Dynamic Positioning Simulator

Interim Report

Jalitha Wills
1104551
j.d.wills@student.tudelft.nl

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TU Delft
Technische Universiteit Delft

VSTEP
Virtual Safety Training & Education Platform

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Preface

This report is written as a result of the literature study for my Master's Thesis project for Applied Mathematics of Delft University of Technology, carried out at VSTEP BV. Since March 2007 I have been working on Dynamic Positioning of vessels, especially for use in a training simulator.

Jalitha Wills

Rotterdam, June 2007

Used symbols and units

Quantities

In this section a few tables can be found with the definition of the quantities used.[1]

Quantity	Unit symbol
Length	m
Mass	kg
Time	s

Table 1: Introduction on (Physical) Quantities and their units: Basic Units.

Physical Quantity	Unit symbol
Plane angle	rad
Force	$N = kg \cdot m/s^2$
Work	$J = N \cdot m$
Power	$W = J/s$
Frequency	$Hz = s^{-1}$
Pressure	$Pa = N/m^2$

Table 2: Introduction on (Physical) Quantities and their units: Special Quantities.

[1] From [9, 10].

Physical Quantity	Unit symbol
Area	m^2
Volume	m^3
Density	kg/m^3
Velocity	m/s
Acceleration	m/s^2
Angular Velocity	rad/s
Angular Acceleration	rad/s^2
Dynamic Viscosity	Ns/m^2
Kinematic Viscosity	m^2/s

Table 3: Introduction on (Physical) Quantities and their units: Other Units.

Symbols

In this paper several symbols are used and defined. In this section an overview on them.

Symbols from Chapter 5

Roman Symbols
A_L: cross section of the wind area. (longitudinal projection)
A_{LS}: cross section of the submerged area. (longitudinal projection)
a_r: resistance augmentation factor.
A_T: section of the wind area. (transverse projection)
A_{TS}: section of the submerged area. (transverse projection)
c: wave velocity.
C_D: dimensionless drag coefficient.
C_L: dimensionless lift coefficient.
C_{*_c}: α_c dependent current load coefficient.
C_{*_w}: α_{rw} dependent wind load coefficient.
$C_{*_{wd}}$: α_{wd} and f_{wd} dependent wave load coefficient.
C_{Q_0}: torque coefficient of the propeller in open water.
C_{Q_b}: torque coefficient of the propeller working in the wake of the ship.
C_T: thrust coefficient.
D: cylinder diameter.
D_p: diameter propeller.
F_D: drag force.
F_l: lift force.
F_∇: buoyant force.
f_v: vortex shedding frequency.
f_{wd}: regular wave frequency.
G: center of gravity
g: gravitational constant.
h: distance below the water surface.
$H_{1/3}$: significant wave height.
J: advance number from propeller.
k: wave number.
L: length of the ship.
m: mass of the ship.
n: number of revolutions per minute of propeller.
N_c: current moment.
N_w: wind moment.
N_{wd}: wave moment.
$\hat{\nabla}$ (nabla): volume of the submerged part of the ship.
O: origin of earth-bound coordinate system.
p: hydrostatic pressure.
P_E: effective power of the propulsion.
P_T: power from the thrust.
Re: Reynolds number.
Q_p: torque from propeller.
R_T: calm-water resistance of the ship.

S: origin of steady translating coordinate system.
T: wave period.
t: time.
t_d: thrust deduction factor.
T_e: wave elevation period.
T_p: thrust from propeller.
U: undisturbed flow velocity.
u: velocity of the flow in x-direction.
V: velocity of the steady translating coordinate system.
v: velocity of the flow in y-direction.
V_A: average inflow speed to the propeller.
V_c: relative velocity of the current.
V_{rw}: relative wind velocity (relative to ship velocity).
V_s: velocity of the ship. V_w: wind velocity.
w: velocity of the flow in z-direction.
w_r: wake fraction.
x: steadily translating coordinate.
x_a: surge coefficient.
$*_0$: earth-bound coordinate. [2]
$*_b$: body-bound coordinate.
X_c: longitudinal current force.
X_w: longitudinal wind force.
X_{wd}: longitudinal wave drift force.
y: steadily translating coordinate.
y_a: sway coefficient.
Y_c: transverse current force.
Y_w: transverse wind force.
Y_{wd}: transverse wave drift force.
z: steadily translating coordinate.
z_a: heave coefficient.

Greek Symbols
α_c: current direction.
α_{rw}: relative wind direction.
α_{wd}: wave drift direction.
ϵ_{F_l}: phase shift of lift force.
$\epsilon_{x\zeta}$: phase angle for surge movement.
$\epsilon_{y\zeta}$: phase angle for sway movement.
$\epsilon_{z\zeta}$: phase angle for heave movement.
$\epsilon_{\phi\zeta}$: phase angle for roll movement.
$\epsilon_{\theta\zeta}$: phase angle for pitch movement.
$\epsilon_{\psi\zeta}$: phase angle for yaw movement.
ζ_a: wave amplitude.
η_0: open water efficiency.
η_H: hull efficiency.
η_r: relative rotative efficiency.
θ_a: pitch coefficient.
λ: wave length.

[2] with $* \in \{x, y, z\}$.

μ: rotation angle of coordinate system.
ν: kinematic viscosity of the fluid.
ρ: density of water.
ρ_{air}: density of air.
Φ: potential function.
ϕ_a: roll coefficient.
ψ_a: yaw coefficient.
ω: circular wave frequency.
ω_e: wave frequency of encounter.

Symbols from Chapter 6

Roman Symbols
$c_1(i)$:part of transverse force of $(n-1)^{\text{th}}$ thruster (as a coefficient of (n-1) longitudinal forces).
$c_2(i)$:part of transverse force of $(n-1)^{\text{th}}$ thruster (as a coefficient of (n-2) transverse forces).
F^*: thruster configuration (which satisfies the forces constraints) with the least energy.
F_{max}: vector of maximum forces of thrusters.
F_x: total of external forces in longitudinal direction.
$F_{x_{current}}$: current force in longitudinal direction.
$(F_x)_i$: see $(F_x)_{thruster_i}$.
$(F_x)_{now}$: total longitudinal force produced by present configuration of thrusters.
$(F_x)_{thruster_i}$: longitudinal force produced by i^{th} thruster.
$F_{x_{wanted}}$: longitudinal force demanded by skipper.
$F_{x_{wave}}$: wave force in longitudinal direction.
$F_{x_{wind}}$: wind force in longitudinal direction.
F_y: total of external forces in transverse direction.
$F_{y_{current}}$: current force in transverse direction.
$(F_y)_i$: see $(F_y)_{thruster_i}$.
$(F_y)_{now}$: total transverse force produced by present configuration of thrusters.
$(F_y)_{thruster_i}$: longitudinal force produced by i^{th} thruster.
$F_{y_{wanted}}$: longitudinal force demanded by skipper.
$F_{y_{wave}}$: wave force in longitudinal direction.
$F_{y_{wind}}$: wind force in longitudinal direction.
F_z: total of external forces in z-direction.
$g\left((F_x)_1, \ldots, (F_x)_{n-1}, (F_y)_1, \ldots, (F_y)_{n-2}\right)$: energy function of the thruster forces.
M: total of external moments.
$M_{current}$: current moment.
M_{now}: total moment produced by present configuration of thrusters.
M_{wanted}: moment demanded by skipper.
M_{wave}: wave moment.
M_{wind}: wind moment.
s: part of transverse force of $(n-1)^{\text{th}}$ thruster.
x_i: x-position of i^{th} thruster.
y_i: y-position of i^{th} thruster.

Greek symbols
ϵ_1: error tolerance for longitudinal force thrusters.
ϵ_2: error tolerance for transverse force thrusters.

Chapter 1

Introduction

"Dynamic Positioning may be defined as a system which automatically controls a vessel to maintain her position and heading exclusively by means of active thrust."[1]

This definition speaks of two special modes which are available for Dynamic Positioned ships. More special modes are available and almost every Dynamic Positioned ship has 2 or 3 different modes installed. The main advantage of Dynamic Positioning is that the system corrects for the external forces working on the ship, namely wind, wave and current forces. A small setback is that the system has high startup costs, but there will be a save of money through lower fuel costs and no expenditures on tugboats.
Ships that are equipped with Dynamic Positioning often have azimuth thrusters, but not necessarily so. These rotating propellers allow for greater manoeuvring flexibility than standard fixed propeller-rudder arrangements. Under standard sailing conditions, these are steered with rotating engine handles. But for precise manoeuvring at low speeds, the skipper can often switch over to joystick control. With joystick control, the ship can be moved and rotated by moving the joystick in the desired direction. The ships control system has built in logic to bring over the joystick steering to the correct rotation and rpm of the engines.
A problem with the Dynamic Positioning system is that training is still very expensive. The training is very elaborate, because the skipper should learn to work with the system and should learn what to do in case of failure or erroneous input from the sensors on the ship.
A Dynamic Positioning Simulator should be able to train the skipper in different weather situations and in different operational situations.
Of importance in creating such a simulator are the hydromechanics involved in sailing a ship and the mathematical model behind Dynamic Positioning. Knowing all the forces acting on the ship, it is necessary to determine the configuration of the thrusters in such a way that minimal power is needed. Algorithms like Steepest Descent with Linesearch and BFGS method are implemented in Matlab to achieve this.

[1]From: [3]

VSTEP is interested in creating a Dynamic Positioning Simulator and already has a lot of expertise in developing Training Simulators, since they have created a Driving Simulator, whereby people learn to steer a car while learning the different traffic regulations, as well as a (Professional) Ship Simulator, whereby people learn to steer various ships in increasingly difficult conditions. This Ship Simulator game draws much attention also from the professional nautical schools and related companies.
VSTEP is a company with locations in Rotterdam (headquarters) and Oxford (UK). VSTEP means Virtual Safety Training and Education Platform. They work in 3 fields: Scenario Training, Procedure Training and Training Simulators.

This report will start with an introduction of ships in Chapter 2. After this in Chapter 3 an introduction about Dynamic Positioning will be given. The necessity of training will be discussed as well. Then in Chapter 4 the programs that are used to create the Ship Simulator game will be briefly introduced. In Chapter 5 the hydromechanics involved will be explicated. After this, in Chapter 6, the different models are described. In Chapter 7 the results are discussed. Finally, in Chapter 8 an overview of the first three months and the future goals will be given.

Chapter 2

Basics about Ships

From 1800 until after the WWII trade and passenger travel with the Dutch colonies in the East and the West and the stream of emigrants who moved to mostly North America led to a growing demand for developments in shipbuilding. The technology in this area endured a slow, but radical development. This development was characterized by:

- transition from wooden ships to iron and later on steel ships,
- transition from sail ships to steam ships and later on motor vessels,
- emergence of new types of ships as tankers and refrigerated and freezer vessels,
- a partially increase in the speed of the vessels, the size of the vessels and their safety.

After the WWII the world trade and with that the shipping changed very fast. Everlasting more and bigger ships were needed. Transport of passengers, oil, containers, heavy loads, animals and (dry) bulk with one and the same ship was normal until about 1970. Most of the modern multi-purpose ships are not able to transport so much different types of load.
The transport of passengers on long distant trips is now almost completely elbowed out by air traffic. But the number of passengerships that are specialized in vacation trips (cruises) has grown strongly.[1]
In the next sections a general introduction will be given about ships, the movements, and the type of ships in the *Ship Simulator 2007* game. In Appendix A figure A.4 a nice picture of a ship can be found with the terms used for the different parts of the ship.

[1] This introduction to the chapter is based on Chapter 1 from [5], p.50-51

2.1 Moves

The coordinate system of the ship is given in figure 2.1.

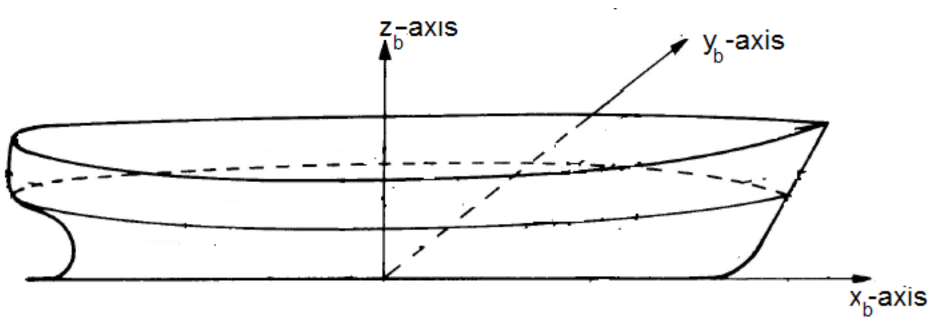


Figure 2.1: The coordinate system of a ship. The origin is in the ship's center of gravity.

There are six standard movements a ship can make and those are divided in movements and rotations.
The three movements:

- Surge: a horizontal oscillating movement in x-direction, i.e. a transient motion in a fore and aft direction;
- Sway: a horizontal oscillating movement in y-direction, i.e. a motion from side to side;
- Heave: a vertical oscillating movement in z-direction, i.e. a transient up-and-down motion;

The three rotations:

- Yaw: an oscillating rotation around z-axis, i.e. the bow yaws from side to side;
- Pitch: an oscillating rotation around y-axis, i.e. the bow pitches up and down;
- Roll: an oscillating rotation around x-axis, i.e. the vessel rotates from side to side, about the fore-aft axis;

2.2 Types of Ships

There are different kind of ships in the simulator game. Some of them will be described in this chapter. Offshore structures like oil rigs will be in the game, and with this supply vessels like the Fairmount Sherpa and Fjell ships. Cruiseships, push boats, rescue boats, tankers, life rafts, containerships and tugboats can also be found in the game.

Containerships

Since 1960 the transport of containers is increased and it still is increasing. Containers can be transported from customer to customer and not only from harbour to harbour. On land the transport is with trains or trucks, so a great part of this transportation goes by other means than ships. There are large, intercontinental containerships that only pass the large harbours and smaller or mediocre containerships, that transport containers from the smaller to bigger harbours, and provide services that are not profitable for larger ships. The big containerships from the game should be able to load and unload containers with a crane. [2]

Cruiseships

Nowadays the passengerships are exclusively meant for making very luxurious trips to tropical countries and harbours. These ships mostly have swimming pools, cinemas, bars, casinos and more on board.[3] There are stabilisation fins to make sure the wiggles of the ship are not too extreme. The Titanic[4], which is simulated in the game, was a luxurious ship that was launched May 31, 1911. On the first trip of this ship (April 10, 1912), it crashed into an iceberg and it sank within 3 hours. The vessel was 46,328 tons, had a length of 882.6 feet and a breadth of 92.6 feet. The boat deck and bridge were 70 feet above the waterline.[5]

Offshore structures

Drilling for oil and gas on the seas is nowadays done in more than 40 countries, 100 of kilometres from the coasts, with increasing depths. There are different types of offshore units build. In this game a semi-submersible drilling unit is available. This can be used for drilling in waters with waterdepth between 150 and 2500 metres. Those platforms have Dynamic Positioning Systems (see Chapter 3) in almost all cases.
Such units also need supply ships and oiltankers can come to retrieve the oil. [6]

Tankers

There are different kind of tankers, namely gas tankers, (crude) oil tankers, chemicals tankers and tankers with other substances, such as finished oil products like motor fuel or diesel oil, but also other fluids like drinking water and wine. In the game there is a crude oil tanker from the type VLCC (Very Large Crude Carrier) which can contain about 200,000-300,000 DWT (dead weight

[2]From: [5] p. 52-54
[3]From: [5] p. 58
[4]Text about the Titanic is from website: http://www.encyclopedia-titanica.org/index.php
[5]http://www.titanicinquiry.org/
[6]From: [5] p. 64-73

tonnage). Oil tankers can be load and unload through a flexible hose or a pipe arm.[7]

Tugboats

A characteristic of tugboats is the low stern. This is necessary to give the wire enough freedom of movement. The point were the force of the wire engages to the ship, should be near midship in such a way that this force does not influences the steering of the ship. The towing winch, which should translate the complete towing force from the propeller to the wire, is very important. Tugboats are used for salvage, towing, anchoring on offshore applications and assistence to ships in the state of emergency.[8]

2.3 Thrusters

There are three types of thrusters that make up the majority of units found in DP vessels, which are: main propellers, tunnel thrusters and azimuth thrusters.[9] Combinations of these thrusters can be found on different kind of ships. A semi-submersible drillship for instance can come with six azimuth thrusters.

2.3.1 Main propellers

Main propellers, sometimes in combination with rudder systems, provide for the main propulsion. In some vessels these rudders are also part of the DP system. If the rudders are not part of the DP system, then it is important that the rudders are amidships, because else they can lead to sway or yaw movements when the propellers are switched on.

