Image Reconstruction in MRI

The Possibilities of Portable, Low-cost MRI Scanners

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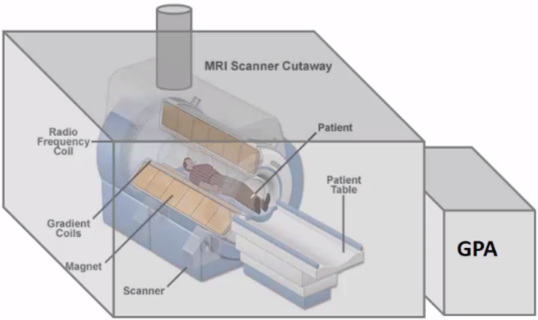
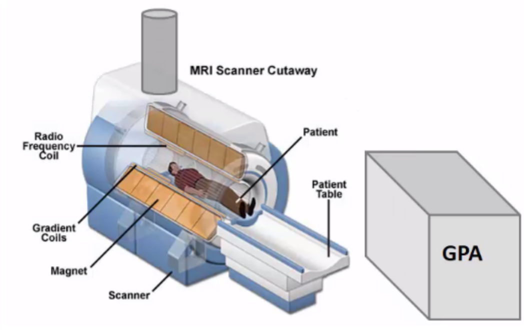
Research goal

To develop a sufficiently accurate image reconstruction algorithm that is able to effectively process signals produced by a low-cost MRI scanner.

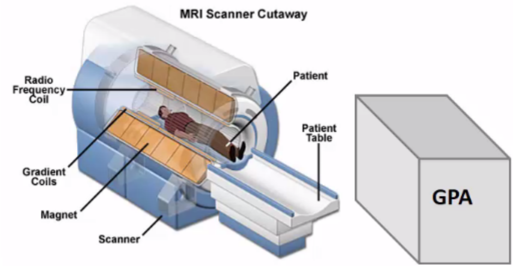
Standard MRI



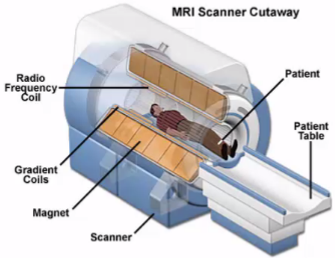
Standard MRI



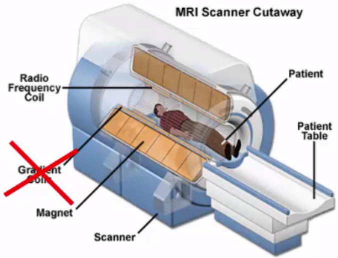
Standard MRI



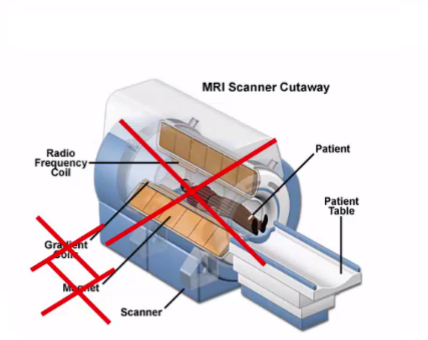
Standard MRI



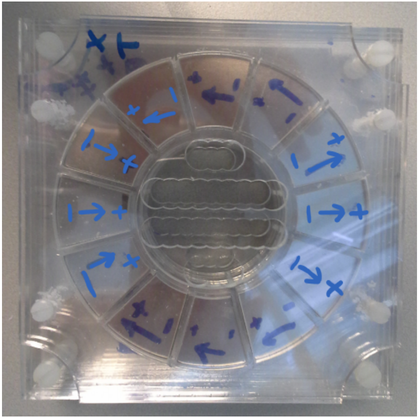
Standard MRI

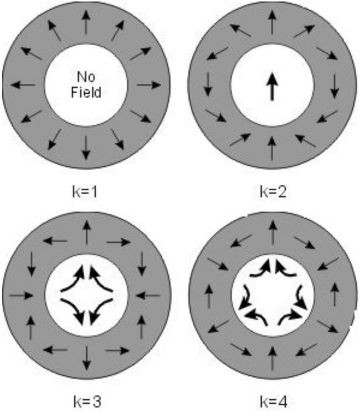


Standard MRI



Main Magnet





Magnetic Flux Density Field

(Loading video)

