Using CONTACT in dynamical simulations

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Outline

About VORtech

- Founded in 1996 in Delft.
- Specialized in mathematical consultancy and development of high performance scientific software.
- Broad range of customers.

CONTACT

- Software that solves contact problems between two objects (e.g. train wheel & rails).
- Aims to be the worlds fastest detailed contact software.
- Originally developed by Prof.dr.ir. J.J. Kalker of TU Delft.
- Taken over by VORtech, now further developed by Dr.ir. E.A.H. Vollebregt.

Goal of this project

- CONTACT focusses on stationary problems.
- Research: how can CONTACT be used for dynamical contact problems?
- Main problem: simulation of a train over a bridge.

Elasticity theory

- Displacement $\mathbf{u}(x, y, z, t)$: Particle originally located at (x, y, z) moves to $(x, y, z) + \mathbf{u}(x, y, z, t)$.

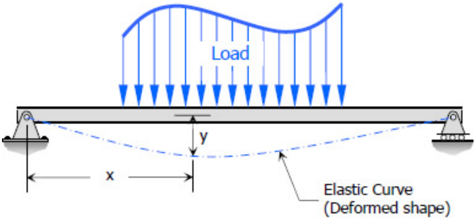

Figure: Elastic curve

The elasticity equation

$$G\Delta\mathbf{u} + (\lambda + G)\nabla(\nabla \cdot \mathbf{u}) + \mathbf{F} = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}$$

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Initial conditions:

$$u(x,y,z,0) = u_0(x,y,z)$$
$$\frac{\partial u}{\partial t}(x,y,z,0) = v_0(x,y,z)$$

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Example boundary conditions:

$$u(x, y, z, t) = 0 \text{ for } (x, y, z) \in \partial\Omega_1$$
$$\sigma\mathbf{n} = p \text{ for } (x, y, z) \in \partial\Omega_2$$

Contact theory

- Describes pressure distribution at the boundary of two objects.
- Different contact models:
  - Hertz model
  - Johnson-Kendall-Roberts (JKR) model
  - Derjaguin-Muller-Toporov (DMT) model
  - Maugis-Dugdale model (MD) model

Time integration schemes

General form of Newton's equation of motion

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What time integration scheme should be used? Possibilities:

- Runge-Kutta / Radau methods
- Verlet method
- Newmark-Beta method

Time integration schemes

General form of Newton's equation of motion

$$M(\mathbf{x})\ddot{\mathbf{x}} + P(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{F}(t)$$

Let $\mathbf{y} = \dot{\mathbf{x}}$, so that

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{y} \\ \dot{\mathbf{y}} &= M^{-1}(\mathbf{x})(\mathbf{F}(t) - P(\mathbf{x}, \mathbf{y})) \end{aligned} \tag{1}$$

which is of the form

$$\dot{\mathbf{z}} = \mathbf{g}(t, \mathbf{z})$$

for $\mathbf{z} = \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix}$.

Runge-Kutta / Radau methods

All Runge-Kutta methods have the form

$$\mathbf{z}_{k+1} = \mathbf{z}_k + \sum_{i=1}^{n} b_i \mathbf{k}_i, \text{ where}$$

$$\mathbf{k}_i = \Delta t \mathbf{g}(t_k + c_i \Delta t, \mathbf{z}_k + \sum_{j=1}^{n} a_{ij} \mathbf{k}_j)$$

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$$\begin{array}{c|cccc} c_1 & a_{11} & a_{12} & \dots & a_{1n} \\ c_2 & a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ c_b & a_{n1} & a_{n2} & \dots & a_{nn} \\ \hline & b_1 & b_2 & \dots & b_n \end{array} = \begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ \hline & 1/6 & 1/3 & 1/3 & 1/6 \end{array}$$