2.3.2 Tunnel thrusters

The effective thrust of a tunnel thrusters depends on the depth of immersion of the propeller, the length of the tunnel and the speed of the vessel. The tunnel creates an increase in thrust in most of the cases. The flow of the water through the tunnel gives the water inside the tunnel a higher velocity than the water outside the tunnel and this results in a pressure difference, which results in an increase in thrust force. The noise from the propellers is also reduced.
If the propeller is too near the surface then it is less efficient, because there is air resistance, due to the air on the surface, and also because this can lead to cavitation, which is a result from air bubbles -due to pressure differences- that implode and lead to damage of the propeller-blade.
The effective thrust decreases also when the speed of the vessel is too high (this bound depends on the kind of ship). The higher the speed is, the higher the

[7] From: [5] p. 55-56
[8] From: [5] p. 59-60
[9] Information for this chapter from [3] chapter 4, [5] chapter 12 and [15]

frictional resistance to the tunnel is, and if this increase in friction is bigger than the improve in thrust, then there is a reduction in efficiency.
A long tunnel causes increased friction losses, and a too short tunnel causes losses due to turbulence.

2.3.3 Azimuth thrusters

An azimuth thruster is, as the name says, a unit able to generate thrust in any direction. These units may be fitted to provide the desired manoeuvrability, or sometimes as a substitute for the main propellers. The most important advantage of azimuth thruster is an optimal thrust in every direction.
In some configurations these thrusters are positioned right underneath the hull of the vessel, in which case the azimuth thrusters are vulnerable to grounding damage and one should take precautions this will not occur. The thrusters can be placed higher on the ship too, but in this case they are closer to the surface and then they are less effective.
There are also azimuth thrusters that are retractable, horizontally in the hull or vertically. The first one takes up less space. When the thruster is retracted it cannot be used.
Some azimuth thrusters are removable. This might be interesting for vessels where retraction of the thrusters is not possible (when there is not enough space for this for instance) or when it is too expensive to install. This is nice for ships that spend most of the time in deep water; when travelling to shallow water one can remove 'simply' the thrusters.
An azimuth thruster can work in the ahead and the astern direction. In the astern direction the amount of thrust available falls to only 60%, but since sailing in reverse is faster than rotating the unit through 180° and since sometimes there is not enough space for rotation, this still is an interesting possibility. But although this is true, some DP systems will rotate the vessel when the effectivity is that low, and other DP systems force the thruster to operate in the ahead mode, so it is important to keep this in mind.
In configurations where the azimuth thrusters are located near each other, it is necessary to ensure that they do not intervene with each other, i.e. that the stream of one will not affect the other thruster, because the other thruster can then overspeed or become less effective. This is also true if the other thruster is a main propeller or tunnel thruster. To cope with this, the thrusters can be barred, which means the range of movement is restricted. This can also be done when it is undesirable to have a thrust in a certain direction, for instance when there are divers in that region. Such a possibility is called a thruster inhibit. If this is not available, then it might be necessary to shut down the thruster.

Chapter 3

Dynamic Positioning

Dynamic positioning was first discovered as a way of positioning ships in the early 1960s. It rapidly growed and now dynamic positioning is established as a reliable way of maintaining a vessel's location or heading over a long period of time. Dynamic positioning systems perform well in deep as well as shallow water and the costs do not increase significantly with water depth. Other advantages are better station-keeping accuracy and flexibility.[1]

3.1 Definitions

There are many definitions for Dynamic Positioning. From [17]:

A means of holding a vessel in relatively fixed position with respect to the ocean floor, without using anchors accomplished by two or more propulsive devices controlled by inputs from sonic instruments on the sea bottom and on the vessel, by gyrocompass, by satellite navigation or by other means.

Another nice definition, from [3], is:

Dynamic Positioning may be defined as a system which automatically controls a vessel to maintain her position and heading exclusively by means of active thrust.

And finally one other definition, from [5]:

With Dynamic Positioning the ship will be held in position within prescribed, in advance given tolerances, in spite of current and wind influences.

3.2 Why Dynamic Positioning?

This is an interesting question. Dynamic Positioning has some advantages when comparing with other systems, but also disadvantages. But there are a lot of

[1] From [3] p.1-4

possibilities with this system. First a short introduction on Dynamic Positioning training to understand the importance of training.

3.2.1 Dynamic Positioning training

With a Dynamic Positioning system the skipper, when steering, does not need to correct for the wind, current and wave influences. The system will make such a correction. So why is training necessary one can ask then? First of all the skipper should learn to work with the system, what are the different modes possible (see section 3.2.3), and he should learn how to steer the ship. Most DP systems work with joystick steering, he should learn how to handle this. This is especially important in applications that need a lot of precision, like anchoring to an oil rig or in a harbour.
Furthermore it is important for the skipper to learn what to do in case of an emergency. What if a part of the systems falls out, or what if the entire system fails? And of course he need to see when the wind or current sensors return an unusual value that might be wrong. He might need to overrule the system then. Also extreme weather situations should be practised.
Target cases will be defined with several conditions, which are important to learn.

3.2.2 Comparison of Dynamic Positioning to other systems

To see why Dynamic Positioning (from now on referred to as DP) is so interesting for position control besides the obvious advantages, this system is compared to other systems.[2]

- Jack-up barge: in this system there is no need for power, thrusters, extra generators or controllers. This means that the system is immune to power blackouts or system failures. Another advantage is that there will be no underwater hazards from thrusters, so this is nice for boats with divers. It is also true that once on location, the vessel will not need position references anymore.
  But of course this system also has some disadvantages. There is in the first place a limitation to the water depth this system is applicable to. Some books speak of limited water depts of up to about 60m, others speak of 150m, but both are very limited depths. When spudded in, the manoeuvrability is gone too. A last disadvantage is that for rigmoves[3] one needs tugs.
- Spread mooring to anchor pattern (Anchoring): again we have no need for complex systems with thrusters, extra generators and controllers, so the system is immune to power blackouts or system failures. Also for this system it holds that when on location there are no position references necessary anymore. Without thrusters we have again no underwater hazards.

[2] From [3] p.5-6
[3] This is a move of a rig, for instance an oil rig.

The advantages are the same.
The disadvantages are pretty much the same. Only for this system there is limited manoeuvrability, when moored, instead of none. The waterdepth is also less limited, up to 600m. For rigmoves and for laying of mooring tugs are necessary. And, this is also an important disadvantage, excessive time is required for setting up the anchor pattern.

- DP: this system has lots of advantages, but also disadvantages. One very important advantage is that tugs are not necessary anymore. The set-up on location is quick, and during operation the vessel is very manoeuvrable. If the weather changes, or there are other changes in the situation on the water, the system can react on this very fast. If the operation requirements change, the system can adapt to this also easily. Task can be completed fast too, so this just means that it is cheaper. The water depth does not matter anymore, the system can work in any water depth, although in shallow water there might be problems. This has also less risk for the environment, since there are no mooring lines on the seabed.
  The main disadvantages are vulnerability to power failures and shortages, thruster failure and electronics failures (since DP is a very complex system). Since the system needs a lot of power, the fuel costs are higher too. The initial costs are also high. Diving can be dangerous with the thrusters under the ship. Position reference is needed continuously. And last but not least, there is need for highly educated staff members, thus training and experience are needed now.

From this we see that DP has many advantages as well as disadvantages, but important is that DP can be used on locations where the other systems can not be applied. Furthermore the manoeuvrability at any time plays an important role in the need for DP.
DP also has many advantages in comparison to conventional steering with turning gas handles. Dynamic Positioning can save power since the system calculates the forces needed to overcome the external forces, while a skipper can make mistakes in estimating these forces. No tugboats are necessary anymore, where as it is to hard in many situations for a skipper to sail himself with the ordinary gas handles. A last interesting advantage is that DP can be used in situations where a lot of precision is needed. A supply vessel that should anchor to an oil rig can do this more easily with DP for instance.

3.2.3 Operating with DP

DP has many applications. There are several modes of operation possible, namely:[4]

- manual/joystick mode: the operator has full control over the vessel;
- auto-heading mode: the system maintains required heading automatically;
- auto-position mode: the system maintains required position automatically;

[4]From: [1] Chapter 10 p.15-16

- auto area position mode: the system maintains automatically within a specified area, while using minimum power;
- auto track mode: the vessel follows a specified track described by a set of way-points;
- autopilot mode: the vessel steers automatically along a pre-defined course;
- follow target mode: the vessel follows a constantly moving target such as an ROV (remotely operated vehicle).

3.2.4 Applications where DP can be used

Most DP systems are equipped with two or three of such applications. With this and the advantages, we can see directly a few examples of applications of DP systems:[5]

- Cable- or pipelay: in this case it is needed that the vessel will sail in a straight line or course;
- Exploration/production drilling: the water can be deep and the ship should stay stable during this kind of operation;
- Oceanographical research: the vessel should be able to move a lot of times, and lay still on other times, so anchoring every time is not an option. This is also better for the environment, since the vessel will not be anchored into the seabed. And with this system the vessel is able to follow a predefined course;
- Wreck survey, salvage and removal: a ship equipped with DP can search the area of the wreck to find it (if the exact position is not available) and then stay stable there while the wreck is recovered;
- Cruiseliners: the ship does not need tugboats to moor in tight harbours.
- Dredging: DP is used here for track keeping.
- Search for sea mines: DP is used here for track keeping. The ship should follow a predefined course in search of the mines.

In the offshore industry, which is of most interest to VSTEP at the moment, the following applications can be found:

- Supply vessel: for supplying offshore platforms.
- Anchor handling: an offshore rig should be anchored as fast as possible and this should be done very accurately as well.
- Floating crane operations: the crane vessel should keep the correct heading and position.

[5] From: [3] p.3

3.3 Dynamic Positioning System

Vessels that need to be dynamically positioned need to have Dynamic Positioning systems that compensate for wind forces, waves and current, each of which can have a different direction and force onto the vessel. Worst-case combinations can occur when wind, current and waves come from different directions.
There is a need for sensors on the ship that measure the velocities and direction of wind, waves and current. A mathematical model will then calculate the power and direction of the thrusters to stay on course. An example of a schematic drawing of this system is given in figure 3.1.
Ships that are equipped with Dynamic Positioning equipment often have azimuth thrusters, but not necessarily so. A combination of twin screws at the back with conventional rudder blades plus a bow and/or stern thrusters also allows for straight sideways movements.
There are two Dynamic Positioning systems available: systems based on PID regulator, and systems based on model control. The first type can only correct after the ship gets off course, the other type can predict deviation and react on this on beforehand.[6]

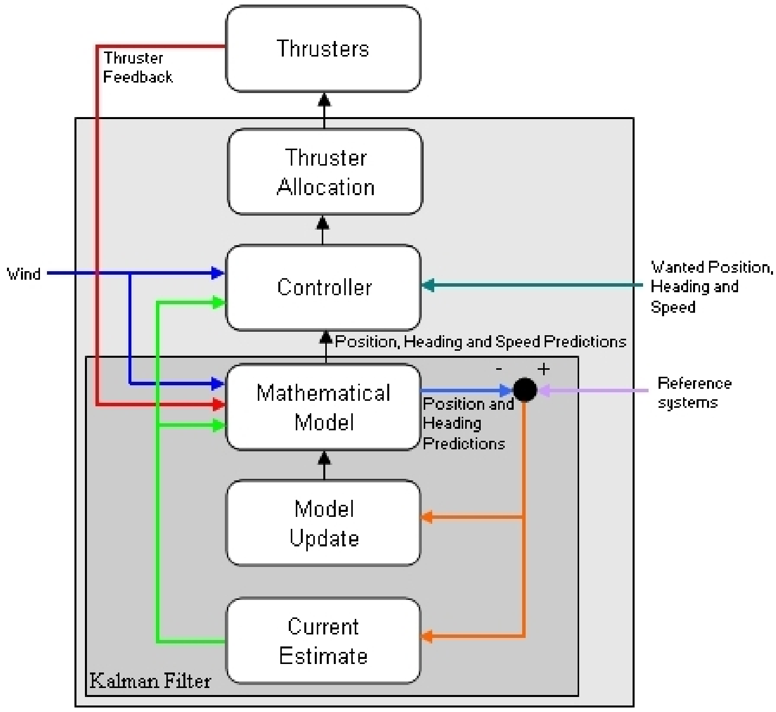


Figure 3.1: This is a schematic drawing of a dynamic positioning system.[7]

[6]more in OE 5663
[7]source: http://content.answers.com/main/content/wp/en/f/fc/Controller.jpg

3.3.1 Mathematical model

The model behind this DP system should calculate the resulting thruster configuration, using the inputsignals from the sensors on the ship. The model should describe the reality as accurate as possible, but one should not forget that there are already errors made when measuring the forces of wind, waves and current. From all the forces the direction and velocity is needed. For waves one also needs the wavelength. From this data all the resulting forces will be estimated. The thruster force is a function of the rotations per minute, the blade area, the interactions between the thruster and the hull, and between thrusters. The model should then calculate the force needed by the thrusters to move the ship in the correct direction (or to hold the ship stable on one location) and minimize the energy produced by these thrusters. So this model holds a combination of various mathematical and physical methods including hydromechanics, wave theory, systems theory and optimization. For the calculations numerical methods can be used to approximate reality.
In Appendix A, figure A.6, an example of such a mathematical model is given.

3.3.2 Position Reference Systems

To know the position of the ship there are several reference systems possible.
Some of them are:

- Taut-wire/rise inclinometers: these systems are particularly useful when the vessel may spend long periods on a static location, where the water depth is limited. Tautwire-based reference may also be used when the vessel need to maintain a location relative to a moving vehicle.
  There are a number of configurations possible for a taut-wire system, but the most commonly used system lowers a wire with a weight on the end of the wire to the seabed. Then when in operation the angle of the wire is measured. The length of the wire together with the angle define the position of the ship. Strong currents degrade the accuracy of the system, especially in deep water, and in shallow water the movement is limited, because the angle of the wire can not become too large.[8]

- Hydro-acoustic Position Reference systems: these systems need a transponder positioned on the seabed and a transponder on the ship. An acoustic signal is send from the ship to the seabed and the time needed for the signal to return to the ship is measured. With that time one can measure the position of the ship relative to the transponder on the seabed. It is important though, not to forget that there should be a compensation for the roll and pitch moves of the ship.
  There are also systems, called Long Baseline acoustic position reference systems, that have multiple transponders on the seabed (not all are used, some are for redundancy). This will improve accuracy. No angle measures are required because of the multiple signals that are received. Different kinds of systems like this are of course possible.

[8] From [3] p.89-94

- (Differential) Global Positioning Systems: this system works with data that is received from different satellites. From these the system calculates the position of the vessel. With differential GPS this data is combined with data from a reference station which position is know exactely.

Chapter 4

Overview of Ship simulator

In this chapter the ship editor of VSTEP will be introduced.

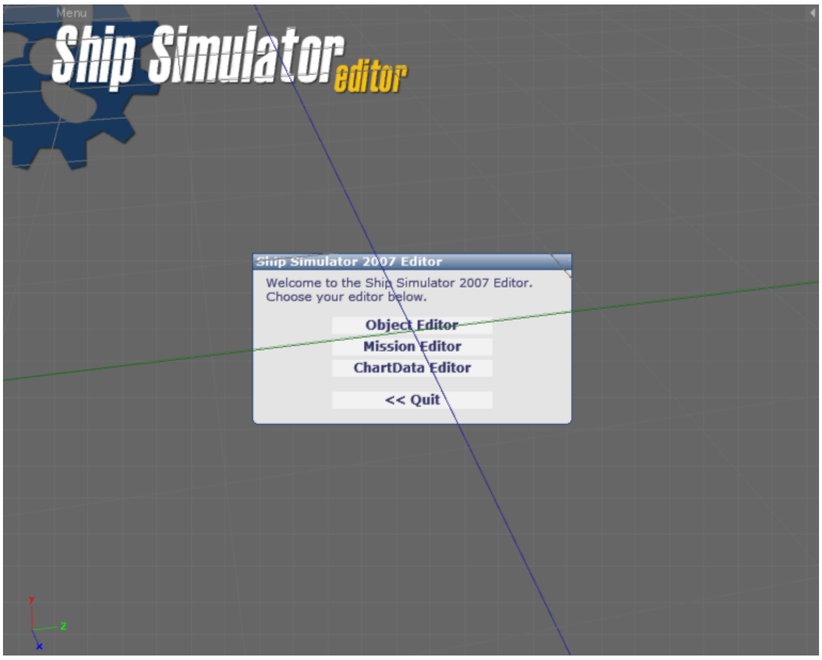


Figure 4.1: Screenshot from the ship editor.

Two examples of ships can be found in figure (4.2) and figure (4.3).

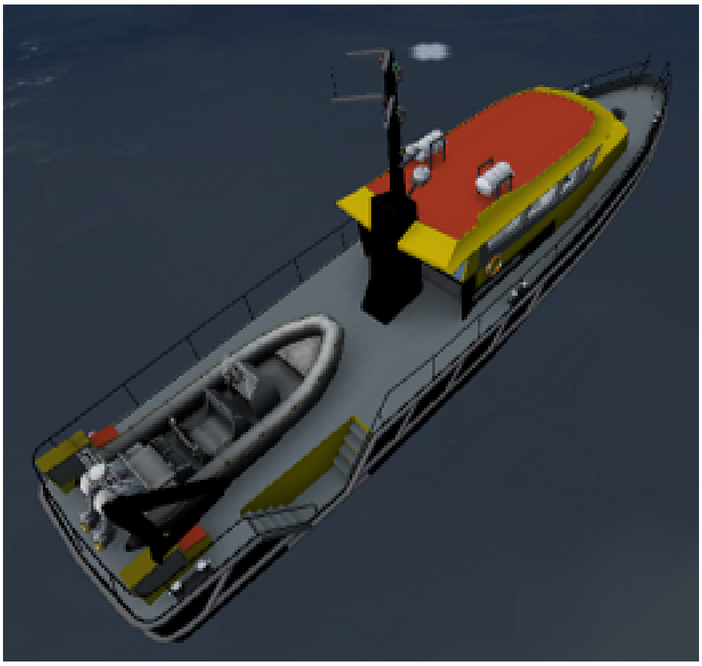


Figure 4.2: Screenshot from the ship editor. This is a harbour patrol boat.