Signal Equation

$$S(t) = \int_{\mathbf{r}\in\mathbb{D}} c(\mathbf{r})\omega_0(\mathbf{r})e^{-t/T_2(\mathbf{r})}e^{-\mathrm{i}\phi(\mathbf{r},t)}f(\mathbf{r})d\mathbf{r}$$

Matrix form

$$\mathbf{y} \quad = \quad A \quad \mathbf{f}$$

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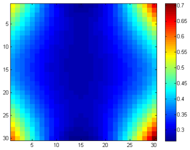
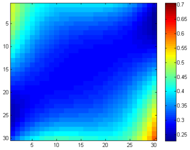
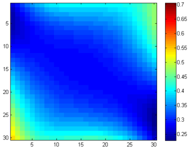


Rotation 1



Rotation 2



Rotation 3

$\vdots$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_{n_p} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_N \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_{n_p} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_N \end{bmatrix}$$

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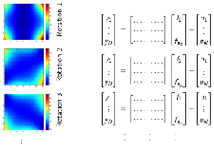
$$\vdots \qquad \vdots \qquad \vdots$$

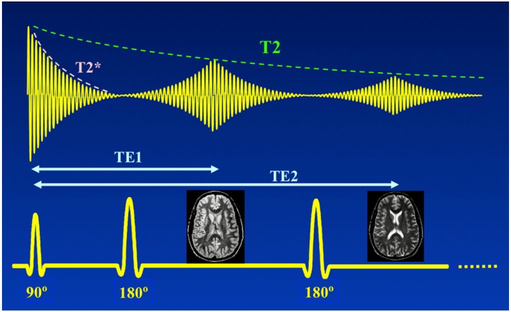
System of equations



$$\begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_{n_r} \end{bmatrix} = \begin{bmatrix} A_1 \\ \vdots \\ A_{n_r} \end{bmatrix} \mathbf{f} + \begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_{n_r} \end{bmatrix}$$

$\mathbf{y} \in \mathbb{C}^{Nn_r \times 1}$, $A \in \mathbb{C}^{Nn_r \times n_p}$, $\mathbf{f} \in \mathbb{C}^{n_p \times 1}$ and $\mathbf{n} \in \mathbb{C}^{Nn_r \times 1}$.