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$$\begin{array}{c|ccc} c_1 & a_{11} & a_{12} & a_{13} \\ c_2 & a_{21} & a_{22} & a_{23} \\ c_3 & a_{31} & a_{32} & a_{33} \\ \hline & b_1 & b_2 & b_3 \end{array} = \begin{array}{r|ccc} 0 & \frac{1}{9} & \frac{-1-\sqrt{6}}{18} & \frac{-1+\sqrt{6}}{18} \\ \frac{3}{5} - \frac{\sqrt{6}}{10} & \frac{1}{9} & \frac{11}{45} + \frac{7\sqrt{6}}{360} & \frac{11}{45} - \frac{43\sqrt{6}}{360} \\ \frac{3}{5} + \frac{\sqrt{6}}{10} & \frac{1}{9} & \frac{11}{45} + \frac{43\sqrt{6}}{360} & \frac{11}{45} - \frac{7\sqrt{6}}{360} \\ \hline & \frac{1}{9} & \frac{4}{9} + \frac{\sqrt{6}}{36} & \frac{4}{9} - \frac{\sqrt{6}}{36} \end{array}$$

Runge-Kutta / Radau methods

- Explicit (Runge-Kutta) methods:
  - Forward Euler
  - Runge-Kutta 4
- Implicit (Radau) methods:
  - Backward Euler
  - Radau5

The Verlet method

Applicable for problems in the form $\ddot{\mathbf{x}} = \mathbf{g}(\mathbf{x})$.

$$\mathbf{x}_1 = \mathbf{x}_0 + \mathbf{v}_0 \Delta t + \frac{(\Delta t)^2}{2} \mathbf{g}(\mathbf{x}_0), \text{ and}$$

$$\mathbf{x}_{k+1} = 2\mathbf{x}_k - \mathbf{x}_{k-1} + (\Delta t)^2 \mathbf{g}(\mathbf{x}_k) \qquad \text{for } k \geq 1$$

Newmark's method

Implicit algorithm described by

$$
\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{x}_k + \Delta t \mathbf{v}_k + \frac{(\Delta t)^2}{2}((1-2\beta)\mathbf{a}_k + 2\beta\mathbf{a}_{k+1}) \\
\mathbf{v}_{k+1} &= \mathbf{v}_k + \Delta t((1-\gamma)\mathbf{a}_k + \gamma\mathbf{a}_{k+1}
\end{aligned}
$$

The acceleration $\mathbf{a}_{k+1}$ should be derived from the equations of motion:

$$
M(\mathbf{x}_{k+1})\mathbf{a}_{k+1} + P(\mathbf{x}_{k+1}, \mathbf{v}_{k+1}) = \mathbf{F}(t_{k+1})
$$

Contact between sphere and elastic half space

- Sphere is dropped from a height of 1m.
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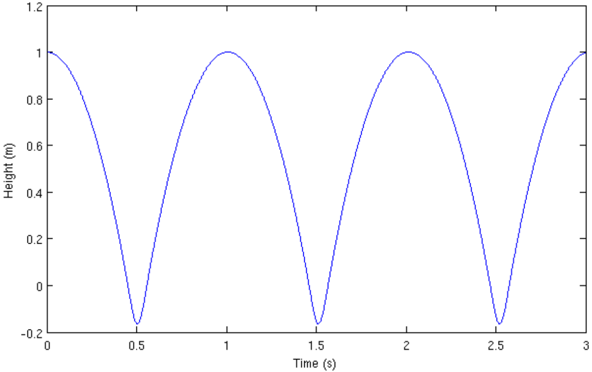
By Newton's second law:

$$\ddot{z} = \frac{4}{3m} E^* R^{1/2} \max(0, -z)^{3/2} - g$$

Contact between sphere and elastic half space

Using Radau5:



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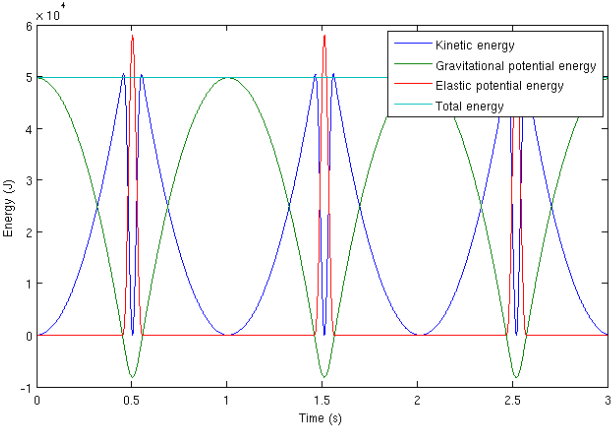