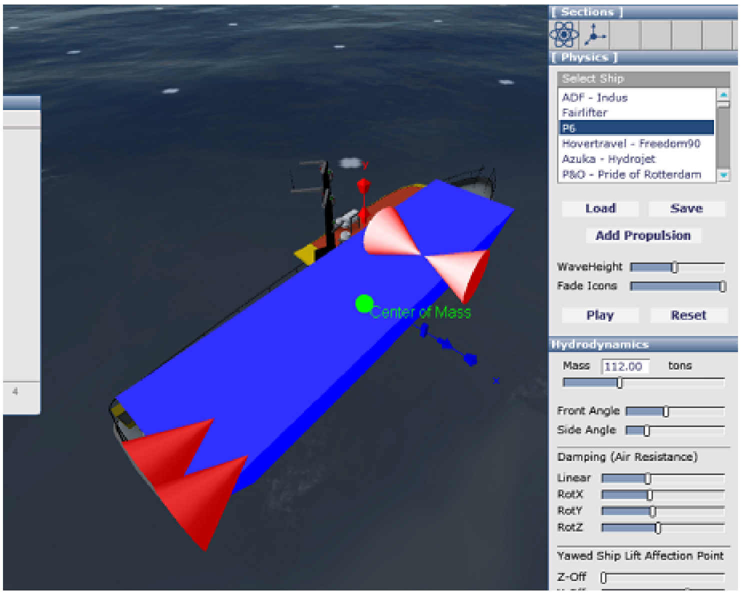
Figure 4.3: Screenshot from the ship editor. This is a hovercraft.

A ship has a complex shape. To calculate all the forces working on the ship, one should take its shape into account. Since this is a complex process, one can also try to approximate the shape. In this ship editor the ship is approximated by a block. This block can take any size necessary and can be rotated. In this way it is possible for the block to fit the ship very well. This can be seen in figure (4.4(a)) and (4.4(b)).

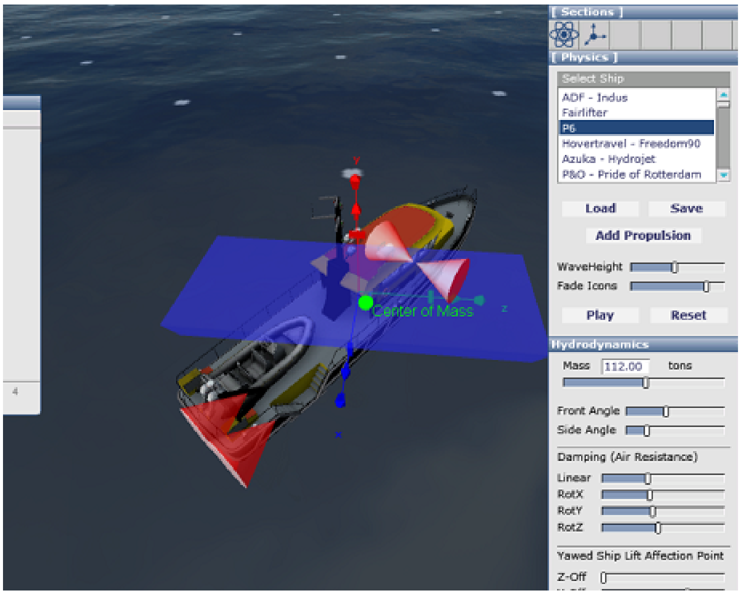
As is known several forces work on a ship in the water. Even when the ship lies still in the water the buoyancy force and gravity work on the ship. The gravity works in this editor on the center of mass, which can be seen in for instance picture (4.4(b)). This can be seen in pictures (4.5(a)), (4.5(b)) and (4.5(c)), where the mass of the ship is increased and the ship starts to sink, because the buoyancy force is not big enough to overcome the gravity.[1]

A lot of properties of the ship and thrusters are not calculated, but read by the program from curves. Other properties are calculated, but since the shape of the ship is approximated, these properties are not correct, because the shape of the ship is more complex. Now some of those properties can be corrected then by a curve to take this complex shape into account. Some of these curves can be found in figures (4.6), (4.7), (4.8), (4.9) and (4.10). These curves show the different kind of possibilities in this editor. Although the forces, resistances and thrust are not calculated exactly, they are modified to make it more realistic.

[1]This is explained in Chapter 5.

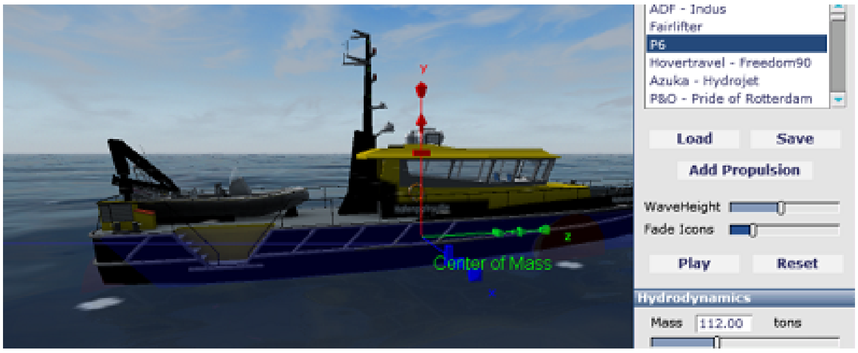


(a) Ship approximated by a block.

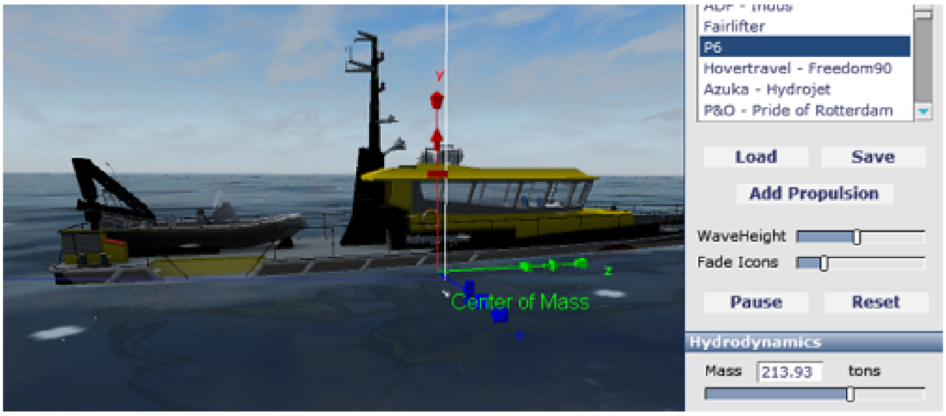


(b) The block can be rotated.

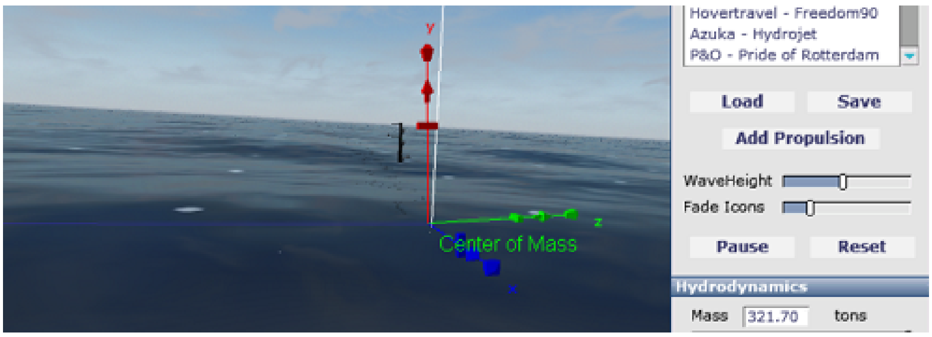
Figure 4.4: In the upper screenshot one can see that the ship is approximated by a block. In the lower screenshot one can see that it is possible to rotate and move this block. From the last picture the orientation and origin of the axis are also clear. In this editor the axis have different names than defined in this paper.



(a) Harbour patrol boat with normal mass.



(b) Harbour patrol boat with higher mass.



(c) Harbour patrol boat with even higher mass

Figure 4.5: The effect of the mass of a ship. In the upper graph the harbour patrol boat has a normal mass. In the center picture the harbour patrol has a higher mass. The ship lies lower in the water, but still buoyancy is higher than gravity. In the lower graph this is not the case, the gravity has won and the harbour patrol boat can not be seen anymore, only the antenna is above water.

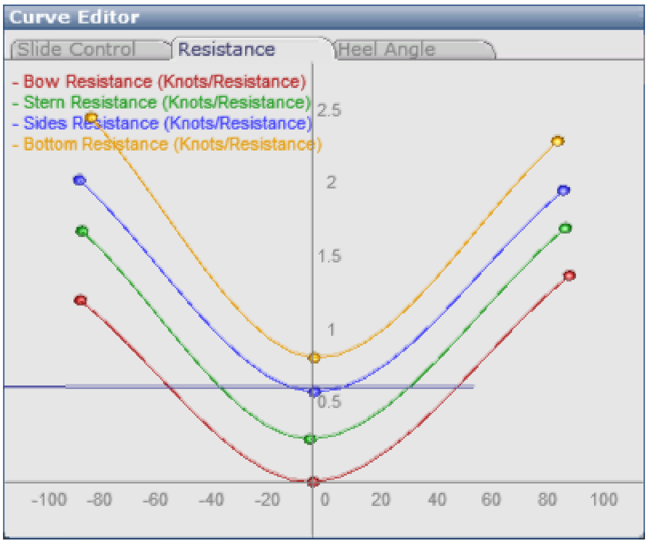


Figure 4.6: Curve of the correction of the resistance of different parts of the ship. The resistance is calculated by the ship editor, but since this is calculated on a simple block, the result should be scaled to take the complex shape into account. The scaling can be found from the graphs, where on the x-axis the speed in knots can be found and on the y-axis the scaling factor.

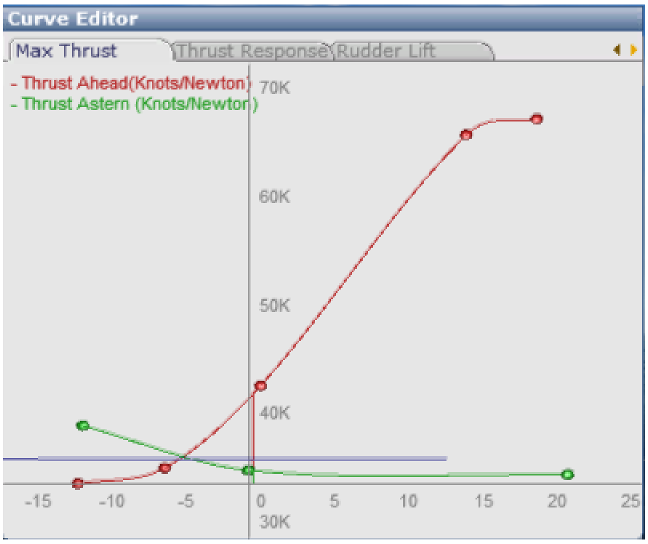


Figure 4.7: Curve of the thrust at different speeds. The thrust is the force the propeller produces.

Figure 4.8: At lower velocities the thrusters can adjust to a higher speed easier than at higher speeds. This is illustrated in this curve.

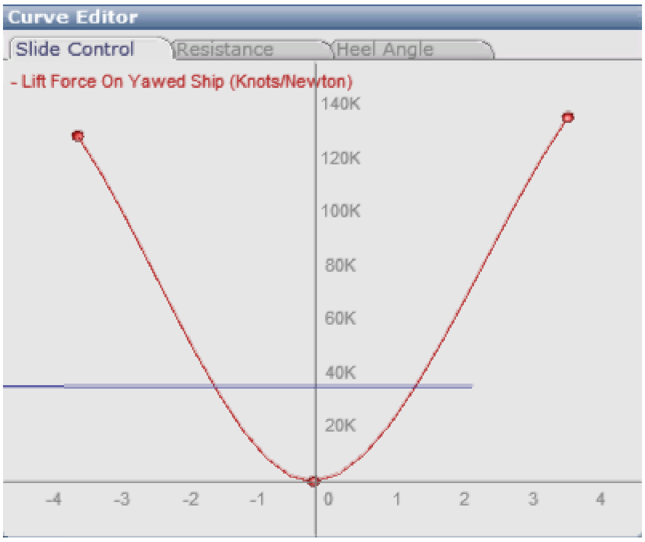


Figure 4.9: Curve of the liftforce that is generated by yawing movements of the ship.

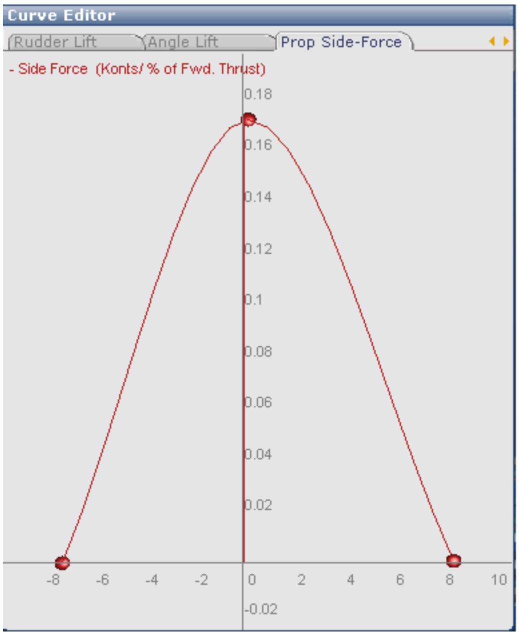


Figure 4.10: When the ships starts to accelerate, it banks over a bit and this is caused by a sideward directed force, also called the torque of the propeller. This force is not calculated in the ship editor, but read from this curve.

The results of all these curves can be seen in a few screenshots from the editor, where the ship is sailing. In figure (4.12) the ship is sailing forward and one can see the white, long arrow indicating the thrust in forward direction. In figure (4.11) the ship is sailing sidewards and there is a white arrow in that direction as well.

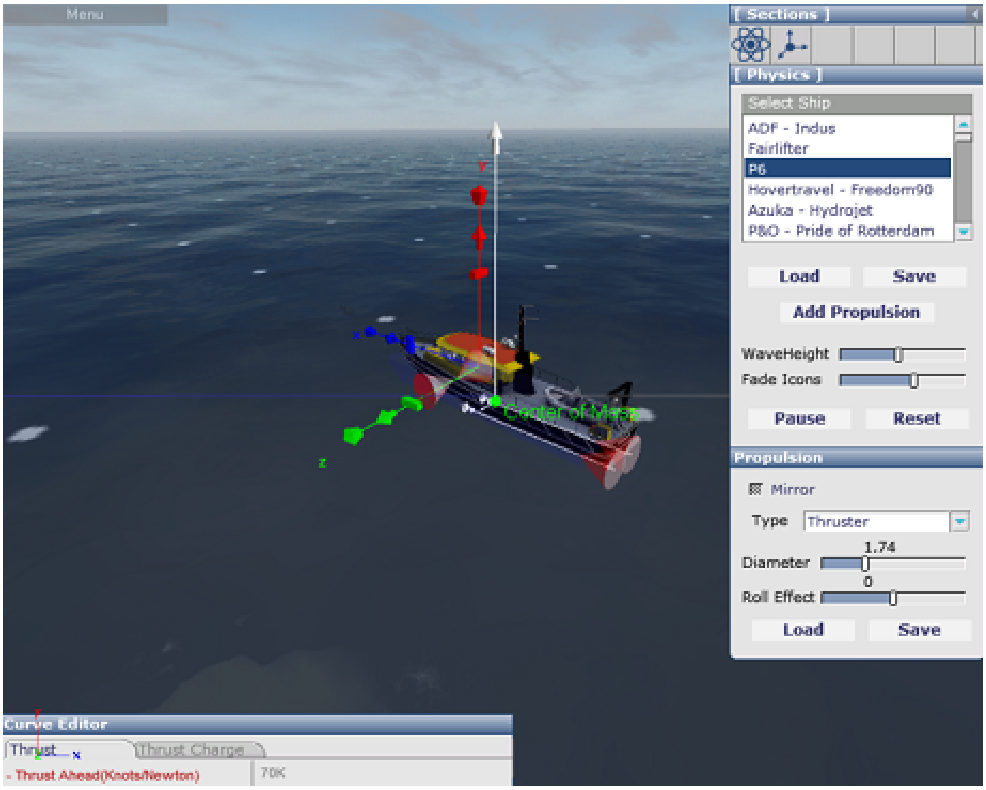


Figure 4.11: Screenshot of the ship sailing forward. The white arrows at the thrusters show the thruster forces working on the ship.

