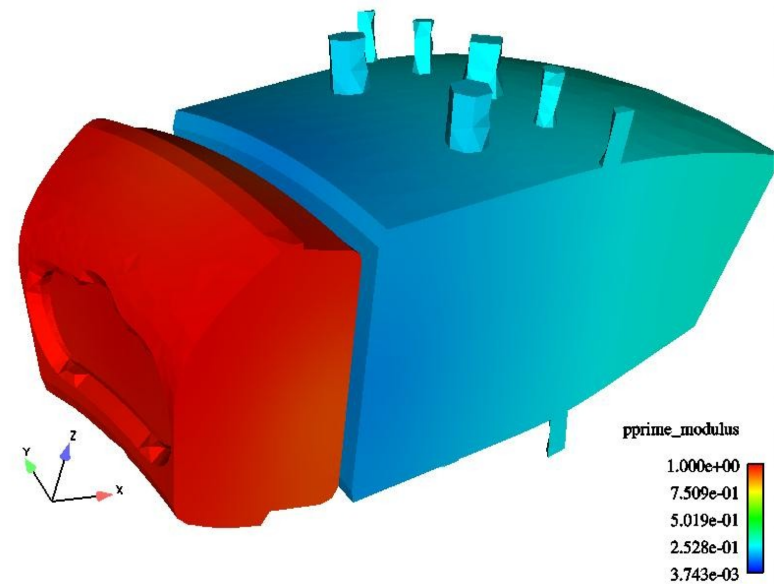
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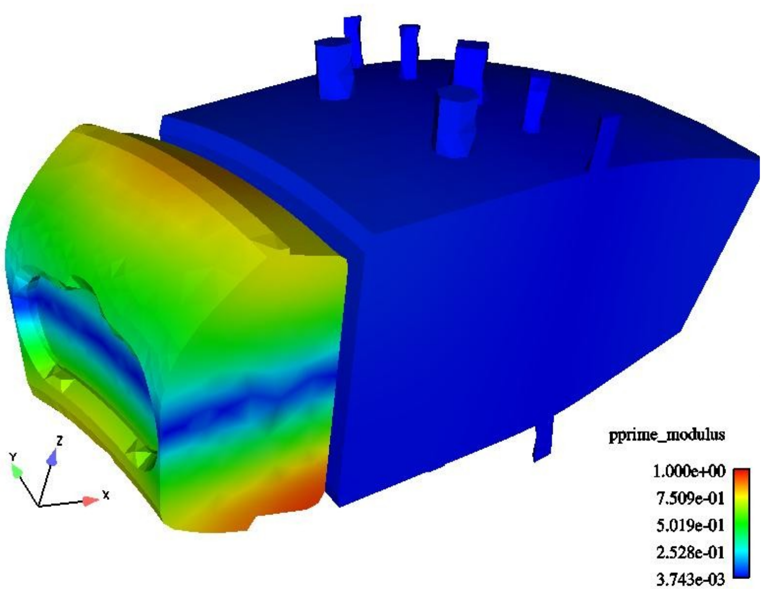
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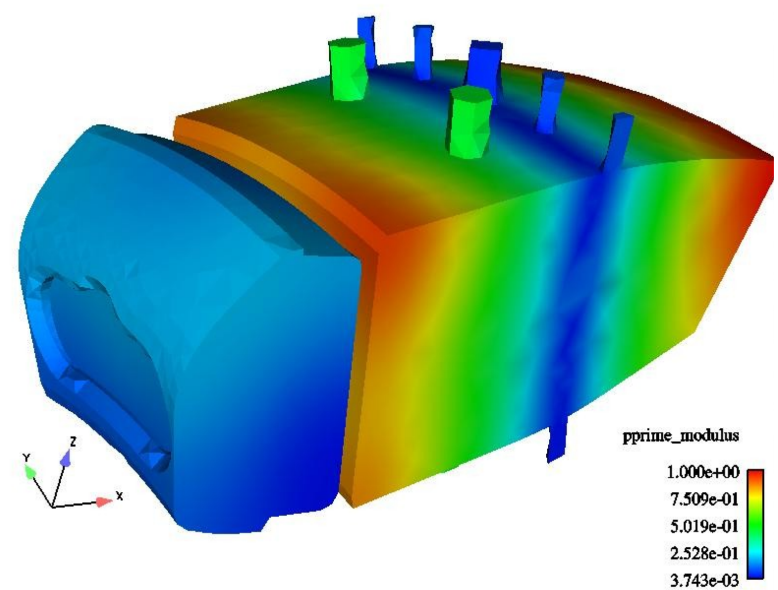
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# Non-Linear Eigenvalue Problems

- Equation:
  - $T(\lambda)x=0$
- Methods
  - Arnoldi
  - Jacobi-Davidson

# Approaches for the solution of combustion problems

- Method A: Large Eddy Simulation
  - > Too complex
- Method B: Simplified Geometry
  - > Too simple
- Method C: Helmholtz equation
  - > Nonlinear Eigenvalue problems

# The Helmholtz Equation

Wave equation

↓

Helmholtz equation + B.C.'s

↓

Discretization (Galerkin Method)

↓

Matrix form: $(A + wB(w) + w^2C)\ P = 0$

With flame response effect:

$(A + D(w) + wB(w) + w^2C)\ P = 0$

# Solving Eigenvalue Problems

- Standard eigenvalue problem: $Ax = \lambda x$
- Power method: $Ax_k = x_{k+1}/c_k$
- Subspace methods: $W^*AVy = \theta W^*Vy$
- Arnoldi: Krylov subspace: $\mathrm{span}(x, Ax, A^2x, ..., A^kx)$
- Jacobi-Davidson:
  - $\theta_{k+1} = x_k^*Ax_k$
  - Solve for t: $(I-u_ku_k^*)(A - \theta_{k+1}I)(I-u_ku_k^*)t=-r_k$

# Non-Linear Eigenvalue Problems

- Linearization:
  - Fixed point method creates quadratic problem, linearization increases problem size
- Arnoldi-type:
  - Original Arnoldi can't be extended, different subspace expansion needed
- Jacobi-Davidson
  - Easily extended: Same correction equation based on Shift-and-Invert with preconditioning

# Performance

## Test problem provided by CERFACS

- Linearized Arnoldi
  - 135 s
  - Finds 5 eigenvalues out of 10
- Jacobi Davidson
  - 21 s
  - Finds the same 5 eigenvalues

# Current solution method for Combustion Problem

- Linearization:
  - Impedance is modeled
  - Fixed point: $(A + D(w_{k-1}) + w_k B + w_k^2 C)\, P = 0$
  - Quadratic problem written as:

$$\begin{bmatrix} 0 & -I \\ A+D & B \end{bmatrix} \begin{bmatrix} P \\ w_k P \end{bmatrix} + w_k \begin{bmatrix} I & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} P \\ w_k P \end{bmatrix} = 0$$

# Next Phase: Goals

- Improve current iterative solution method (Arnoldi-type) for Combustion problem
- Implement Jacobi-Davidson for test problems
- Compare results with Arnoldi
- Improve Jacobi-Davidson for Nonlinear eigenvalue problems

# Summary

- Combustion Modes
- Nonlinear Eigenvalue problems
- Methods
  - Arnoldi
  - Jacobi-Davidson
- Next phase
  - Comparison of methods
  - Improvement of methods