Literature Report

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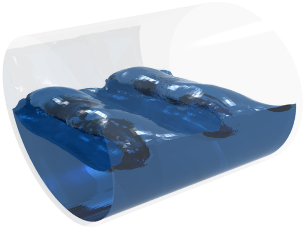
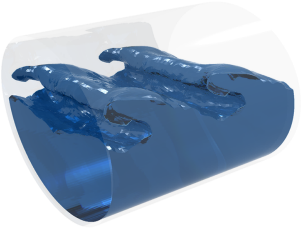
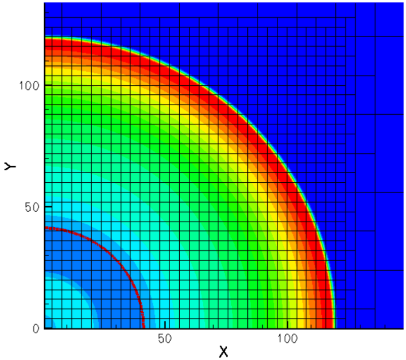
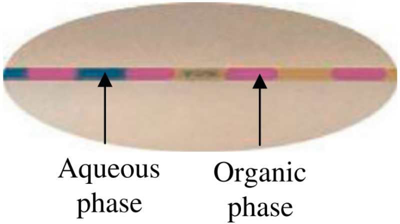
Applications









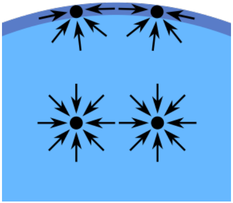
Two-phase flow model

The evolution of the momentum field in a two phase flow problem is given by the Navier-Stokes equations:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\nabla \cdot \boldsymbol{u} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\nabla \cdot \mu(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^t) + \boldsymbol{g}$$

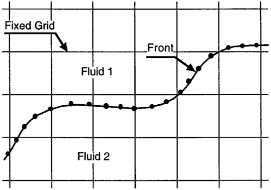
The term $\boldsymbol{g}$ includes surface tension.

Handling surface tension, along with interface jump conditions requires knowledge of the location of the phase interface.



Interface tracking and interface capturing

Interface tracking methods track only the interface location.
⇒ Topology changes are difficult to handle.



Tryggvason, G., et al. "A front-tracking method for the computations of multiphase flow."

Interface capturing methods differentiate between phases with an indicator function.
⇒ Movement of the interface is directly described by changes in fluid.

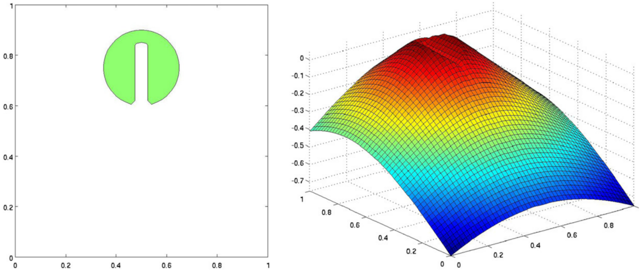
Interface capturing methods have no problems with topology changes of the interface.

Level set method

The level set method uses the signed distance function to the interface.

The interface is the zero level set of this function.

The phase indicator is a smooth function, so an advection method can be applied directly.



Kuzmin, D. "An optimization-based approach to enforcing mass conservation in level set methods."

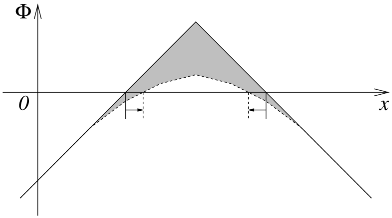
Level set method

Benefits:

- Direct application of scheme for a hyperbolic conservation law

Drawbacks:

- Conservation of level set function $\not\Rightarrow$ conserved area enclosed by interface.



van der Pijl, S. P. Computation of bubbly flows with a mass-conserving level-set method

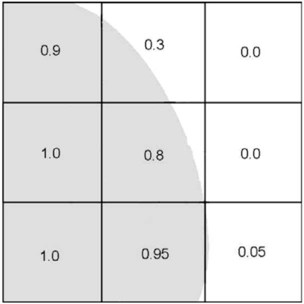
Mass conservation is improved somewhat by using more accurate discretisations.