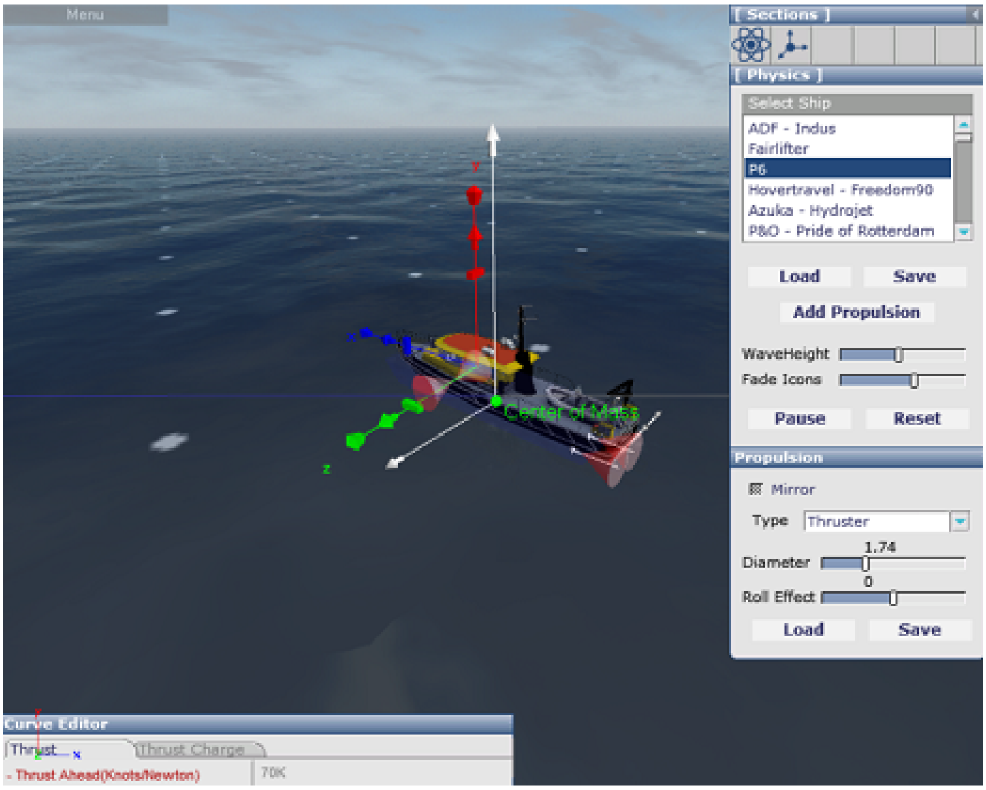


Figure 4.12: Screenshot of the ship sailing sideward. The white arrows at the thrusters are showing the side forces working on the ship. There is also a force working on the ship near the center of gravity which is due to the ships sideward movement through the water.

Chapter 5

Hydromechanics

In this chapter the basics of hydromechanics will be described.
As an introduction a few laws and basic equations will be mentioned before getting into details.

5.1 Introduction

At first there is **Archimedes' law**:

$$m = \rho \hat{\nabla}, \tag{5.1}$$

where m is the mass of the ship, ρ is the density of the water and $\hat{\nabla}$ is the volume of the submerged part of the ship. This leads to the **law of buoyancy**:

$$F_{\nabla} = \rho g \hat{\nabla}, \tag{5.2}$$

where F_{∇} is the buoyant force working on the ship, ρ is the density of the water, g is the gravitational constant and $\hat{\nabla}$ is the volume of the submerged part of the ship. This law states basically that when a solid is immersed in a liquid, it experiences an upthrust equal to the weight of the fluid displaced.

The **hydrostatic pressure** follows from this equation:

$$p = \rho g h, \tag{5.3}$$

where p is the hydrostatic pressure, ρ is the density of water, g is the gravitational constant and h is the distance below the water surface.

5.2 Flow

The **continuity equation**:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0, \tag{5.4}$$

where is u the velocity of the flow in the x-direction, v the velocity of the flow in the y-direction and w the velocity of the flow in z-direction.

If the fluid is treated as incompressible, then ρ is constant and this leads to the **simplified continuity equation**:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{5.5}$$

Definition 5.2.1 (Potential Function)**.** A potential function, Φ, associated with a potential flow field is a mathematical expression having the convenient property that at any point in the flow, the velocity component in any chosen direction is simply the derivative of this potential function in that chosen direction:

$$u = \frac{\partial \Phi}{\partial x} \quad v = \frac{\partial \Phi}{\partial y} \quad w = \frac{\partial \Phi}{\partial z} \quad .$$

From this definition the new continuity equation for incompressible flows becomes the **Laplace equation**:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0. \tag{5.6}$$

Very important in the fluid dynamics are the **Navier-Stokes equations** (here for incompressible flows):

$$\begin{aligned}
\rho(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}) &= \rho f_1 - \frac{\partial p}{\partial x} + \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}),\\
\rho(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}) &= \rho f_2 - \frac{\partial p}{\partial y} + \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}),\\
\rho(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}) &= \rho f_3 - \frac{\partial p}{\partial z} + \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}).
\end{aligned} \tag{5.7}$$

These equations can be simplified by neglecting the volumetric forces (f_i) and assuming that the flow is non-viscous ($\mu = 0$). This leads then to the **Euler equations**:

$$\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial x},\\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial y},\\
\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial z}.
\end{aligned} \tag{5.8}$$

From Definition (5.2.1) the Euler equations can be transformed. Using:

$$\begin{aligned}
u\frac{\partial u}{\partial x} &= \frac{\partial \Phi}{\partial x}\frac{\partial^2 \Phi}{\partial x^2} = \frac{1}{2}\frac{\partial}{\partial x}(\frac{\partial \Phi}{\partial x})^2,\\
v\frac{\partial u}{\partial y} &= \frac{\partial \Phi}{\partial y}\frac{\partial^2 \Phi}{\partial x \partial y} = \frac{1}{2}\frac{\partial}{\partial x}(\frac{\partial \Phi}{\partial y})^2,\\
w\frac{\partial u}{\partial z} &= \frac{\partial \Phi}{\partial z}\frac{\partial^2 \Phi}{\partial x \partial z} = \frac{1}{2}\frac{\partial}{\partial x}(\frac{\partial \Phi}{\partial z})^2,
\end{aligned}$$

$$u\frac{\partial v}{\partial x} = \frac{\partial \Phi}{\partial x}\frac{\partial^2 \Phi}{\partial x \partial y} = \frac{1}{2}\frac{\partial}{\partial y}(\frac{\partial \Phi}{\partial x})^2,$$
$$v\frac{\partial v}{\partial y} = \frac{\partial \Phi}{\partial y}\frac{\partial^2 \Phi}{\partial y^2} = \frac{1}{2}\frac{\partial}{\partial y}(\frac{\partial \Phi}{\partial y})^2,$$
$$w\frac{\partial v}{\partial z} = \frac{\partial \Phi}{\partial z}\frac{\partial^2 \Phi}{\partial y \partial z} = \frac{1}{2}\frac{\partial}{\partial y}(\frac{\partial \Phi}{\partial z})^2,$$

$$u\frac{\partial w}{\partial x} = \frac{\partial \Phi}{\partial x}\frac{\partial^2 \Phi}{\partial x \partial z} = \frac{1}{2}\frac{\partial}{\partial z}(\frac{\partial \Phi}{\partial x})^2,$$
$$v\frac{\partial w}{\partial y} = \frac{\partial \Phi}{\partial y}\frac{\partial^2 \Phi}{\partial y \partial z} = \frac{1}{2}\frac{\partial}{\partial z}(\frac{\partial \Phi}{\partial y})^2,$$
$$w\frac{\partial w}{\partial z} = \frac{\partial \Phi}{\partial z}\frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{2}\frac{\partial}{\partial z}(\frac{\partial \Phi}{\partial z})^2.$$

the Euler equations (5.8) become:

$$\begin{aligned}
\frac{\partial}{\partial x}\left\{\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2\right] - \frac{p}{\rho}\right\} &= 0,\\
\frac{\partial}{\partial y}\left\{\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2\right] - \frac{p}{\rho}\right\} &= 0,\\
\frac{\partial}{\partial z}\left\{\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2\right] - \frac{p}{\rho}\right\} &= 0.
\end{aligned} \tag{5.9}$$

Since the differentations of the expressions between braces $\{\ldots\}$ with respect to x, y and z gives zero, which means that these expressions are functions of only time, $C(t)$, which then provides the **Bernoulli equation** for an instationary flow:

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2\right] + \frac{p}{\rho} + gz = C(t), \tag{5.10}$$

where the term gz is the gravity which was neglected at first, but returns in this equation.

5.3 Forces

A flow disturbance results in a **drag force** (F_D) -by definition parallel to the flow direction- on the object. When the ship is seen as a cylinder, as illustrated in figure 5.1, this force (on a unit length of cylinder) can be written as:

$$F_D = \frac{1}{2}\rho U^2 C_D D, \tag{5.11}$$

with ρ mass density of the fluid, U the undisturbed flow velocity, C_D dimensionless drag coefficient and D the cylinder diameter.[1]

[1] This is from [12] Chapter 4 p8-11

A **lift force** (F_l) is defined as a force component acting perpendicular to the undisturbed flow velocity, so this force is also perpendicular to the drag force. There is no lift force when the cylinder is placed in a completely symmetric flow. When the velocity gets higher and the Reynolds number, $Re = \frac{Vd}{\nu}$, with V the velocity of the flow, d the diameter of the cylinder and ν the kinematic viscosity of the fluid, exceeds 90, then the flow is disturbed too much and a Von Karman vortex street appears.[2] When vortices are shed behind the cylinder, there is a resulting force directed toward the vortex from high velocities in the wake and low local pressures. The lift force can be seen in figure 5.1. This lift force (on a unit length of the cylinder) is defined as:

$$F_l = \frac{1}{2}\rho U^2 D C_L \sin(2\pi f_v t + \epsilon_{F_l}), \quad (5.12)$$

with ρ mass density of the fluid, U the undisturbed flow velocity, D the cylinder diameter, C_L dimensionless lift coefficient, f_v vortex shedding frequency, t time, ϵ_{F_l} phase shift.[3]

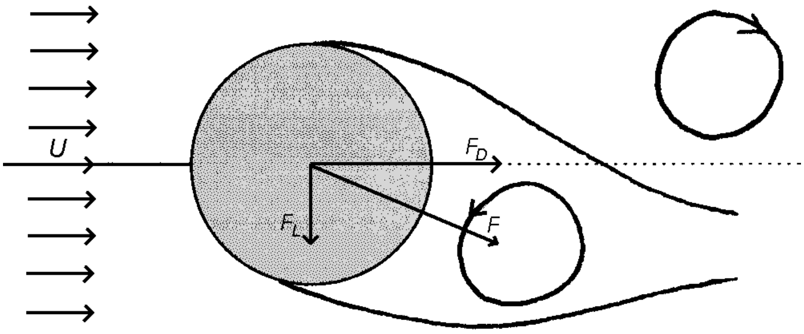


Figure 5.1: Drag and lift force are illustrated in this figure, where the ship is seen as an cylinder and U is the velocity of the flow. F is the resulting force from drag and lift force.[4]

Wind has a stochastic nature which depends on time and location. At sea the variation in the mean wind velocity is small compared to the wave period. The wind can be considered as steady, both in magnitude and direction, which result in constant forces and moment on a fixed floating or sailing body. The wind plays two roles in the behaviour of a floating body. First it exerts a force on the part of the structure exposed to the air, for the calculation of this force only local winds are necessary. Secondly, wind generates waves and currents and through these influence the ship too. To determine these effect, wind conditions in a much larger area are needed. The **direct wind forces** and **moment** can be computed as follows:

$$X_w = \frac{1}{2}\rho_{air} V_{rw}^2 C_{X_w}(\alpha_{rw}) A_T,$$

[2] See for instance: http://en.wikipedia.org/wiki/Von_K%C3%A1rm%C3%A1n_vortex_street for information on the Von Karman vortex street.
[3] This is from [12] Chapter 4 p12-13
[4] From [12] Chapter 4 p.13.

$$Y_w = \frac{1}{2}\rho_{air}V_{rw}^2 C_{Y_w}(\alpha_{rw})A_L, \tag{5.13}$$
$$N_w = \frac{1}{2}\rho_{air}V_{rw}^2 C_{N_w}(\alpha_{rw})A_L L,$$

with X_w steady longitudinal wind force, Y_w steady transverse wind force, N_w steady horizontal wind moment, ρ_{air} density of air, V_{rw} relative wind velocity, α_{rw} relative wind direction, A_T transverse projected wind area, A_L longitudinal projected wind area[5], L length of the ship, $C_{*w}(\alpha_{rw})$ α_{rw} dependent wind load coefficient.[6]
There is actually another approach to calculate the wind force. The ship is then split into different common shapes as circles, rectangles, etc. The total wind force is then the sum of the forces on the various shapes at different locations at this height. For now this will not be done, but the formulas from equation (5.13) will be applied.[7] The wind velocity can be calculated using the power law profile:

$$V_w(z) = V_w(z = 10m) \cdot \left(\frac{z}{10}\right)^{\frac{1}{8}}, \tag{5.14}$$

where z is the height at which the velocity is needed, $V_w(z = 10m)$ is the velocity at 10m (which is the typical measurement height) and $V_w(z)$ is the velocity at z.[8]
There are also dynamic wind effects. The wind velocity is typically defined as the 1-hour sustained velocity, but there are also 10-minute and 1-minute velocities, which can be up to 25% and 40% higher than the 1-hour velocities. There are various spectra used to describe this, namely Harris, Ochi-Shin, DNV and Wills spectrum.

The forces and moment exerted by a current on a floating object are composed of two parts. A viscous part, due to friction between the fluid and the structure. For blunt bodies it is small compared to the viscous pressure drag, so this can be neglected. The second part is a potential part, with a component due to circulation around the object, and with a component from the free water surface wave resistance, but this last component is small in comparison with the first in most cases and will be neglected. These are the equations for the **current forces** and **moment**:

$$X_c = \frac{1}{2}\rho V_c^2 C_{X_c}(\alpha_c)A_{TS},$$
$$Y_c = \frac{1}{2}\rho V_c^2 C_{Y_c}(\alpha_c)A_{LS}, \tag{5.15}$$
$$N_c = \frac{1}{2}\rho V_c^2 C_{N_c}(\alpha_c)A_{LS}L,$$

with X_c steady longitudinal current force, Y_c steady transverse current force, N_c steady yaw current moment, ρ density of water, V_c current velocity, α_c current direction, A_{TS} submerged transverse projected area, A_{LS} submerged longitudinal projected area[5], L length of the ship, $C_{*c}(\alpha_c)$ α_c dependent current

[5] For transverse and longitudinal projection, see figure (5.2).
[6] This is from [12] Chapter 4 p23-25
[7] From [14] p71-72
[8] This is from [14] p71

load coefficient.[9]

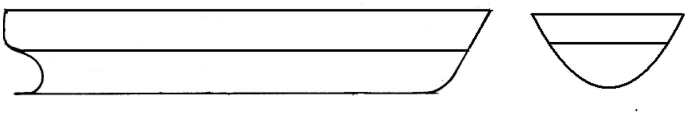


Figure 5.2: On the left is the transverse projection of the ship and on the right is the longitudinal projection of the ship.

There are a lot of books about wave theory. Different ways of estimating the forces and added resistance are explained in them. For the sake of simplicity in the first instance only a small part will be studied and applied, but later on this will be extended.
In DP cyclic wave forces are not important, since they do not result in a net displacement of the vessel. These waves have short periods (think of 20 seconds or shorter) so would require also a lot of power from the thrusters, which is not necessary, because the average position of the vessel is not affected by these waves.
The other type of wave force is the wave drift force. This force is relatively recently understood. The forces were reported at first in 1924, but the effects were not understood until 1960. A good example of wave drift forces is that they can be used to retrieve an object from the water for instance, so now it is clear what is discussed here. The mean wave drift force can be broken down into four components (five in shallow water), namely:

- contribution due to relative wave height;
- contribution due to drop of dynamic pressures due to the submerged hull;
- contribution due to pressure due to first order vessel motions;
- contribution due to combined rotation with translational inertia;
- contribution due to a second order wave potential (only in shallow water).

The first of these forces is the dominant one. For high frequency waves, the third and fourth term disappear, because these waves do not result in significant first order vessel motions. In regular waves, the wave drift forces are constant and dependent on the wave frequency.
Because the first term is the dominant wave drift force, the **wave drift forces** and **wave drift moment** are often defined as:

$$\begin{aligned} X_{wd} &= C_{X_{wd}}(\alpha_{wd}, f_{wd})\frac{1}{8}\rho g H_{1/3}^2 L, \\ Y_{wd} &= C_{Y_{wd}}(\alpha_{wd}, f_{wd})\frac{1}{8}\rho g H_{1/3}^2 L, \qquad (5.16) \\ N_{wd} &= C_{N_{wd}}(\alpha_{wd}, f_{wd})\frac{1}{8}\rho g H_{1/3}^2 L^2, \end{aligned}$$

(5.17)

[9]This is from [12] Chapter 4 p29-30

with X_{wd} steady longitudinal wave drift force, Y_{wd} steady transverse wave drift force, N_{wd} steady yaw wave drift moment, ρ density of water, α_{wd} wave direction, f_{wd} the regular wave frequency, $C_{*wd}(\alpha_{wd}, f_{wd})$ α_{wd} and f_{wd} dependent wave drift coefficient, g is the gravity coefficient, $H_{1/3}$ is the significant wave height and L the length of the ship.[10]

Reaction forces occur as a result of the vessel moving through the water and air. Since the reaction forces due to water are much larger than those due to air, the latter is neglected in commonly. The water reaction forces are hard to calculate, this will be another extension for the future.

