Master Thesis Project (WI5005)
Section: Numerical Analysis

Overtopping Failure in Levees

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Project description

Grass covers offer a level of erosion protection against hydrodynamic loads induced by overtopping waves. The development of an accurate failure criterion for grass covers has been hampered by the lack of understanding of how grass fails.

Methods have been developed to predict the hydrodynamic loads that act on the levee surface and consequently are transmitted to the porous grass cover and subsoil [1]. However, it is unknown how these hydrodynamic surface loads on the soil are distributed amongst the pore water pressures and effective stresses. Models that assume an undrained behaviour of the porous media fail to capture the correct pressure profile in the soil without applying an artificial compressibility of water [2]. The unknown distribution between the effective stresses and pore water pressures offers a significant challenge for Hydraulic engineers to simulate the transmission of stresses into the porous grass cover. Little is thereby also known on how grass responds to hydrodynamic loads and when it would fail.

A new 2D continuum based modelling method for porous media which only requires for the total stresses and shear stresses at the levee surface to be determined by the user has been formulated (see the Appendix, unpublished). Based on this information the model determines how the distribution of the surface stresses into pore water pressures and effective stresses changes with time. The model should thereby be able to resolve any time dependent and spatially distributed surface load.

In this project you will analyse the proposed model, investigate which numerical techniques can be used to obtain approximate solutions to the model, calculate these approximate solutions and investigate the approximate solutions.

Tasks

During this project several tasks have to be completed, which are (at least):

1. Literature research on the proposed model;
2. Literature research on current state-of-the-art numerical solving techniques;
3. Literature report;
4. Selection and application of a solving technique;
5. Analysis of approximate solutions;
6. Master Thesis.

Appendix - The proposed model

Within the model the following variables are present:

- t Time in seconds
- x Horizontal location in meters
- z Vertical location in meters
- u_x Horizontal displacement in meters
- u_z Vertical displacement in meters
- P Pressure in Pascals
- ϵ Volumetric strain (no unit)
- ω Vorticity (no unit)

The levee occupies the region

$$\Omega = \left\{(x, z) \in \mathbb{R}^2 : 0 \leq x \leq L, -Z \leq z \leq 0\right\},$$

where $z = 0$ reresents the top of the levee (the "maaiveld"). Note that we consider a cross section of a complete levee.

Within the levee the physics are governed by the following partial differential equations:

$$(1-p)\frac{\partial^2 \epsilon}{\partial t^2} + \frac{\gamma_w}{\rho_p K_s}\frac{\partial \epsilon}{\partial t} - \frac{3\beta - 2a}{\rho_p}\frac{\partial^2 \epsilon}{\partial x^2} - \frac{3\beta - 2a}{\rho_p}\frac{\partial^2 \epsilon}{\partial z^2} = 0$$

$$\rho_p(1-p)\frac{\partial^2 \omega}{\partial t^2} - \frac{a}{2}\frac{\partial^2 \omega}{\partial x^2} - \frac{a}{2}\frac{\partial^2 \omega}{\partial z^2} = 0$$

$$-\frac{\partial^2 P}{\partial x^2} - \frac{\partial^2 P}{\partial z^2} + \frac{\gamma_w}{K_s}\frac{\partial \epsilon}{\partial t} + \rho_w \frac{\partial^2 \epsilon}{\partial t^2} = 0$$

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} - \epsilon = 0$$

$$\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z} - \omega = 0$$

At the left ($x = 0$) and right ($x = L$) boundary of Ω the following boundary conditions are imposed:

$$u_x = 0 \qquad \frac{\partial u_z}{\partial x} = 0 \qquad \frac{\partial \epsilon}{\partial x} = 0 \qquad \frac{\partial P}{\partial x} = 0$$

Deep in the levee ($z = -Z$) the following boundary conditions are imposed:

$$\frac{\partial u_x}{\partial z} = 0 \qquad u_z = 0 \qquad \frac{\partial P}{\partial z} = 0$$

At the top of the levee ($z = 0$) the following boundary conditions are imposed:

$$-\frac{\alpha}{2}\omega - \alpha\frac{\partial u_z}{\partial x} = \tau$$

$$-\beta\epsilon - \alpha\frac{\partial u_z}{\partial z} + P = \sigma$$

$$-\beta\frac{\partial \epsilon}{\partial z} - \alpha\frac{\partial^2 u_z}{\partial z^2} + \frac{\partial P}{\partial z} = -\frac{\partial \tau}{\partial x}$$

where τ and σ are known functions and β and α follow from the shear modulus and compressibility modulus.

Referenties

[1] Van Bergeijk, V.M.. Warmink, J.J., Van Gent, M.R.A. and Hulscher, S.J.M.H. *An analytical model of wave overtopping flow velocities on dike crests and landward slopes*, Coastal Engineering, 149, pp 28 – 38, 2019.

[2] Ye, J. and Jeng, D.-S. *Effects of bottom shear stresses on wave-induced dynamic response in a porous seabed: PORO-WSSI (shear) model*, Acta.Mech.Sin. 27(6), pp 898 – 910, 2011.