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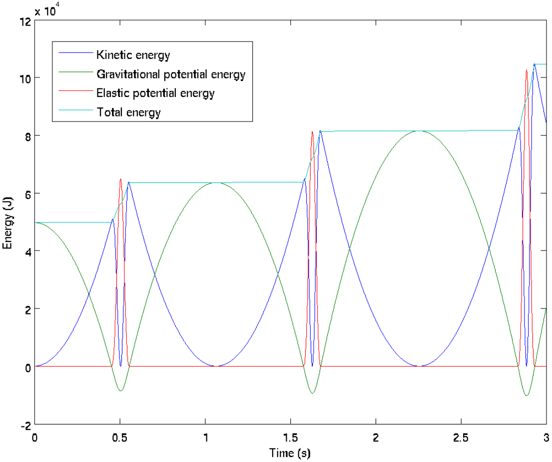
Using Forward Euler:



Deformation of a bridge

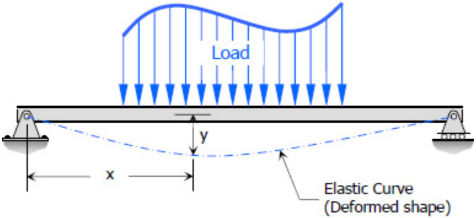


Figure: Elastic curve

Euler-Bernoulli beam equation (1D):

$$EI\frac{\partial^4 u}{\partial x^4} = -\rho\frac{\partial^2 u}{\partial t^2} + p(x,t) \tag{2}$$

$$u(0,t) = \frac{\partial u}{\partial x}(0,t) = 0 \tag{3}$$

$$u(L,t) = \frac{\partial u}{\partial x}(L,t) = 0 \tag{4}$$

Deformation of a bridge

Can be discretised using Finite Differences.

$$\frac{\partial^4}{\partial x^4}u_i(t) \approx \frac{u_{i-2}(t) - 4u_{i-1}(t) + 6u_i(t) - 4u_{i+1}(t) + u_{i+2}(t)}{\Delta x^4}$$

Results into equation

$$\ddot{\mathbf{u}} = A\mathbf{u} + p(\mathbf{u}, t)$$

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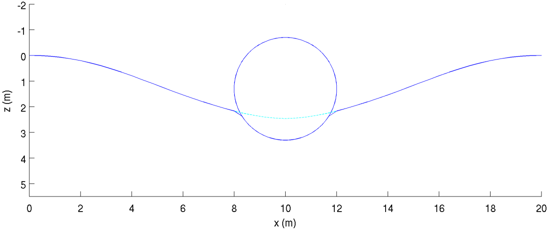
Problem: CFL condition for the Verlet method:

$$\frac{EI}{\rho} \frac{\Delta t^2}{\Delta x^4} \leq \frac{1}{4}$$

Combining global and local deformations

A bridge can deform both globally and locally:



How can both phenomenons be taken into account?

Combining global and local deformations

Possibilities:

- Complete Finite Element model
  - Computationally expensive, but accurate.
- Combining the beam equation (for global deformations) and CONTACT (for local deformations).
  - Hard, since both phenomenons are not independent of each other.

Conclusion & Discussion

Literature study:

- Elasticity theory
- Contact theory & models
- Time integration methods for elastodynamics, tested on two different problems:
  - Dynamical contact between sphere and elastic half-space.
  - Global deformation of a beam.
- Implicit methods like Radau5 are preferred.

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Research:

- Research how CONTACT can be used for dynamical contact problems.
- Apply the Finite Element method.
- Long-term goal: perform train/bridge simulation, taking both global and local deformations into account.