Volume of fluid method

The VoF method assigns colours to the different phases

$$\chi = \begin{cases} 0, & \text{fluid `0'} \\ 1, & \text{fluid `1'} \end{cases} \quad \Rightarrow \quad \psi = \frac{1}{|\Omega|} \int_\Omega \chi \, d\Omega$$

The colour function is discontinuous at the interface so advection methods cannot be applied on the volume fractions.

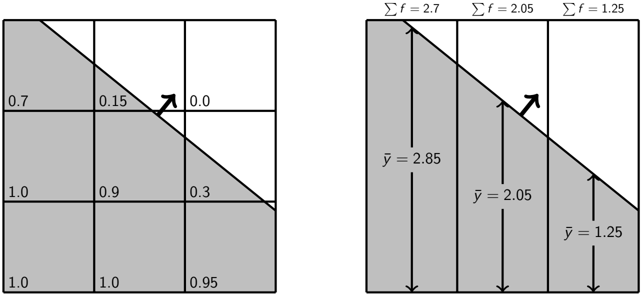


Volume of fluid method

Interface capturing

Interface orientation is extracted from average fluid heights.
Interface must not cross the top or bottom of the cell.

Approximates the non-existent derivatives of the discontinuous colour function.

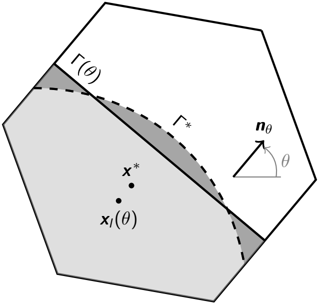


Volume of fluid method

Interface capturing

Using centres of mass only requires local data.

Interface only defined by angle ⇒ minimise 1D cost function.
Cost function = distance between fluid centres.

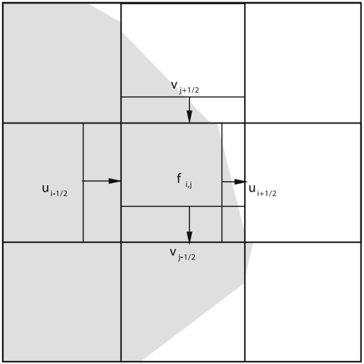


Volume of fluid method

Interface advection

'Donating regions' determine flux.

This requires dimensional splitting, and a condition on time step to avoid overfilling.

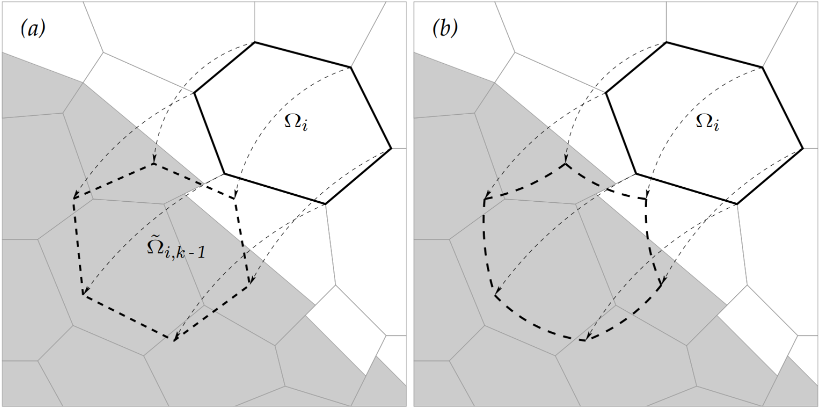


Volume of fluid method

Interface advection

Pre-image of cells do not have straight edges ⇒ possible volume errors.

