MSc. Project: Towards Robust Numerical Solvers for Nuclear Fusion Simulations Using JOREK

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If current times have taught us anything, it is that the need for alternative energy sources is becoming more and more imperative and many research initiatives are trying to bring us one step closer towards sustainability. One such initiative is the concept of controlled fusion, where we try to imitate at smaller scale the nuclear fusion processes taking place in the centre of stars that form helium nuclei. This process results in a release of energy, which can serve as a source of power generation for society.

While this ambition seems promising in theory, many practical engineering and simulation obstacles need to be resolved before we can economically realize nuclear fusion. Building the largest operational reactor (ITER), which is a global joint initiative, requires tremendous financial and computational resources. Given the unprecedented scale and scope of the project, many preliminary design choices for the reactor and direction of fusion rely on large-scale simulations of non-linear *magneto-hydrodynamic* (MHD) equations, which model the macroscopic plasma equilibrium and stability.

Numerical simulations of these phenomena require the solution of large linear systems, making the use of direct solution methods infeasible. Moreover, the coefficient matrix is often severly ill-conditioned. As a result, iterative methods with tailored physics-based preconditioning are indispensible in simulating and understanding these non-linear dynamics.

One of the most advanced and established codes, *JOREK*, is widely used to simulate non-linear MHD in tokamaks. The code is developed by an international community with strong contributions by the leading research institute on numerics for nuclear fusion, the *Max Planck Institute for Plasma Physics* (IPP) in Garching, Germany. The solver behind the code relies on a physics-based preconditioner using the Generalized Minimal Resdiual (GMRES) method. The preconditioner is effective in improving the general convergence. It moreover leads to reduced memory and computational costs. However, GMRES is notoriously known to become more and more expensive when many iterations are needed and the overall reduction in the number of iterations dictates the efficiency of the solver.

The aim and ambition of this joint project between *Delft Insitute of Applied Mathematics* (DIAM) and IPP is to design and work towards a state-of-the-art numerical solver for fusion simulations. In particular, the development of a robust preconditioner is envisioned such that large-scale instabilities can be simulated accurately and within a reasonable amount of time.

The main contribution of this project is to understand and mathematically define the convergence behavior of the current preconditioner in JOREK. Using these insights, future projects will be developed where we work towards an interdisciplinary approach in designing robust numerical solvers for plasma fusion simulations.

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Research overview

The MSc. project will develop an understanding of the following research questions and contains the following phases:

Literature phase (+/- 3 months)

- Study MHD equations
- Study linearized time discretization
- Study literature on JOREK and structure of the solver
- Study literature on related convergence issues (ill-conditioning for GMRES)
- Write literature report and give an overview presentation
- Define concrete research questions towards designing robust preconditioning

Research phase (+/- 4-5 months)

- Obtain data from IPP – example of a linear system dealing with a particular use-case
- Research properties of the linear system
- Stress-test behaviour of GMRES against these properties
- Differentiate convergence properties of the matrix between linear and non-linear transition
- Document results in thesis form

Valorization phase (+/- 1-2 months)

- Finalize thesis
- Present results to IPP containing recommendations for the preconditioner
- Implement/construct initial framework for potential recommendations
- Discuss how the recommendations can be generalized to other use-cases

Profile

A strong background in numerical methods and programming experience with MPI is recommended. During the project data of an example from IPP, resources from TU Delft and the DelftBlue supercomputer will be available to you.

References

1. Hoelzl, M., Huijsmans, et. al. (2021). The JOREK non-linear extended MHD code and applications to large-scale instabilities and their control in magnetically confined fusion plasmas. *Nuclear Fusion*, *61*(6), 065001
2. Holod, I., Hoelzl, M., Verma, P. S., Huijsmans, G. T. A., Nies, R., & JOREK Team. (2021). Enhanced preconditioner for JOREK MHD solver. *Plasma Physics and Controlled Fusion*, *63*(11), 114002

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