5.4 Ship movements

The coordinate system defined before in Chapter 2 and figure (2.1) was a body-fixed coordinate system, where the origin is in the ship's center of gravity (G). There are also two other coordinate system important when discussing ship movements, namely the steadily translating coordinate system and the earth-fixed coordinate system. The importance of the steadily translating coordinate system is found in the fact that the ship carries out oscillations around this system. For the calculation of the forces the body-fixed coordinate system is important and position reference systems return earth-fixed coordinates for the position of the ship.
The steadily translating coordinate system is moving forward with a constant ship speed V. If the ship is stationary the direction of this coordinate system are in the same direction as those of the body-fixed system. The origin is at, above, or under the time-averaged position of the center of gravity. The earth-fixed system has the positive x-axis in the direction of the wave propagation. The positive z-axis is directed upwards. The (x, y)-plan lies in the still water surface and can be rotated at a horizontal angle μ relative to the translating coordinate system. This can be seen in figure (5.3).

As mentioned before the circular waves are not important in DP situations, but since this paper handles also situations without DP, here a short introduction of these waves.
Now all the conventions are know, one can define the harmonic elevation of the wave surface ζ in the earth-bound coordinate system by:

$$\zeta = \zeta_a \cos(\omega t - kx_0), \tag{5.18}$$

where ζ_a is the wave amplitude, $k = \frac{2\pi}{\lambda}$ is the wave number, λ is the wave length, ω is the circular wave frequency, t is the time.
The wave speed is defined as:

$$c = \frac{\omega}{k} = \frac{\lambda}{T}.$$

This wave speed has angle of direction μ. The steadily translating coordinate system is moving forward at the ship's speed V which leads to:

$$x_0 = Vt\cos\mu + x\cos\mu + y\sin\mu. \tag{5.19}$$

[10] This part about the wave forces is from [14] p75-78
[11] From [12] Chapter 6 p.2.

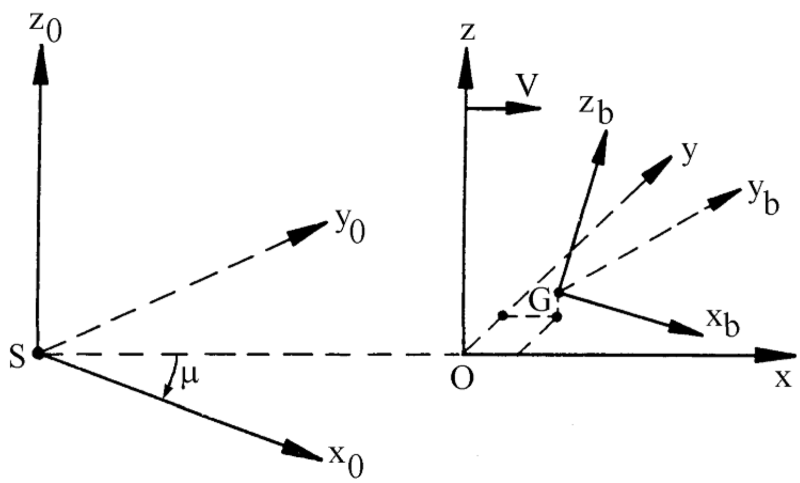


Figure 5.3: On the left is the earth-fixed coordinate system. On the right are the steadily translating and the body-fixed system.[11]

In figure (5.4) a graph can be seen from the ship which encounters the wave.

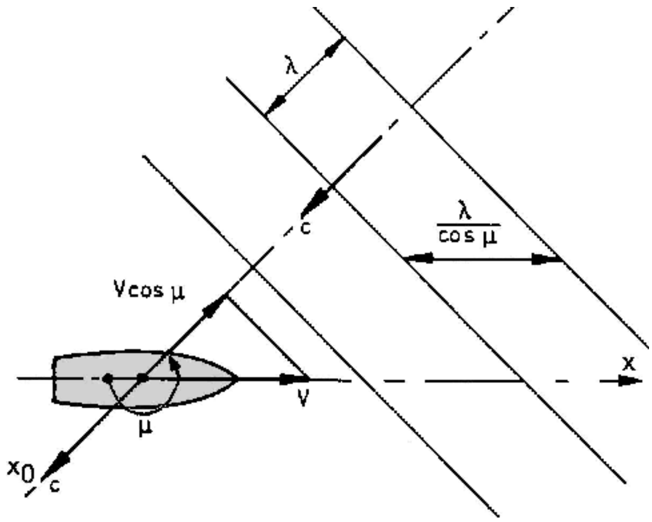


Figure 5.4: Ship encounters the waves.[12]

The period at which this happens is:

$$T_e = \frac{\lambda}{c + V\cos(\mu - \pi)} = \frac{\lambda}{c - V\cos\mu}.$$

From this the circular frequency of encounter, ω_e, becomes:

$$\omega_e = \frac{2\pi}{T_e} = \frac{2\pi(c - V\cos\mu)}{\lambda} = k(c - V\cos\mu).$$

[12]From [12] Chapter 6 p.3.

Since $kc = \omega$ the relation between ω and ω_e is clear:

$$\omega_e = \omega - kV \cos\mu. \tag{5.20}$$

Substituting equations (5.19) and (5.20) in (5.18) leads to the following equation for the wave elevation:

$$\begin{aligned} \zeta &= \zeta_a \cos\left((\omega_e + kV\cos\mu)t - k(Vt\cos\mu + x\cos\mu + y\sin\mu)\right) \\ &= \zeta_a \cos\left(\omega_e t - kx\cos\mu - ky\sin\mu\right). \end{aligned} \tag{5.21}$$

The resulting ship movements are:

$$\begin{aligned} \text{Surge}: &\quad x = x_a \cos\left(\omega_e t + \epsilon_{x\zeta}\right), \\ \text{Sway}: &\quad y = y_a \cos\left(\omega_e t + \epsilon_{y\zeta}\right), \\ \text{Heave}: &\quad z = z_a \cos\left(\omega_e t + \epsilon_{z\zeta}\right), \\ \text{Roll}: &\quad \phi = \phi_a \cos\left(\omega_e t + \epsilon_{\phi\zeta}\right), \\ \text{Pitch}: &\quad \theta = \theta_a \cos\left(\omega_e t + \epsilon_{\theta\zeta}\right), \\ \text{Yaw}: &\quad \psi = \psi_a \cos\left(\omega_e t + \epsilon_{\psi\zeta}\right), \end{aligned} \tag{5.22}$$

in which each of the ϵ values is a different phase angle.
To find the motions of a certain point, one needs to transform this point at first from the body-bound coordinate system to the steadily translating coordinate system. The angles of rotation ϕ, θ and ψ are assumed to be small, so one can apply linearizations: $\sin\phi \approx \phi$ and $\cos\phi \approx 1$. With this the transformation matrix becomes:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{pmatrix} \cdot \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} \tag{5.23}$$

So with this the motions of a point P are given by:

$$\begin{aligned} x_P &= x - y_b\psi + z_b\theta, \\ y_P &= y + x_b\psi - z_b, \phi \\ z_P &= z - x_b\theta + y_b, \phi \end{aligned} \tag{5.24}$$

where x, y, z, ϕ, θ and ψ are the motions of and about the center of gravity, G.As can be seen the motion of the point in x-direction is made up of surge, pitch and yaw contributions, the motion in y-direction of sway, roll and yaw contributions and the motion in z-direction of heave, roll and pitch contributions.

5.5 Thrusters

The thrust T_p and the torque Q_p, which are illustrated in figure 5.5, of a propeller are expressed as functions of the rpm n as:[13]

$$T_p = C_T \rho n^2 D_p^4, \tag{5.25}$$

$$Q_p = C_{Q_0} \rho n^2 D_p^5, \tag{5.26}$$

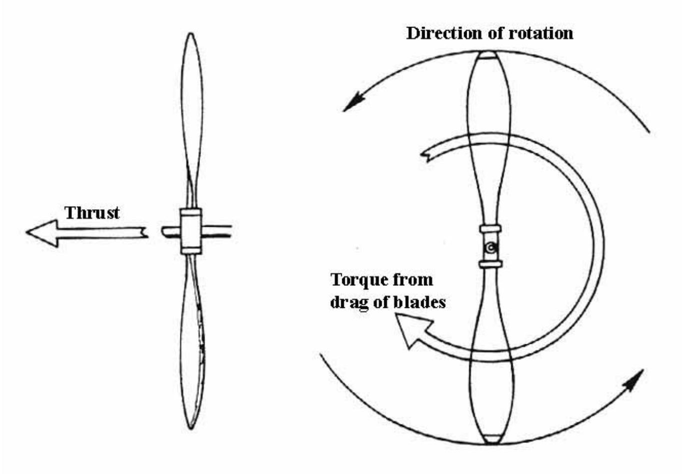


Figure 5.5: The Thrust and Torque are illustrated here.[14]

where C_T and C_{Q_0} are thrust resp. torque coefficients, n is the number of revolutions per minute and D_p is the diameter of the propeller.
Now for the efficiency of the propeller, define the advance number J:

$$J = \frac{V_A}{nD_p}, \tag{5.27}$$

where V_A is the average inflow speed to the propeller.
The open water efficiency η_0 is defined as the ratio of the work done by the propeller in developing a thrust force T_pV_A divided by the work required to overcome the torque $2\pi nQ_p$. With equations (5.25), (5.26) and (5.27) one can deduct the following formula for the efficiency:[15]

$$\eta_0 = \frac{T_pV_A}{2\pi nQ_p} \tag{5.28}$$

$$= \frac{C_T\rho n^2 D_p^4}{C_{Q_0}\rho n^2 D_p^5} \cdot \frac{V_A}{2\pi n}$$

$$= \frac{C_T}{C_{Q_0}} \frac{J}{2\pi} \tag{5.29}$$

5.5.1 Interaction

The performance of a thruster will be influenced by the position of the thruster, the shape of the ship and other nearby thrusters. In this section the interaction between ship and propeller will be discussed. For now the thruster-thruster interaction will be neglected.
The propulsive efficiency is split in three parts: the open water efficiency, the

[13] From [2, 8]
[14] From: http://www.centennialofflight.gov/essay/Theories_of_Flight/props/TH18G2.htm
[15] From [2] p.40 and [8] p.26

hull efficiency and the relative rotative efficiency. The effective power is defined as:

$$P_E = R_T V_s, \tag{5.30}$$

where R_T is the total calm-water resistance of the ship, excluding resistance the propellers yield, and V_s is the speed of the ship, P_E then denotes the power needed to tow the ship without propellers.
The power from the thrust is:

$$P_T = T_p V_A, \tag{5.31}$$

where T_p is the thrust, and V_A the speed of advance of the propeller. The speed of advance of the propeller is defined as the propeller inflow.
The effective power is thus the power to get the ship through the water at a certain speed, when it is pulled through the water and the power from the thrust is the power to get the ship through the water at a certain speed with propellers on the ship. When the ship is fitted with a propeller now and is traveling at the same speed, the pressure field around the hull changes due to the action of the propeller. This creates a pressure difference between the hull surface and the after part of the hull and thus increases the resistance of the vessel from that which was measures in the towed resistance case. Now this increase in resistance can be expressed as:

$$T_p = R_T(1 + a_r), \tag{5.32}$$

where a_r is the resistance augmentation factor.
An alternative way to express this equation is by considering the deduction in effective thrust, which leads to the following expressing:

$$R_T = T_p(1 - t_d), \tag{5.33}$$

where t_d is the thrust deduction factor.
The speed of advance of the propeller is generally slower than the ship speed due to the ship's wake. The retardation of the wake is expressed in the wake fraction, defined as:

$$w_r = 1 - \frac{V_A}{V_s}. \tag{5.34}$$

Typically, $0 < w_r < 0.4$; the larger value applies for very full hulls. Now a new efficiency can be defined, the hull efficiency. This is not really an efficiency, since the values can be bigger than one. The hull efficiency, which is the ratio of the effective power to the thrust power, can be defined as:

$$\eta_H = \frac{P_E}{P_T} = \frac{R_T V_s}{T_p V_A} = \frac{1 - t_d}{1 - w_r}. \tag{5.35}$$

The last part of the propulsive efficiency is the relative rotative efficiency, which is again not really an efficiency, since the values can be bigger than one. This accounts for the differences in torque absorption characteristics of a propeller when operating in mixed wake and open water flows. This value lies close to one in most cases and is generally within the range $0.96 \leq \eta_r \leq 1.04$. This efficiency is defined as:

$$\eta_r = \frac{C_{Q_o}}{C_{Q_b}}, \tag{5.36}$$

with C_{Q_o} is the torque coefficient of the propeller in open water and C_{Q_b} is the torque coefficient of the propeller working in the wake of the ship.
Now after all these definition one can define the propulsive efficiency:

$$\eta_D = \eta_H \cdot \eta_0 \cdot \eta_R \tag{5.37}$$

The propulsive efficiency is generally greater than the open-water propeller efficiency.[16]

[16]This section is based on three books which have similar definitions, but all describe the matter in their own way. From: [2] p.37-40,62-64; [8] p.24-26,32-34; [4] p.85-86,290-292.

Chapter 6

Modeling

Two models will be created, one with DP and one without this system. Both models have the external forces as input and all the information of the ship. In the models the thruster configuration will be optimized in such a way that the energy used by the thrusters is minimal. The first model that is created in this three month literature study is not very realistic. There are a lot of limitations in this first model, but the model is created to develop a working algorithm to minimize the power the thrusters produce. From this position the model will be changed and expanded in the last six months of this project.
Which forces are very important and which are less important in different situations can be found in Appendix A figure A.5.

6.1 Power Model

6.1.1 Without DP

Input:

- Current, wind, wave forces (direction and size)
- Current direction and velocity
- Number of Thrusters and position
- Important information of the ship (size, weight etc)
- Direction and velocity ship (from joystick ?)

In Program:

- Optimization least energy thruster configuration
- Thrusters can not give more power than their maximum
- Boundaries of azimuth thruster to prevent loss of efficiency by thrusters that work against each other

Output:

- RPM thrusters
- direction of azimuth thrusters

6.1.2 With DP

It is needed on beforehand which operating modes[1] are possible in this vessel. Only those modes are available in the program.

Input:

- Current, wind wave forces (direction and size)
- Current position vessel
- Number of thrusters, position and type (which are azimuth thrusters?)
- Mode:
  - manual/joystick: model without DP
  - auto-heading: heading needed
  - auto-position: position needed
  - auto area position: area vessel needs to be in
  - auto-track: way-points of course to follow
  - auto-pilot: course to follow
  - follow target: position target (every x seconds refreshed)
- Type of Position Reference System (best might be (D)GPS. Others need receivers on seabed etc.)
- Optional: Max time to get on position/track

In Program:

- Optimization least energy thruster configuration
- Thrusters can not give more power than their maximum
- Boundaries of azimuth thruster to prevent loss of efficiency by thrusters that work against eachother
- Use of PRS
- Updates of changes in environment

[1]see Chapter 3.2.3

Output:

- RPM thrusters
- Direction of Azimuth Thrusters
- Optional: time it will take to get on position/track?

6.2 Programming

6.2.1 A first model

In figure 6.1 the orientation of the axis, the moment and the forces is given. The moment orientation is taken as clockwise. Using this, one can define a model for the ship simulation.

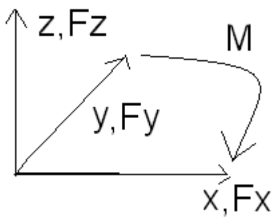


Figure 6.1: The coordinate system.

Input:

- Vector with wind force in x direction, wind force in y-direction and moment of wind.
- Vector with current force in x direction, current force in y-direction and moment of current.
- Vector with wave force in x direction, wave force in y-direction and moment of wave.
- Force (x and y direction, or resulting and angle) and moment that are wanted by the driver of the vessel
- Matrix of position of thrusters and forces now.
- Vector of maximum force that a thruster can give.
- Error tolerance

In program:

1. Calculate the force needed in x-direction:

$$F_x = F_{x_{wanted}} - F_{x_{wind}} - F_{x_{current}} - F_{x_{wave}}. \tag{6.1}$$

2. Calculate the force needed in y-direction:
$$F_y = F_{y_{wanted}} - F_{y_{wind}} - F_{y_{current}} - F_{y_{wave}}. \quad (6.2)$$
3. Calculate the moment needed:
$$M = M_{wanted} - M_{wind} - M_{current} - M_{wave}. \quad (6.3)$$
4. Calculate the force the thrusters provide in the current configuration in x-direction: $(F_x)_{now} = \sum_{i=1}^{n} (F_x)_{thruster_i}$.
5. Calculate the force the thrusters provide in the current configuration in y-direction: $(F_y)_{now} = \sum_{i=1}^{n} (F_y)_{thruster_i}$.
6. Calculate the Moment the thrusters provide in the current configuration: $(M)_{now} = \sum_{i=1}^{n} -y_i \cdot (F_x)_{thruster_i} + x_i \cdot (F_y)_{thruster_i}$.
7. If the forces and moment needed are almost the same as the forces and moment given, then keep this configuration: if $|F_x - (F_x)_{now}| < \epsilon_1$ and $|F_y - (F_y)_{now}| < \epsilon_1$ and $|M - (M)_{now}| < \epsilon_2$.
8. If they are not almost the same then a new configuration must be found.