Calculating the curvature of edges is expensive.



Volume of fluid method

Benefits:

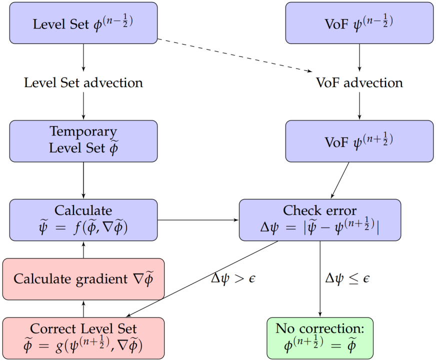
- Mass is conserved when using donating regions

Drawbacks:

- Expensive interface reconstruction method required
- Condition on the time step to ensure mass conservation
- VoF reconstruction and advection requires a rectangular grid

MoF method possibly more useful than VoF reconstruction.

Dual interface method



Oud, G. T. "A dual interface method in cylindrical coordinates for two-phase pipe flows."

Dual interface methods combine level set and volume of fluid methods.

Idea: use level set to improve efficiency VoF reconstruction step.
Use VoF to keep level set mass conserving.

Interface orientation is extracted from the level set field by linear interpolation.

How do MoF and LS reconstruction compare?

Dual interface method

Benefits LS:

- Direct application of scheme for a hyperbolic conservation law

Drawbacks LS:

- No mass conservation

Benefits VoF:

- Mass is conserved when using donating regions

Drawbacks VoF:

- Expensive interface reconstruction method required
- Condition on the time step to ensure mass conservation
- Donating regions require a rectangular grid

Dual interface method

Benefits LS:

- Direct application of scheme for a hyperbolic conservation law

Drawbacks LS:

- ~~No mass conservation~~

Benefits VoF:

- Mass is conserved when using donating regions

Drawbacks VoF:

- ~~Expensive interface reconstruction method required~~
- Condition on the time step to ensure mass conservation
- Donating regions require a rectangular grid

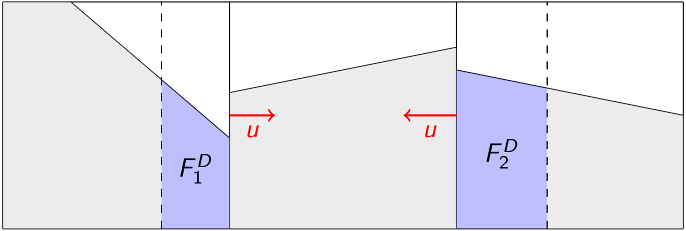
Only drawbacks from donating regions remain.

Solving the problems

The problems in dual methods come from the use of donating regions.
How to advect the interface without using donating regions?

Instead of using donating regions

$$\Psi^{n+1} - \Psi^{n} = - F_1^D\Big| - F_2^D\Big|$$

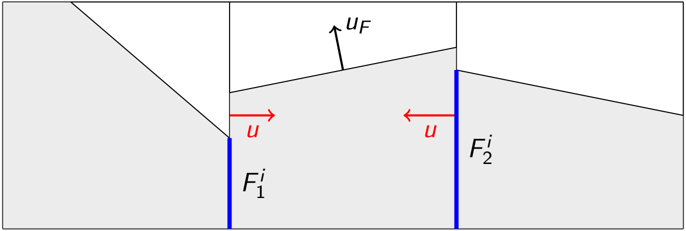


Solving the problems

Use the *momentary fluxes* instead. Wanted: LS field that satisfies

$$\frac{\mathrm{d}\Psi(\Phi)}{\mathrm{d}t} + F_1^i(\Phi^{n+1})\Big| + F_2^i(\Phi^{n+1})\Big| = 0$$

and a mass conserving constraint.
$\Rightarrow$ Good initial guess required!

