Numerical Methods for Differential Algebraic Equations

Project Proposal

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Introduction

DotX Control Solutions BV, a Dutch company based in Alkmaar, is specialized in designing and implementing complex control software for industrial processes. Due to our innovative approach, these processes may occur in a great deal of different applications, therefore, our clients come from a broad spectrum of industries. Over the past years, DotX has worked on projects in water management, steel production, wind turbine design, paper drying and many more. For more information on products, projects and clients please see our website.

One of the specific research fields in which DotX operates is Model-Predictive Control (MPC). MPC typically computes a (sub-) optimal control based on a mathematical model of the process in question. In standard packages, such models usually need to be formulated as dynamical systems with linear in- and outputs. Because of the inherent nonlinear nature of many applications, these standard packages have their limitations. DotX fills this gap by offering tailor-made software.

One of our core products is the DotX Nonlinear Predictive Controller (DNPC), which, as the name suggests, is able to cope with nonlinearities in the underlying model. Obviously, key components in this method are the dynamical systems. Though DotX has made significant progress in nonlinear modeling, there is always a need for even more realistic and complex models. This project focusses on a specific class of these nonlinear systems.

Theoretical background

A simple first-order dynamical model can be defined by

$$\dot{x}(t) = f(x(t), t), \quad x(0) = x_0, \tag{1}$$

on a time span $[0, T]$, where $x : \mathbb{R} \to \mathbb{R}^n$ is an unknown function. In general, such equations are hard to solve, i.e. the function $x(t)$ is hard to find analytically. Hence, one needs to resort to the next best thing: finding a numerical approximation.

Given a series of sample points $\{t_n\}_{0 \leq n \leq N}$ on $[0, T]$, a numerical method typically intends to approximate $\mathbf{x}_n \approx x(t_n)$ by transforming the continuous model to a discrete iteration:

$$\boxed{\begin{array}{c} \dot{x}(t) = f(x(t), t) \\ x(0) = x_0 \end{array}} \quad \Rightarrow \quad \boxed{\begin{array}{c} \mathbf{x}_{n+1} = \mathbf{F}(\mathbf{x}_n, t_n) \\ \mathbf{x}_0 = x_0 \end{array}}$$

Over the years, many of these numerical schemes have been developed (Euler, Adams-Bashford, Runge-Kutta, etc). Recall that the differential equation (1) is one of the most simple models. In many applications however, there are much more complex systems that need to be solved. It turns out that many of the existing methods are hard to be generalized to such complexer models.

For instance, consider an added term to the first-order nonlinear dynamical system:

$$\dot{x}(t) = f(x(t), y(t), t), \tag{2a}$$
$$0 = g(x(t), y(t), t), \tag{2b}$$

where both $x : \mathbb{R} \to \mathbb{R}^n$ and $y : \mathbb{R} \to \mathbb{R}^m$ are unkown functions. Finding an iterative scheme to approximate the solution is not straightforward and has many implications. Systems as these are part of the family of Differential Algebraic Equations (DAEs), and finding numerical approximations to their solutions is the centerpiece of this research project.

Note that, as in many studies in numerical mathematics, this research on numerical solutions to DAEs contains both a mathematical aspect (such as differential equations, numerical stability, optimisation, etc) and a computional side (accuracy, computation speed, etc).

Project Information

The highly mathematical nature of this research problem makes this project ideal for a student in the exact sciences such as mathematics or physics (at both technical or theoretical universities). The student is required to:

A. do an extensive literature review on Differential Algebraic Equations, and in specific numerical methods applied to these equations,

B. find a novel technique, or a best-practice method based on the literature review,

C. provide a mathematical analysis of existing and new methods, focussing on aspects as accuracy and computation time,

D. implement several methods in MATLAB,

E. do a numerical comparison of these methods, based on both theoretical and real life testcases.

Obviously, all these requirements can be altered upon negotation with student and superivisor. DotX on its turn will supervise the student in regular meetings by guiding the research process with our expertise in the subject, assisting with the MATLAB implementation and helping in writing a final document (for instance a project report or thesis).

More information

If you are interested, or in need of more information, please contact Drs. Eelco Nederkoorn at e.nederkoorn@dotxcontrol.com.