6.2.2 Method to find the new configuration

With F_x, F_y and M the forces and moment needed from the thrusters, the following formulas must hold (with n the number of thrusters, with subscript i short for thruster i and with x_i x-coordinate of the position of thruster i and y_i y-coordinate of the position of thruster i.)[1]:

$$F_x = \sum_{i=1}^{n} (F_x)_i, \quad (6.4)$$
$$F_y = \sum_{i=1}^{n} (F_y)_i, \quad (6.5)$$
$$M = \sum_{i=1}^{n} (-y_i \cdot (F_x)_i + x_i \cdot (F_y)_i). \quad (6.6)$$

Rewriting equation (6.4) and (6.5) to express $(F_x)_n$ and $(F_y)_n$ as functions of the other forces leads to:

$$(F_x)_n = F_x - \sum_{i=1}^{n-1} (F_x)_i, \quad (6.7)$$
$$(F_y)_n = F_y - \sum_{i=1}^{n-1} (F_y)_i. \quad (6.8)$$

[1] This section is based on [23, 24].

Substituting (6.7) and (6.8) into (6.6) leads to:

$$M = \sum_{i=1}^{n-1} (-y_i \cdot (F_x)_i + x_i \cdot (F_y)_i) - y_n \cdot \left(F_x - \sum_{i=1}^{n-1} (F_x)_i \right) + x_n \cdot \left(F_y - \sum_{i=1}^{n-1} (F_y)_i \right) =$$
$$\sum_{i=1}^{n-1} ((y_n - y_i) \cdot (F_x)_i + (x_i - x_n) \cdot (F_y)_i) - y_n \cdot F_x + x_n \cdot F_y. \quad (6.9)$$

Now assume that $x_n \neq x_{n-1}$.[2] Now rewrite (6.9) to express $(F_y)_{n-1}$ as a function of the other forces:

$$(F_y)_{n-1} = \frac{M + y_n \cdot F_x - x_n \cdot F_y - \sum_{i=1}^{n-1} ((y_n - y_i) \cdot (F_x)_i) - \sum_{i=1}^{n-2} ((x_i - x_n) \cdot (F_y)_i)}{x_{n-1} - x_n} =$$
$$\underbrace{\frac{M + y_n \cdot F_x - x_n \cdot F_y}{x_{n-1} - x_n}}_{s} + \sum_{i=1}^{n-1} \left(\underbrace{\frac{y_i - y_n}{x_{n-1} - x_n}}_{c_1(i)} \cdot (F_x)_i \right) + \sum_{i=1}^{n-2} \left(\underbrace{\frac{x_n - x_i}{x_{n-1} - x_n}}_{c_2(i)} \cdot (F_y)_i \right). \quad (6.10)$$

So this means:

$$(F_y)_{n-1} = s + \sum_{i=1}^{n-1} c_1(i)(F_x)_i + \sum_{i=1}^{n-2} c_2(i)(F_y)_i, \quad (6.11)$$
$$(F_y)_n = F_y - \sum_{i=1}^{n-2} (1 + c_2(i)) \cdot (F_y)_i - s - \sum_{i=1}^{n-1} c_1(i)(F_x)_i.$$

Now it is important that a thruster configuration is found in which the total power is minimized. The formula for the total power is:

$$g((F_x)_1, \ldots, (F_x)_{n-1}, (F_y)_1, \ldots, (F_y)_{n-2}) = \sum_{i=1}^{n} \sqrt{(F_x)_i^2 + (F_y)_i^2}. \quad (6.12)$$

To minimize this, one needs to find the forces for which the partial derivatives of the power are zero, i.e.:

$$\left\{ \begin{array}{rcl} \frac{\partial g}{\partial (F_x)_1} & = & 0, \\ \vdots & \vdots & \vdots \\ \frac{\partial g}{\partial (F_x)_{n-1}} & = & 0, \\ \frac{\partial g}{\partial (F_y)_1} & = & 0, \\ \vdots & \vdots & \vdots \\ \frac{\partial g}{\partial (F_y)_{n-2}} & = & 0. \end{array} \right. \quad (6.13)$$

[2] If they are the same, one can also take another thruster and just rewrite the order of the thrusters!

From (6.12) it is easy to see that:

$$\frac{\partial g}{\partial (F_x)_i} = \sum_{j=1}^{n} \left(\frac{(F_x)_j \cdot \frac{\partial (F_x)_j}{\partial (F_x)_i}}{\sqrt{(F_x)_j^2 + (F_y)_j^2}} + \frac{(F_y)_j \cdot \frac{\partial (F_y)_j}{\partial (F_x)_i}}{\sqrt{(F_x)_j^2 + (F_y)_j^2}} \right), \tag{6.14}$$

$$\frac{\partial g}{\partial (F_y)_i} = \sum_{j=1}^{n} \left(\frac{(F_x)_j \cdot \frac{\partial (F_x)_j}{\partial (F_y)_i}}{\sqrt{(F_x)_j^2 + (F_y)_j^2}} + \frac{(F_y)_j \cdot \frac{\partial (F_y)_j}{\partial (F_y)_i}}{\sqrt{(F_x)_j^2 + (F_y)_j^2}} \right). \tag{6.15}$$

Equation 6.14 and 6.15 can be simplified since $n-3$ of the derivatives in the summation are zero, as can be seen from the following equations:

$$\text{For } j = 1 \ldots n-2, \quad j \neq i:$$
$$\tfrac{\partial (F_x)_j}{\partial (F_x)_i} = 0 \text{ and } \tfrac{\partial (F_y)_j}{\partial (F_y)_i} = 0.$$
$$\text{For } j = 1 \ldots n-2, \quad j = i:$$
$$\tfrac{\partial (F_x)_j}{\partial (F_x)_i} = \tfrac{\partial (F_x)_i}{\partial (F_x)_i} = 1 \text{ and } \tfrac{\partial (F_y)_j}{\partial (F_y)_i} = \tfrac{\partial (F_y)_i}{\partial (F_y)_i} = 1.$$

$$\text{For } i = 1 \ldots n-2:$$
$$\tfrac{\partial (F_y)_{n-1}}{(F_x)_i} = c_1(i), \quad \tfrac{\partial (F_y)_n}{(F_x)_i} = -c_1(i), \quad \tfrac{\partial (F_y)_{n-1}}{(F_y)_i} = c_2(i) \text{ and } \tfrac{\partial (F_y)_n}{(F_y)_i} = -(1 + c_2(i)).$$
$$\text{For } i = 1 \ldots n-1 (\text{from equation 6.7}):$$
$$\tfrac{\partial (F_x)_n}{(F_x)_i} = -1.$$

This leads to:

For $i = 1 \ldots n-1$:

$$\frac{\partial g}{\partial (F_x)_i} = \frac{(F_x)_i}{\sqrt{(F_x)_i^2 + (F_y)_i^2}} + \frac{c_1(i)(F_y)_{n-1}}{\sqrt{(F_x)_{n-1}^2 + (F_y)_{n-1}^2}} - \frac{(F_x)_n + c_1(i)(F_y)_n}{\sqrt{(F_x)_n^2 + (F_y)_n^2}}, \tag{6.16}$$

For $i = 1 \ldots n-2$:

$$\frac{\partial g}{\partial (F_y)_i} = \frac{(F_y)_i}{\sqrt{(F_x)_i^2 + (F_y)_i^2}} + \frac{c_2(i)(F_y)_{n-1}}{\sqrt{(F_x)_{n-1}^2 + (F_y)_{n-1}^2}} - \frac{(1 + c_2(i))(F_y)_n}{\sqrt{(F_x)_n^2 + (F_y)_n^2}}. \tag{6.17}$$

It is not easy to solve the non-linear system (6.13). How to solve this system is explained later in this chapter, but when the solution has been found it is important to compare the forces that are found with the maximum forces of every thruster. If the maximum force of a thruster is lower than the force needed in the optimal configuration, one should run the algorithm again, but this time with the forces of all the thrusters, that should be higher than their maximum[3] in the optimal configuration, held constant at their maximum. In this way one finds a feasible optimal solution. This can be described as the following iteration scheme:

[3] In a first implementation the maximum force is defined in an unrealistic way as a maximum force in x- and y-direction. In reality this maximum is just a maximum rotations per minute of the thrusters. The method will be extended later on.

1. Minimize $g(F_i)$. Say F^* is the thruster configuration that gives the least total force.
2. Compare F_i^* to $(F_{max})_i$. For all $i \in \{1, 2, \dots, 2n-3\}$: if $F_i^* > (F_{max})_i$: $F_i^* = (F_{max})_i$.
3. If no forces are above max, stop. Else: minimize $g(F_k, F_i)$ where F_k are the forces that are not at their max yet and F_i are the forces that are held constant at their maximum. Now one only finds values for F_k. The F_i are not changed.
4. Repeat step 2.

To solve the non-linear system one can use the BFGS, Broyden-Fletcher-Goldfarb-Shanno, method. This method will be explained in the next subsection.[4]

6.2.3 Minimization methods

BFGS Method

To minimize the non-linear function f, one needs a starting point $x^{(0)}$ and a starting symmetric positive definite matrix, H_0. This matrix is mostly chosen as a positive multiple of the identity matrix.
In fact, this method is a quasi-newton method. In Newton's method one uses the Jacobian to optimize the search direction. It is expensive to compute and invert this Jacobian, so to avoid this, one tries in this BFGS Method to approximate the Jacobian. In every step the approximation gets better. The convergence of Newton's method is quadratic, so this is a very nice characteristic. That is the reason why one tries to approximate this method.[5]
The iteration for this method is:

$$\begin{aligned}
&1. \quad x^{(k+1)} = x^{(k)} - H_k^{-1} \nabla f(x^{(k)}), \\
&2. \quad s^{(k)} = x^{(k+1)} - x^{(k)}, \\
&3. \quad y^{(k)} = \nabla f(x^{(k+1)}) - \nabla f(x^{(k)}), \\
&4. \quad H_{(k+1)} = H_k - \frac{H_k s^{(k)} (s^{(k)})^T (H_k)^T}{(s^{(k)})^T H_k s^{(k)}} + \frac{y^{(k)} (y^{(k)})^T}{(y^{(k)})^T s^{(k)}}
\end{aligned} \tag{6.18}$$

In this method it is also possible to write the first two steps as:

$$\begin{aligned}
&1. \quad H_k s^{(k)} = -\nabla f(x^{(k)}), \\
&2. \quad x^{(k+1)} = x^{(k)} + s^{(k)}.
\end{aligned} \tag{6.19}$$

In this way no inverse has to be calculated and a faster method can be applied to solve the first equation for $s^{(k)}$.
Convergence can be checked in every step by calculating the norm $||\nabla f(x^{(k)}||$. The iteration is finished when the norm has become sufficiently small. One

[4] From: http : //www.math.mtu.edu/ msgocken/ma5630spring2003/lectures/global2/global2/node8.html and http : //en.wikipedia.org/wiki/BFGS_method
[5] See [13] p.80-82

should keep in mind that this method only works when $(s^{(k)})^T y^{(k)} > 0$ holds in every step. This can be ensured by the use of an appropriate linesearch.[6]
In the version of the method described in equation (6.18) no line-search will be applied. With a linesearch the method changes slighty and one should determine $\alpha^{(k)}$ in every step. The determination of $\alpha^{(k)}$ is described later on in this chapter. The adaptation for a linesearch is changing equation (6.19) by:

$$2. \quad x^{(k+1)} = x^{(k)} + \alpha^{(k)} s^{(k)} \tag{6.20}$$

Gradient descent method

Another method that can be used is the gradient descent method. This method is also called steepest descent method.
The method has the following 'scheme':

$$x^{(k+1)} = x^{(k)} - \gamma \nabla f(x^{(k)}). \tag{6.21}$$

This γ can be calculated every step by performing a linesearch, or one can choose one γ for every step, but this might mean that the iteration will take a long time. If γ is chosen the same in every step then the new x is not guaranteed to be nearer a solution than the previous one.

Linesearch

To determine the steplength γ in equation (6.21) is by letting $\gamma = \beta^m$, where $\beta \in (0, 1)$ and $m \geq 0$ is the smallest nonnegative integer such that the following inequality, the Armijo rule, holds:[7]

$$f(x^{(k)} - \gamma \nabla f(x^{(k)})) - f(x^{(k)}) < -\alpha\gamma ||\nabla f(x^{(k)})||^2. \tag{6.22}$$

This parameter α is typically set to 10^{-4}.
A linesearch algorithm is:

1. For $k = 1, \ldots, k_{max}$
   (a) Compute f and ∇f. Test for termination.
   (b) Find the least integer $m \geq 0$ such that equation (6.22) holds for $\gamma = \beta^m$.
   (c) $x = x - \gamma \nabla f$.

2. The algorithm stops when the termination criterion is fulfilled or when k_{max} iterations are done unsuccesfully.

[6] From: http://en.wikipedia.org/wiki/Line_search
[7] This section is from [7] chapter 3.

Gradient descent with linesearch

A nice combination of the previous two methods is proposed by Vuik in [24]. The algorithm is searching for the ω_{opt} such that $x^{new} = x^{old} - \omega \nabla f$ is minimal. The search for ω_{opt} can be defined by the following algorithm:

1. Start with $\omega_{min} = 0$ and $\omega_{max} = 1$.
   Calculate $f_i = f(x^{old} - \omega_i \nabla f(x^{old}))$ with $i = \{min, max\}$.
   If $f_{max} > f_{min}$ STOP. Else set $\omega_{min} = \omega_{max}$ and multiply ω_{max} with 2 until $f_{max} > f_{min}$. Now it holds $\omega_{min} \leq \omega_{opt} \leq \omega_{max}$.

2. Split the interval in two such that ω_{opt} remains between ω_{min} and ω_{max} until the distance between ω_{min} and ω_{max} satisfies the termination criterion.

There are also two other methods possible to find the zero of a function.

Bisection method

One needs to find a zero of $\phi'(\alpha)$.
Input: $\epsilon > 0$ accuracy parameter; $\alpha^{(0)}$, $\alpha^{(1)}$ are given such that $\phi'(\alpha^{(0)}) < 0$ and $\phi'(\alpha^{(1)}) > 0$ (which of course means that a zero is in between if the function is continuous).
Step 1: If $|\alpha^{(0)} - \alpha^{(1)}| < \epsilon$ STOP.
Step 2: Let $\alpha = \frac{1}{2}(\alpha^{(0)} + \alpha^{(1)})$;
Step 3: If $\phi'(\alpha) < 0$ then $\alpha^{(0)} := \alpha$; GOTO Step 1.
Step 4: If $\phi'(\alpha) > 0$ then $\alpha^{(1)} := \alpha$; GOTO Step 1.
The function $\phi'(\alpha)$ does not need to be differentiable, as opposed to the next method.[8]

Newton's method

In this method one needs to find the zero of $\phi'(\alpha)$ or minimize the quadratic function (this yields in most cases the same result): $q(\alpha) = \phi(\alpha^{(k)}) + \phi'(\alpha^{(k)})(\alpha - \alpha^{(k)}) + \frac{1}{2}\phi''(\alpha^{(k)})(\alpha - \alpha^{(k)})^2$.
Input: $\epsilon > 0$ is the accuracy parameter; $\alpha^{(0)}$ is the given initial point; k=0;
Step 1: Let $\alpha^{(k+1)} = \alpha^{(k)} - \frac{\phi'(\alpha^{(k)})}{\phi''(\alpha^{(k)})}$.
Step 2: If $|\alpha^{(k+1)} - \alpha^{(k)}| < \epsilon$ STOP.
Step 3: $k := k + 1$, GOTO Step 1;
The method as presented here might be unstable. The minimum of this approximation can be further from the minimum of ϕ than the previous point itself, but if not too far from the mimimum point, this method converges, as said before, quadratically.[9] This is actually in one dimension, for multiple dimension functions the first step changes to:

[8] For Bisection method see [13] p.82-83
[9] For Newton's method see [13] p.83

Step 1: Let $\alpha^{(k+1)} = \alpha^{(k)} - J(\phi(\alpha^{(k)}))\phi(\alpha^{(k)})$,
where J stands for the jacobian matrix of ϕ, which is:

$$\begin{bmatrix} \frac{\partial\phi_1}{\partial x_1} & \cdots & \frac{\partial\phi_1}{x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial\phi_n}{\partial x_1} & \cdots & \frac{\partial\phi_n}{x_n} \end{bmatrix}$$

In the second step the norm of the difference is taken.