T_2 relaxation

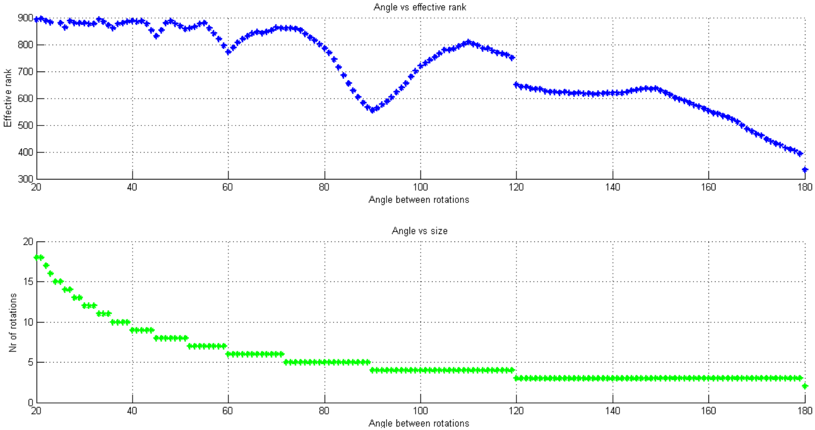


$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

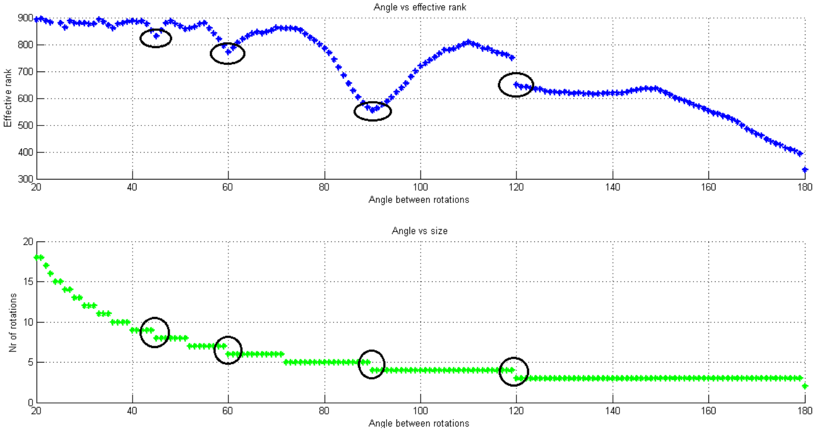
Nyquist

Sampling rate (f_s =bw)	Nyquist rate ($2.2f_s = 2.2$bw)	Upperbound dt
1 MHz	2.2 MHz	$4.5455 \cdot 10^{-7}$ s
2 MHz	4.4 MHz	$2.2727 \cdot 10^{-7}$ s
3 MHz	6.6 MHz	$1.5152 \cdot 10^{-7}$ s

Method of rotating



Method of rotating

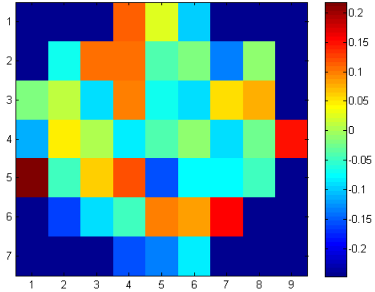


Method of rotating

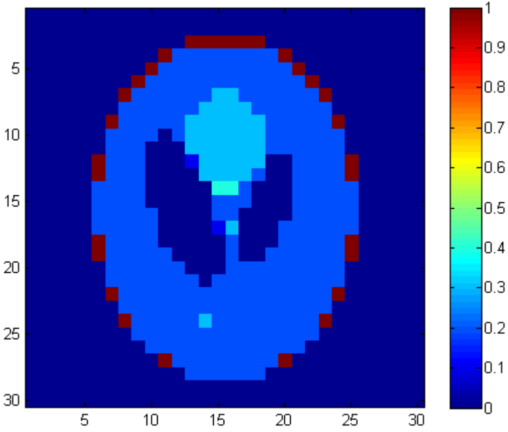
Bandwidth bw	min. number of rotations	max. angles	optimal angles	effective rank
3	5	89	72	860
2	7	59	48	864
1	10	39	33	813

Field perturbations

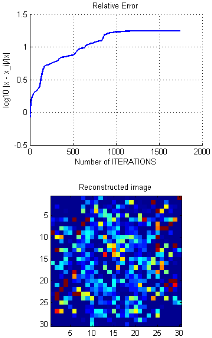
- 10^{-1} mT: accuracy measuring tool
- 1 mT: human mistakes



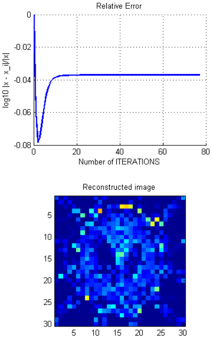
Phantom image



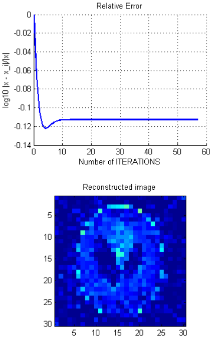
Field perturbations 10^{-1} mT



(a) angles= $70°$

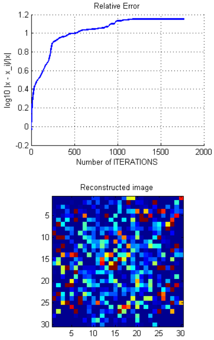


(b) angles= $30°$

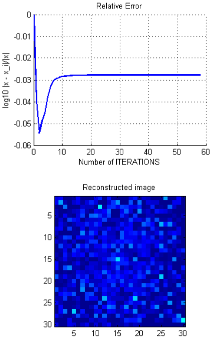


(c) angles= $10°$

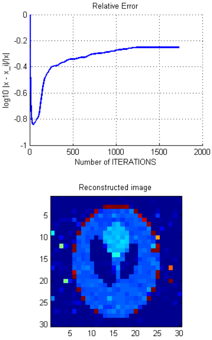
Field perturbations $1\ \mathrm{mT}$ and $10^{-2}\ \mathrm{mT}$