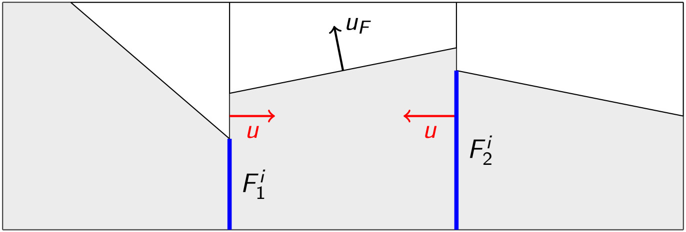


Solving the problems

The change in VoF is linked to the change in the level set field by the length of the interface

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = \frac{l_{if}}{h^2}\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{l_{if}}{h^2}u_F$$

Change in LS matters most near the interface
⇒ Large cells in interior of fluid ⇒ Polygonal grid



Solving the problems

Goal: dual interface method with unrestricted time step that can be applied on unstructured grids. $\Rightarrow$ No donating regions.
Mass conservation must be guaranteed!

FVM are difficult to implement on unstructured grids.
FEM may not be stable for the advection equation.
Combine FVM+FEM $\Rightarrow$ Discontinuous Galerkin

LS field should satisfy nonlinear equation under mass conserving constraint.
$\Rightarrow$ Similar to optimisation-based mass correction for LS.

LS advection equation on polygons requires DG.

Discontinuous Galerkin

Discontinuous Galerkin is a combination of FVM and Galerkin FEM methods.

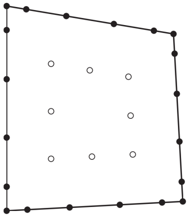
$$\mathcal{M}^k \frac{\mathrm{d}\phi_h^k}{\mathrm{d}t} + \mathcal{S}^k \phi_h^k = \int_{\partial D^k} \hat{\boldsymbol{n}} \cdot \left[\boldsymbol{u}\phi_h^k - \boldsymbol{f}^* \right] N_i^k d\Gamma$$

$$\mathcal{M}_{ij}^k = \int_{D^k} N_i^k N_j^k \, d\Omega, \quad \mathcal{S}_{ij}^k = \int_{D^k} N_i^k \; \boldsymbol{u} \cdot \nabla N_j^k \; d\Omega$$

Adjacent cells are only coupled by the *numerical flux*.
$\Rightarrow$ The value of the level set does not matter if the gradient does not change.

Discontinuous Galerkin

On polygons, evaluate integrals using *nodal points*:



More nodal points $\Rightarrow$ higher order basis functions $\Rightarrow$ higher accuracy

Balance between smaller elements and more nodal points.

Handling unique elements is expensive $\Rightarrow$ use reference elements

Optimisation based mass correction

Avoiding donating regions means that VoF advection likely cannot be mass conserving. Steps similar to level set mass correction steps need to be taken.

Introduce a control variable v and minimise a functional, e.g.

$$J(\phi, v, \widetilde{\phi}) = \frac{1}{2}||\phi - \widetilde{\phi}|| + \frac{\beta}{2}||v||^2$$

under a mass conserving constraint.

Solve the resulting Lagrangian with a fixed-point method.

Optimisation based mass correction

Using mass correction steps on VoF is similar to correcting the level set with independent cells (DG).

Problem: resulting solution is not unique.
$\Rightarrow$ Additional constraints are needed to get a unique solution.

Research goals

The goal is to develop, on a Cartesian grid, a method that

- is mass conserving,
- has no condition on the time step (or a less restrictive one),
- uses only methods that can be extended to polygonal cells.

The following research questions are posed:

(R1) *Is it worthwhile to use moment of fluid interface reconstruction alongside the level set implementation?*

(R2) *Which functional or optimality condition should be used to ensure a unique solution in the optimisation problem?*

(R3) *Is it feasible to implement an improved method on polygonals with 5 or more vertices or should it be limited to triangles/quads?*

(R4) *Is it possible for the improved method to have no stability condition on the time step size?*

Time schedule