Chapter 7

Results

The model from the previous chapter was implemented in Matlab. The maximum thruster forces were not taken into account. Only equation 6.13 was solved, using the method described in Section 6.2.3.
The ship was build considering 20% of the length as the bow. The thrusters are arranged symmetrically. The moments from the thrusters are calculated in relation to the center of gravity (CoG). Some resulting figures can be found in this paragraph, with 3, 4, 5, 6, 10, 20 and 30 thrusters, for a ship with length 100m and width 20m in the figures 7.1-7.7.
The forces working on this ship are:

1. Current -5N in x-direction, 5N in y-direction,
2. Wave: 7N in x-direction, 7N in y-direction,
3. Wind: 3N in x-direction, 8N in y-direction.

The wanted resulting force for the ship is -2N in x-direction and 10N in y-direction.
The external forces are assumed to work on the CoG, so the moments of the external forces are 0. The demanded moment for the ship is also 0.

The energy of the optimal configurations can be found in table 7.1.

Number of Thrusters	Energy in optimal configuration
3	12,34
4	12,21
5	12,22
6	12,44
10	12,37
20	12,22
30	12,91

Table 7.1: Number of thrusters on a ship with length 100m and width 20m versus the energy in the optimal configuration. The configurations are illustrated in the figures 7.1-7.7.

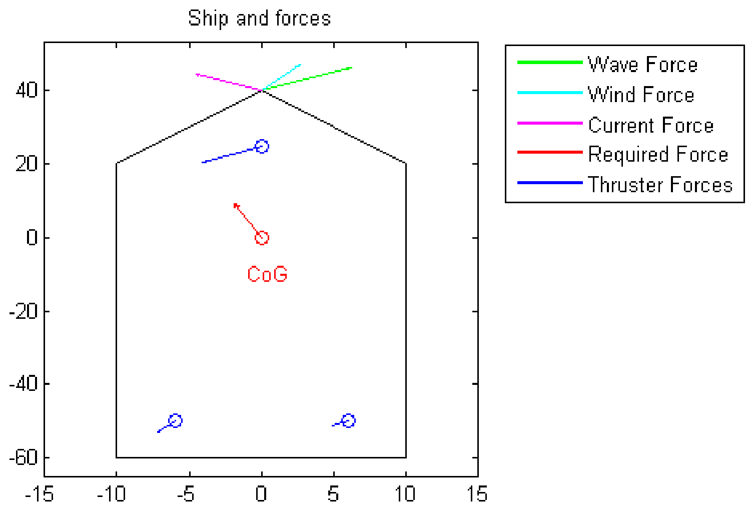


Figure 7.1: Configuration of a ship with dimensions 100x20m with 3 thrusters.

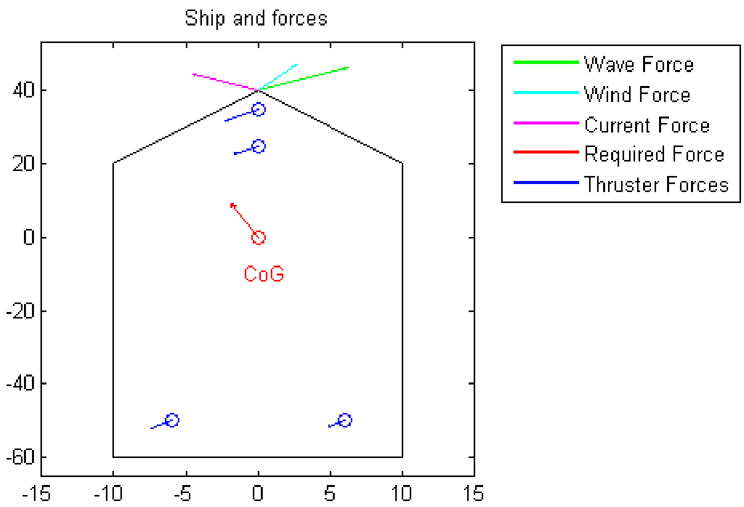


Figure 7.2: Configuration of a ship with dimensions 100x20m with 4 thrusters.

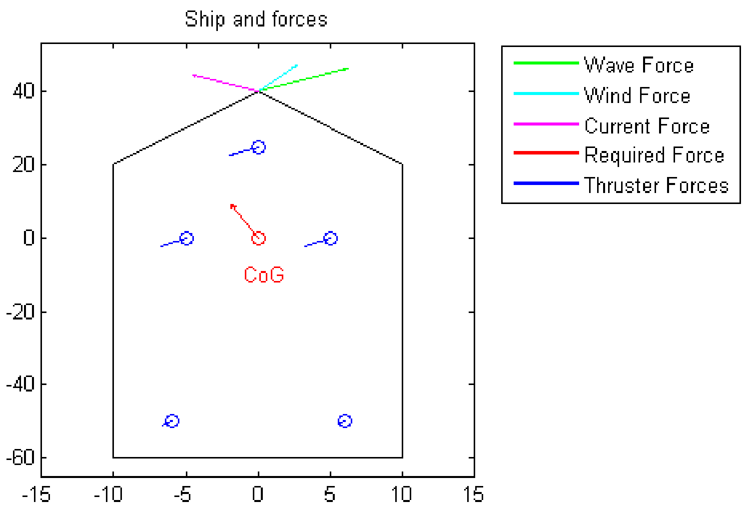


Figure 7.3: Configuration of a ship with dimensions 100x20m with 5 thrusters.

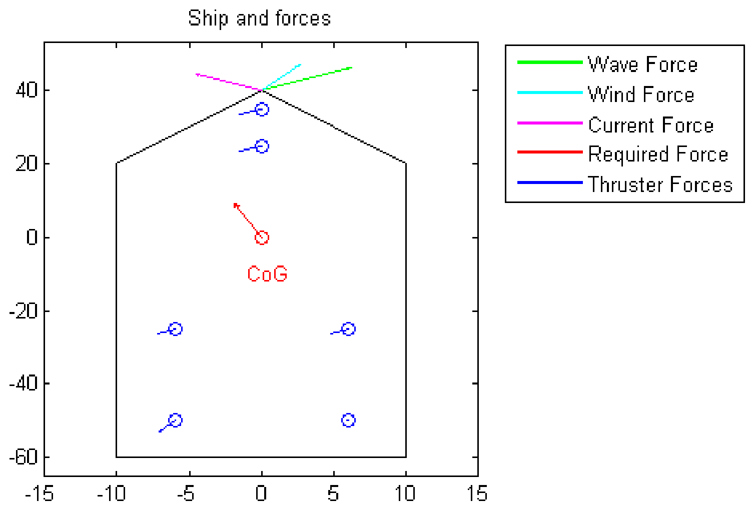


Figure 7.4: Configuration of a ship with dimensions 100x20m with 6 thrusters.

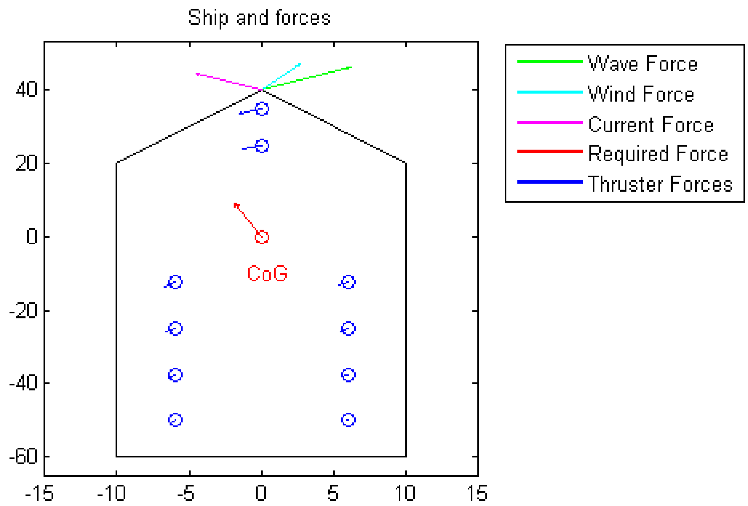


Figure 7.5: Configuration of a ship with dimensions 100x20m with 10 thrusters.

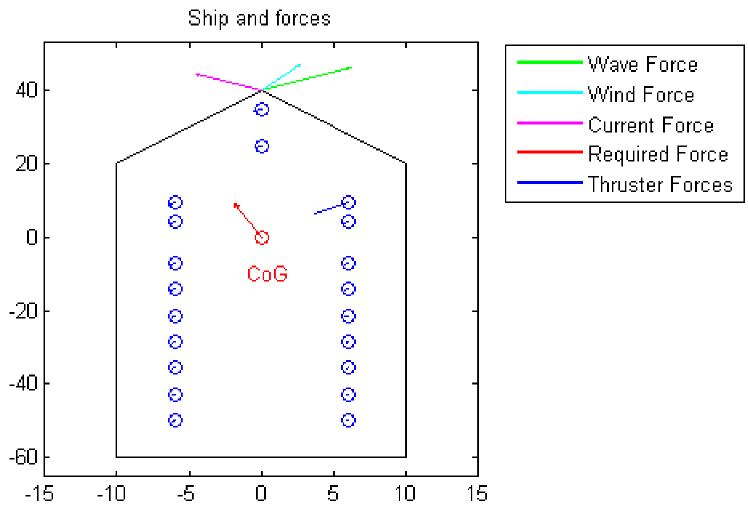


Figure 7.6: Configuration of a ship with dimensions 100x20m with 20 thrusters.

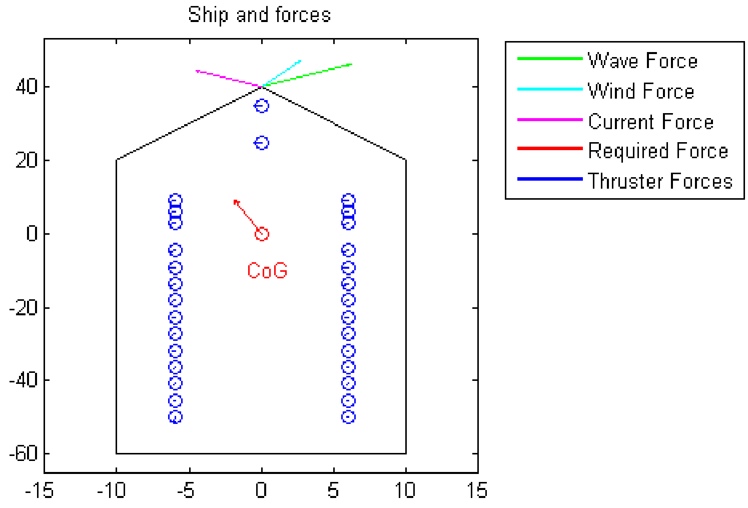


Figure 7.7: Configuration of a ship with dimensions 100x20m with 30 thrusters.

Chapter 8

Evaluation and Future Goals

8.1 Evaluation of first three months

In this 3-month study the formulas for the external forces working on the ship were found. Furthermore, a first model was created in which forces, position of thrusters and ship size where all defined in an artificial way. The thruster configuration was optimized in such a way that the energy of the thrusters was minimal. All the programming was done in Matlab. The ship was assumed to be a pointmass, so none of the external forces was given a moment. Since the thrusters did have a torque, this was a small inconsistency, so the forces were handled as if they worked on the center of gravity, which meant they had no moment.
Also a lot of literature was studied. Some very nice articles were also quoted in the bibliography, but were not cited in this paper; they might be useful in the future.
The first three months a little modeling and implementation was done. This was characterized by fictitious ships with fictitious thruster positions, but the main goal was to create an optimization program that found the optimum configuration in a small amount of CPU time. At first two optimization algorithms were implemented, but these failed to converge for more than 2 thrusters, so finally another method was used. With this the goal has been accomplished. From this position on the programs will be made more realistic in the final six months of the project.

The project description has as objective:

1. Steering ships in the Ship Simulator game with azimuth thrusters in a realistic way;

2. Dynamic Positioning of ships with and without azimuth thrusters in the game and in training applications.

The approach is:

1. Study the basics of ship hydrodynamics, azimuth thrusters propulsion, and DP;
2. Learn Quest3D and Newton Dynamics, as it is implemented in Quest3D as a plug-in called Newton for Quest (NfQ).
3. Investigate the ship dynamics solutions as implemented in Ship Simulator by VSTEP.
4. Investigate current methods and tools of calculating forces on ships that result from current and wind.
5. Design and implement a solution for azimuth thrusters propulsion in Ship Simulator.
6. Design and implement a solution for DP of vessels in Ship Simulator under various weather, wave and current conditions.

The approach from the project description was partly done. The basics of ship hydrodynamics, thruster propulsion and DP were studied. The ship simulator solutions were investigated and methods for calculating forces on ships as well. The tutorials of Quest3D were studied and some features of NfQ were explored. Now it is important to put more realism in the models that are created until now.

8.2 Future Goals

The last six months of this project will be dedicated to make the model more realistic.
From the project description, the following subgoals can be extracted:

1. Define thrust not as thrust in x and y direction, but as a resulting thrust and angle.
2. Define a maximum thrust (speed in rpm) per thruster.
3. Optimization with non azimuth thrusters and combinations. This can be done by defining the angle as a fixed value.
4. Go from Matlab to C++. After this it might be necessary to check certain algorithms with Matlab first, before implementing in C++.
5. Sailing with and without DP. Take the inertia of the ship into account. After the forces are calculated realistically, the sailing should be adjusted of course.
6. Use the formulas for the forces. Find the constants somewhere. These are the formulas from Chapter 5.
7. Finite length of the ship. This means the forces on the ship should integrated or a vector should be defined with values of the forces for different parts of the ship.
8. With the finite length the forces induce also a moment on the ship.
9. In reality it costs time to speed up or slow down the thrusters and also to rotate the azimuthing thrusters. This should be taken into account.
10. Penalty for reverse thrust of every thruster.
11. Implement test cases with real ships and known forces.
12. Create with the programmers from VSTEP an interface with information for the skipper like for instance conning displays (different conning displays can be found in Appendix A figures A.1-A.3).

If there is time enough, the implementation will be extended with more complex subjects such as circular waves. Furthermore, some other optimization techniques will be used if necessary.

Appendix A

Figures

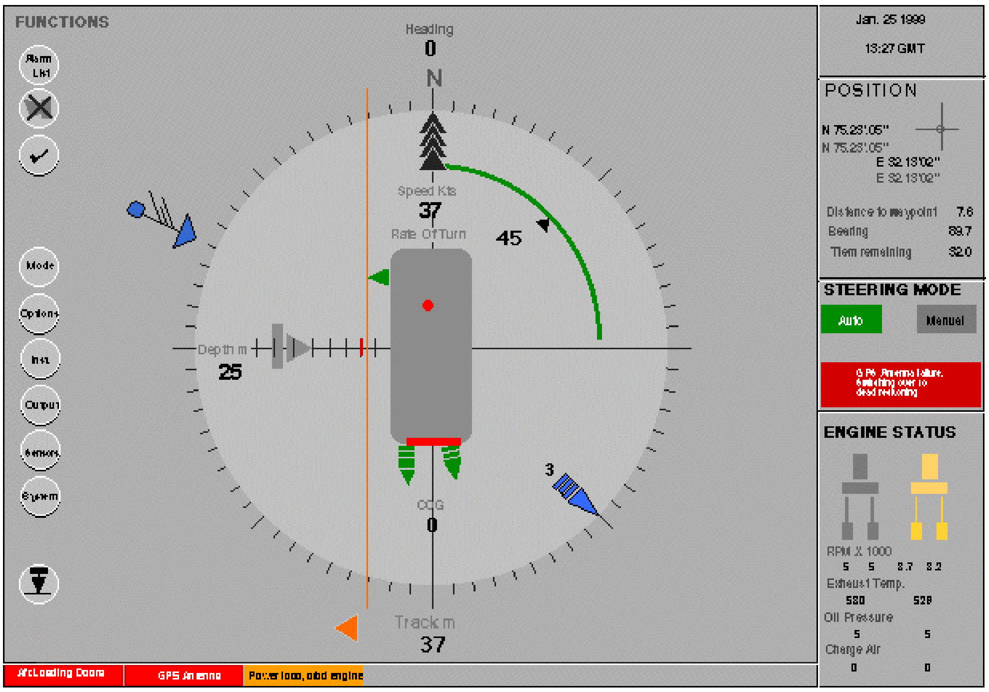


Figure A.1: A picture of a conning display.[1]