(a) angles= $70°$, 1mT

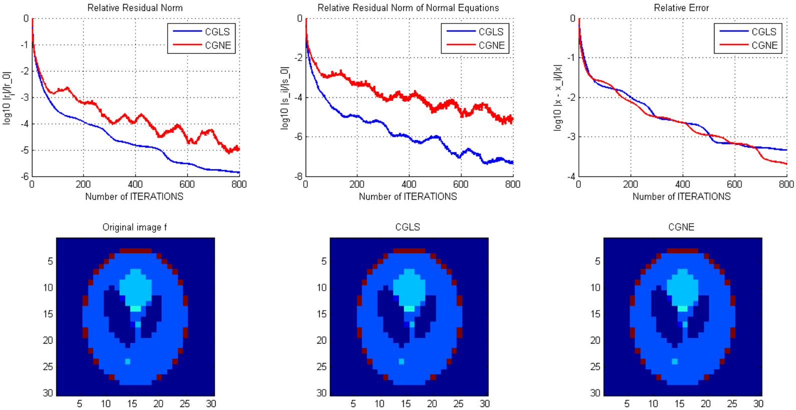


(b) angles= $10°$, 1mT

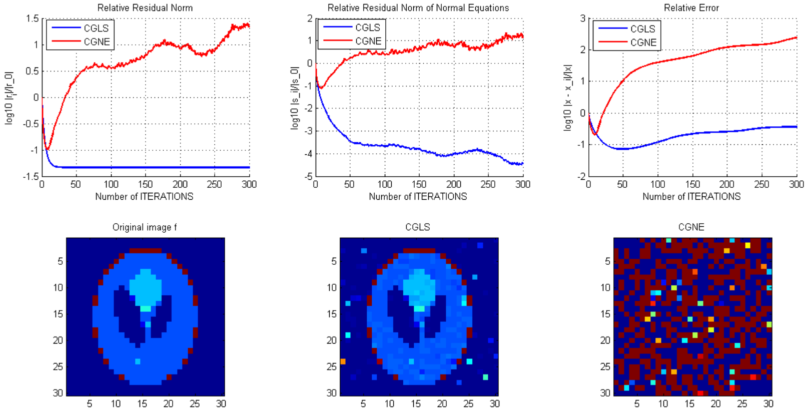


(c) angles= $70°$, 10^{-2}mT

CGLS vs CGNE without noise



CGLS vs CGNE with noise



CGLS with regularisation

$$\arg\min_{\mathbf{f}} ||\mathbf{y} - A\mathbf{f}||_2 + \lambda^2 R(\mathbf{f}), \qquad R(\mathbf{f}) = ||L\mathbf{f}||.$$

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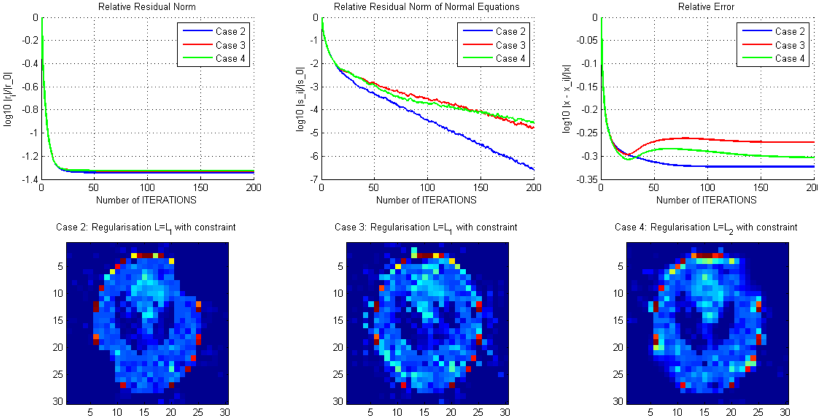
1. regularisation type 0: $L = I$,
2. regularisation type 0 with air constraint: $L = \begin{bmatrix} I & \text{constraint} \end{bmatrix}^T$,
3. regularisation type 1 with air constraint: $L = \begin{bmatrix} L_1 & \text{constraint} \end{bmatrix}^T$,
4. regularisation type 2 with air constraint: $L = \begin{bmatrix} L_2 & \text{constraint} \end{bmatrix}^T$,

Regularisation, case 2,3,4



- Analysis of the model (Nyquist rate, rotations, frequency bandwidth, perturbations)
- T_2^*
- CGLS, Tikhonov, $L = I$

Future research

- Analysis encoding methods
- modeling coil sensitivity and T_2 or T_2^*
- 2D to 3D


Thank you!