On a quantum neural network to compute implied volatility in finance

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1 Introduction

There have been a growing number of computationally challenging problems arising in the filed of computational finance, which require huge computational resources. Deep learning-based algorithms have become increasingly popular to address the computational challenges, for example, pricing financial derivatives [2, 3].

As one of promising next-generation computing technologies, a quantum computer is supposed to outperform classical (electronic) computers and impact the scientific computing in many fields [4]. Quantum computing processes information through the principles of quantum mechanics (e.g., quantum-bits), which can encode an exponentially large amount of data into a linearly large amount of quantum-bits and perform operations on all data at the same time. In recent years financial institutions have been showing great interest into quantum computing, for example, JP Morgan [5] reviewed quantum algorithms for various financial tasks and the authors of [6] developed quantum Monte Carlo methods to price financial derivatives.

The combination of quantum computing and artificial neural networks forms the concept of quantum deep learning. It could be in the form of quantum computers emulating the exact computations of deep neural networks, or trainable quantum circuits resembling multi-layer perceptron structure. Recent advancements of training quantum neural networks can be found in [1]. This MSc thesis aims to explore quantum neural networks to compute the implied volatility from American-style and European-style options, as an extension of the paper [3].

Implied volatility is an important quantity in finance, which represents a specific measure of the future price uncertainty from the viewpoint of market practitioners. Computing the implied volatility is to search for a volatility for the Black-Scholes pricing model, so that the Black-Scholes model can produce an option price which exactly matches the observed option price. Mathematically, given an observed market option price V^{mkt} (European or American-style option), the implied volatility σ^* is defined by

$$BS(\sigma^*; S_0, K, T, r, q) = V^{mkt}, \tag{1}$$

where $BS(\cdot)$ represents the Black-Scholes pricing model, and S_0 represents underlying asset price at time $t = 0$, with the maturity time T, risk-free interest rate r, strike price K, continuous dividend yield q, A closed-form expression of Eq. (2) is unavailable for either European-style or American-style options. This is a numerically challenging inverse problem.

Different from traditionally relying on iterative root-finding numerical methods (like Newton-Raphon), the paper [2] developed a neural-networks-based algorithm to solve the inverse function for implied volatility, as follows,

$$\sigma^* = BS^{-1}(V^{mkt}, S_0, K, T, r, q), \tag{2}$$

which speeds up the computation by orders of magnitude, especially when dealing with a large amount of option prices simultaneously. The author [7] replaced the classical neural network with a quantum neural network to compute the implied volatility in the case of only one input variable, that is, the European-style market option price. This thesis will employ quantum deep neural networks to process multiple input variables, as shown in Eq. (2), and further compute the implied volatility from American-style options (much more complicated than European-style options).

2 Objectives

- Literature review on quantum computing and quantum machine learning for financial derivatives.
- Conversion of the neural network-based algorithms in [3] into the language of quantum computing.

- Implementation of quantum neural networks (e.g., IBM's quantum computer), comparison between classical and quantum neural networks.
- Numerical experiments and analysis.

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