[1]From: http://www.msidesign.se/designs/conning.jpg

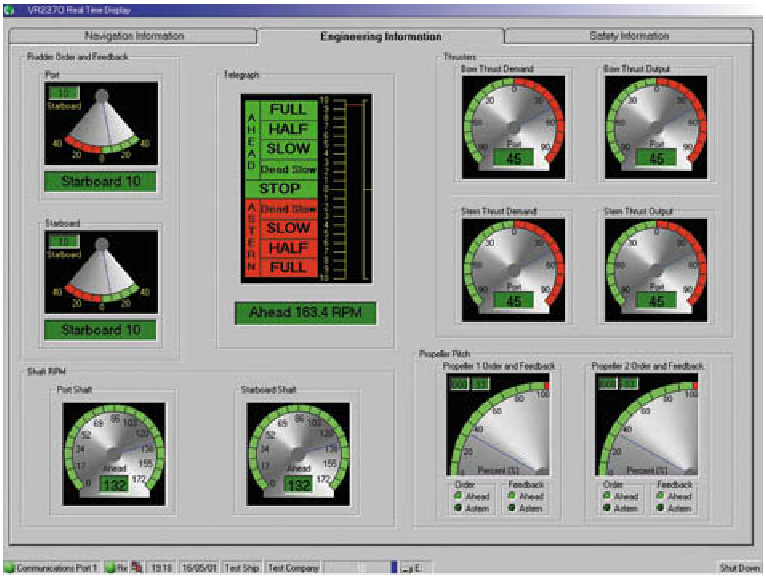


Figure A.2: A picture of a conning display.[2]

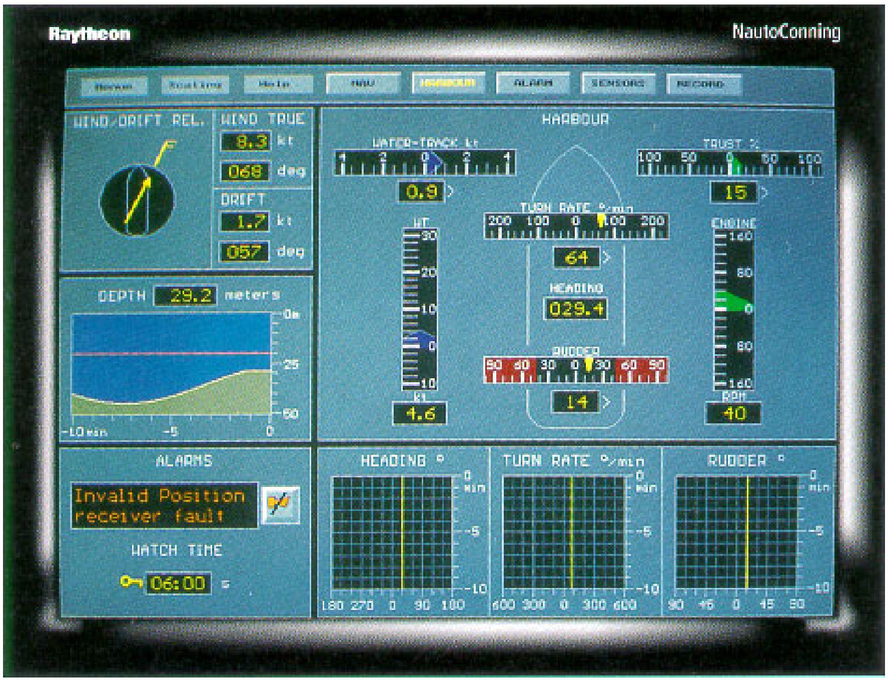


Figure A.3: A picture of a conning display.[3]

[2]From: http://www.ami-gfv.com/image2/display1.jpg
[3]From: http://www.l-3klein.com/navigation/nautoconning/conning_display.jpg

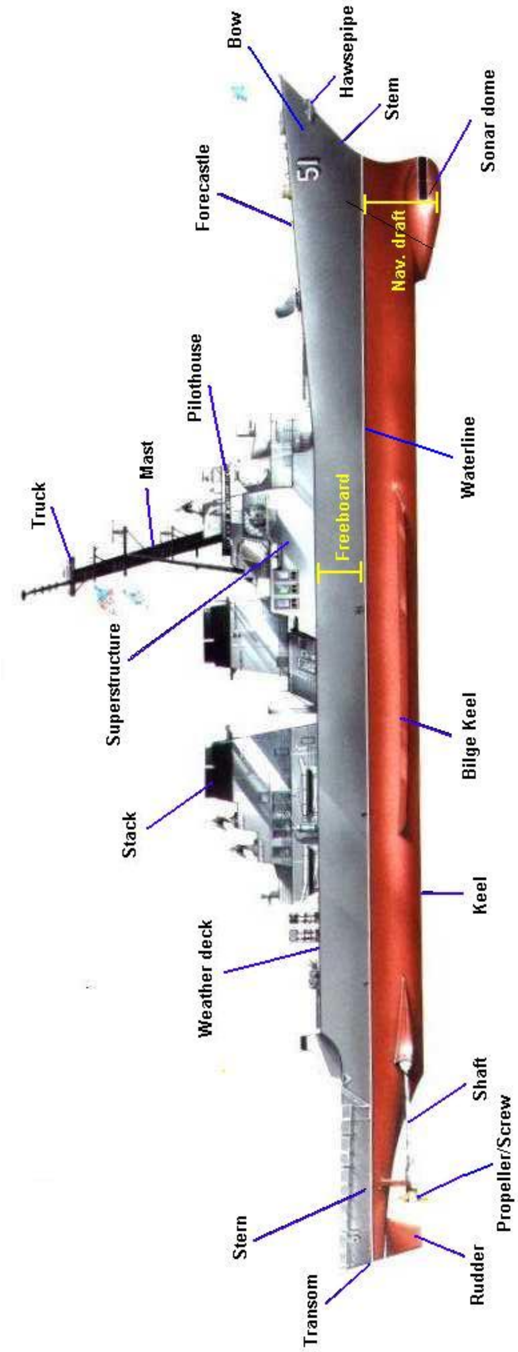


Figure A.4: The terminology used often when speaking of ships is illustrated in this picture.[4]

[4]From: http://navsci.berkeley.edu/ns12b/Presentations/Ship%20Operations/K%20-%20Shiphandling.ppt

	Current	Wind	Waves	
			Circular	Drift
River	+	+-	-	+-
Sailing	+	+-	-	+-
Turning in small sidearm	++	++	+	+
Ocean	+	++-	-	+
Sailing	+	+	-	+
Lay still near oil rig [(un)loading]	+	++	+	++
Approaching oil rig	+	++	-	+
Harbour				
No important flows	-	+*	-	+-
Accelerate	-	+*	-	+-
Anchoring	-	++*	+	+
Unloading	-	++*	+	+
With important flows	+	+*	-	+-
Accelerate	+	+*	-	+-
Anchoring	++	++*	+	+
Unloading	++	++*	+	+

* Some harbours have windshields, so the wind influence is less important there.

Rating	**Meaning**
-	Not important
+-	Not very important
+	Important
++	Very important

Figure A.5: The importance of the different external influences in specific sailing conditions and applications.

[5] Scan from [3], p. 15

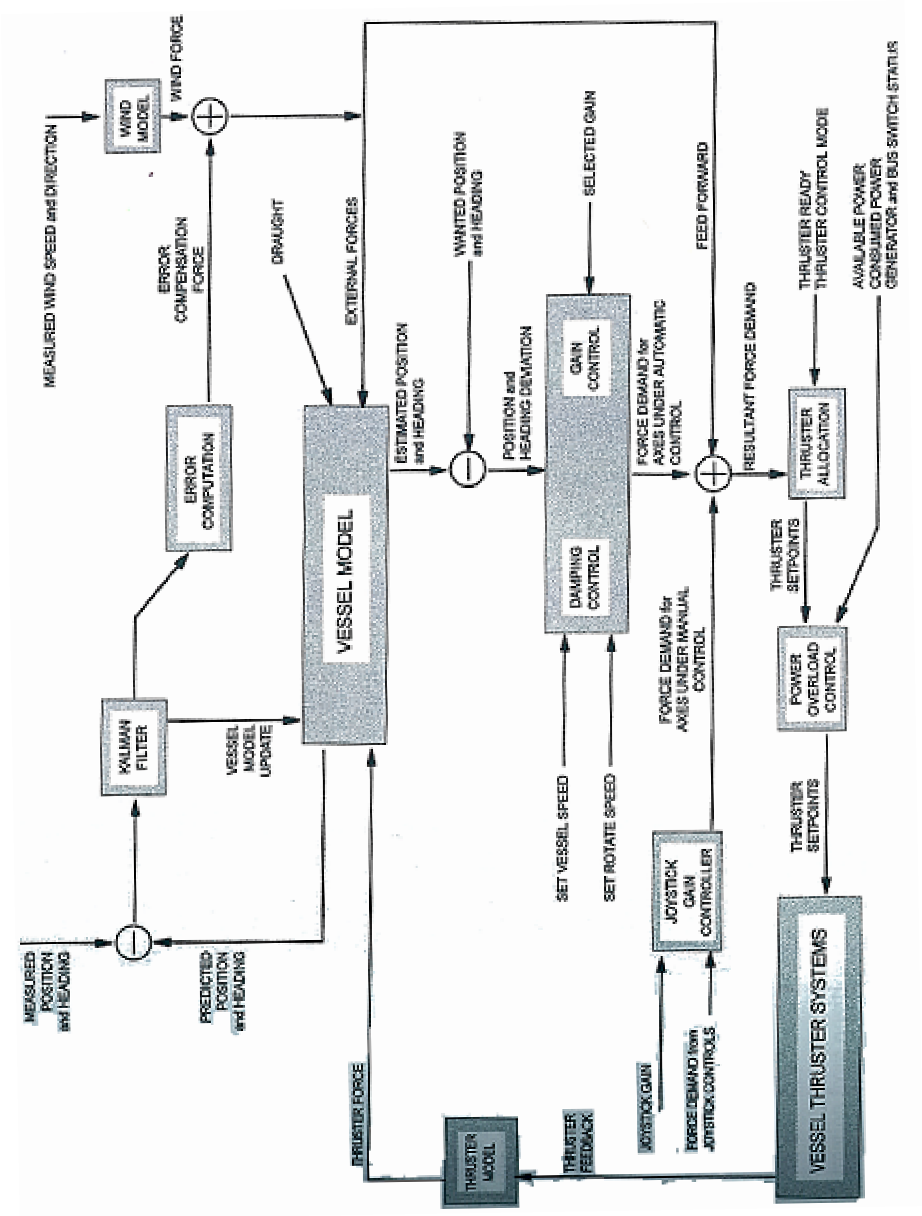


Figure A.6: This is an example of a mathematical model behind DP.[5]

Appendix B

Testproblem

In this testproblem the following forces are working on the ship:
In x-direction: wind = 3, current = -5 and wave = 7.
In y-direction: wind = 8, current = 5 and wave = 7.
The moment for all the external forces is taken 0 for simplicity.
The force demanded by the skipper is -2 in x-direction and 10 in y-direction. The moment is 0.
There are two thrusters and their positions are: (-6,-50) and (6,-50).
The coordinate system of importance here is the body-fixed coordinate system.
Now from equations 6.1, 6.2 and 6.3 one can calculate F_x, F_y and M:

$$\begin{aligned} F_x &= F_{x_{wanted}} - F_{x_{wind}} - F_{x_{current}} - F_{x_{wave}} = -2 - 3 - (-5) - 7 = -7, \\ F_y &= F_{y_{wanted}} - F_{y_{wind}} - F_{y_{current}} - F_{y_{wave}} = 10 - 8 - 5 - 7 = -10, \\ M &= M_{wanted} - M_{wind} - M_{current} - M_{wave} = 0 - 0 - 0 - 0 = 0. \end{aligned}$$

For the moment the following formula should hold: $M = -y_1 \cdot (F_x)_1 + x_1 \cdot (F_y)_1 - y_2 \cdot (F_x)_2 + x_2 \cdot (F_y)_2 = 50 \cdot (F_x)_1 - 6 \cdot (F_y)_1 + 50 \cdot (F_x)_2 + 6 \cdot (F_y)_2$. With this the following minimization problem can be defined to minimize the energy of the thrusters:

$$\min \quad \sqrt{(F_x)_1^2 + (F_y)_1^2} + \sqrt{(F_x)_2^2 + (F_y)_2^2}. \tag{B.1}$$

With :

$$(F_x)_1 + (F_x)_2 = F_x = -7, \tag{B.2}$$

$$(F_y)_1 + (F_y)_2 = F_y = -10, \tag{B.3}$$

$$50 \cdot (F_x)_1 - 6 \cdot (F_y)_1 + 50 \cdot (F_x)_2 + 6 \cdot (F_y)_2 = M = 0. \tag{B.4}$$

Multiplying (B.2) with -50 and add this up with (B.4) yields:

$$\begin{aligned} -50(F_x)_1 - 50(F_x)_2 &= 350 \\ 50(F_x)_1 + 50(F_x)_2 - 6(F_y)_1 + 6(F_y)_2 &= 0 \quad + \\ \hline -6(F_y)_1 + 6(F_y)_2 &= 350. \end{aligned} \tag{B.5}$$

Multiplying (B.3) with 6 and add this up with (B.5) yields:

$$\begin{array}{rcll} -6(F_y)_1 + 6(F_y)_2 & = & 350 & \\ 6(F_y)_1 + 6(F_y)_2 & = & -60 & + \\ \hline 12(F_y)_2 & = & 290 & \rightarrow (F_y)_2 = \dfrac{290}{12}. \end{array} \tag{B.6}$$

Substituting (B.6) into (B.3) yields:

$$(F_y)_2 + \frac{290}{12} = -10 \quad \rightarrow (F_y)_1 = -\frac{410}{12}. \tag{B.7}$$

One can rewrite equation (B.2):

$$(F_x)_1 = -7 - (F_x)_2. \tag{B.8}$$

Now substitute (B.6), (B.7) and (B.8) into (B.1) and the minimization problem is transformed into:

$$\begin{array}{ll} \min & \sqrt{(-7-(F_x)_2)^2 + (\frac{410}{12})^2} + \sqrt{(F_x)_2^2 + (\frac{290}{12})^2} \rightarrow \\ \min & \sqrt{(F_x)_2^2 + 14(F_x)_2 + \frac{43789}{36}} + \sqrt{(F_x)_2^2 + \frac{21025}{36}} \end{array} \tag{B.9}$$

To minimize this function, one should search for the point were the derivative is zero. Taking the derivative of (B.9) leads to:

$$\frac{(F_x)_2 + 7}{\sqrt{(F_x)_2^2 + 14(F_x)_2 + \frac{43789}{36}}} + \frac{(F_x)_2}{\sqrt{(F_x)_2^2 + \frac{21025}{36}}} = 0. \tag{B.10}$$

To find this zero one could use algorithms like Newton's method or bisection method. In this case equation (B.10) was solved by maple, yielding as an answer: $(F_x)_2 = -\frac{29}{10}$. Using this answer in (B.8) leads to: $(F_x)_1 = -7 + \frac{29}{10} = -\frac{41}{10}$. The optimal configuration of the thrusters is then:

$$\begin{array}{rcl} F_1 & = & \left(-\dfrac{41}{10}, -\dfrac{410}{12}\right) \approx (-4.1, -34.16666667)\,, \\ F_2 & = & \left(-\dfrac{29}{10}, \dfrac{290}{12}\right) \approx (-2.9, 24.16666667)\,. \end{array}$$

With the minimization methods from Chapter 6.2.3 code from Matlab is created. From this Matlab calculates:

$$\begin{array}{rcl} F_1 & = & (-4.0937, -34.1667)\,, \\ F_2 & = & (-2,9067, 24.1667)\,. \end{array}$$

As can be seen the configurations are almost the same, the difference is approximately 0.01.
The configuration can also be seen in the plot from the matlab program in figure (B.1).

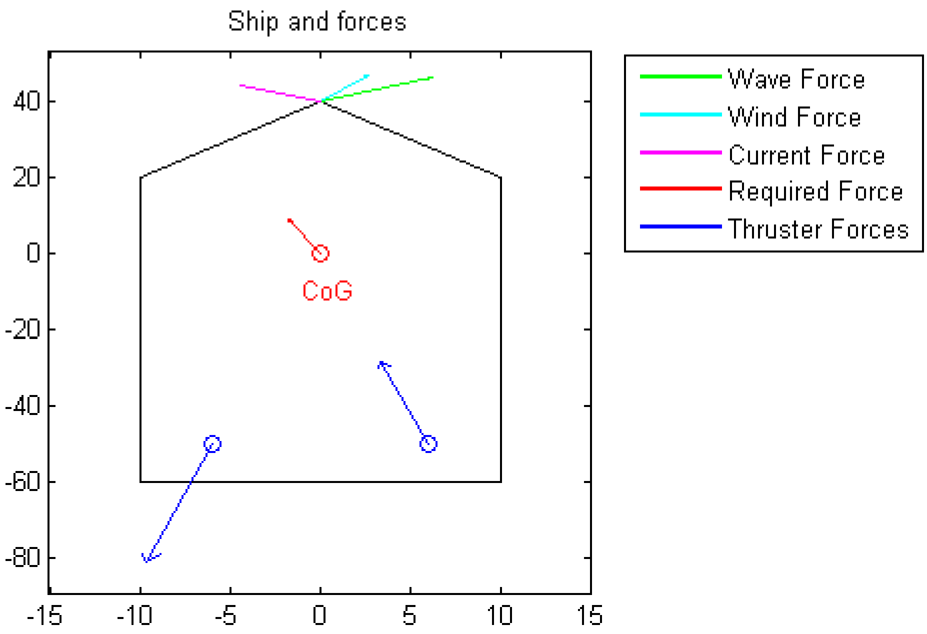


Figure B.1: Optimal solution from Matlab program with two thrusters.

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