
# Performance of SIMPLE-type Preconditioners in CFD Applications for Maritime Industry

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# Maritime Research Institute Netherlands

Located in Wageningen, Ede and Houston

Agents in Spain and Brasil

Joint Venture in China

330 employees

Foundation

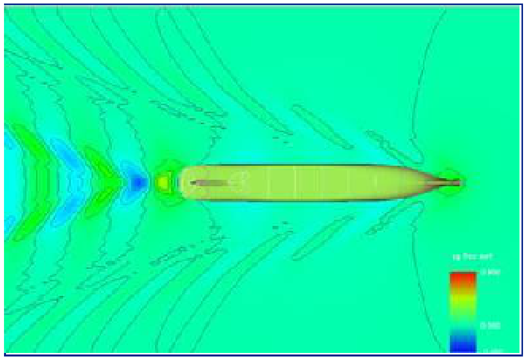
Non-profit

Since 1932

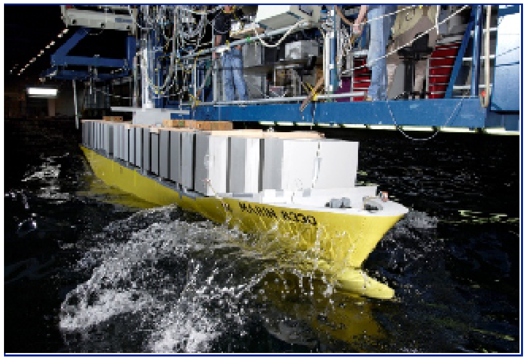
9200 models

7100 propellers

# Activities



Simulations



Model testing

Full scale

Training

## Overview

**Problem description:** maritime applications require large, unstructured grids

- matrix-free approach for coupled Navier-Stokes system
- only compact stencil for velocity and pressure sub-systems

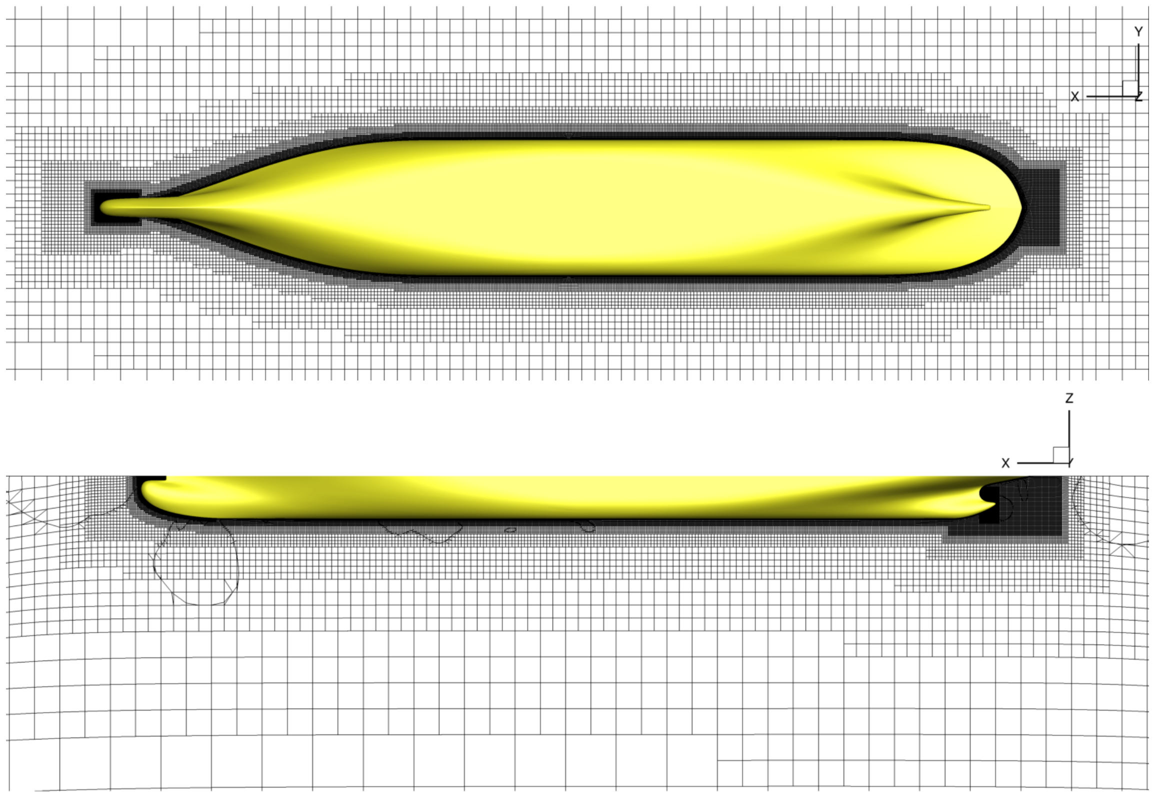
**Proposed solution:** solve coupled system with Krylov subspace method and SIMPLE-type preconditioner

- coupled matrix not needed to build preconditioner
- special treatment of stabilization

**Evaluation:** SIMPLE as solver versus SIMPLE as preconditioner

- reduction in number of non-linear iterations and wall-clock time?

# Container vessel (unstructured grid)



RaNS equations

$k$-$\omega$ turbulence model

$y^+ \approx 1$

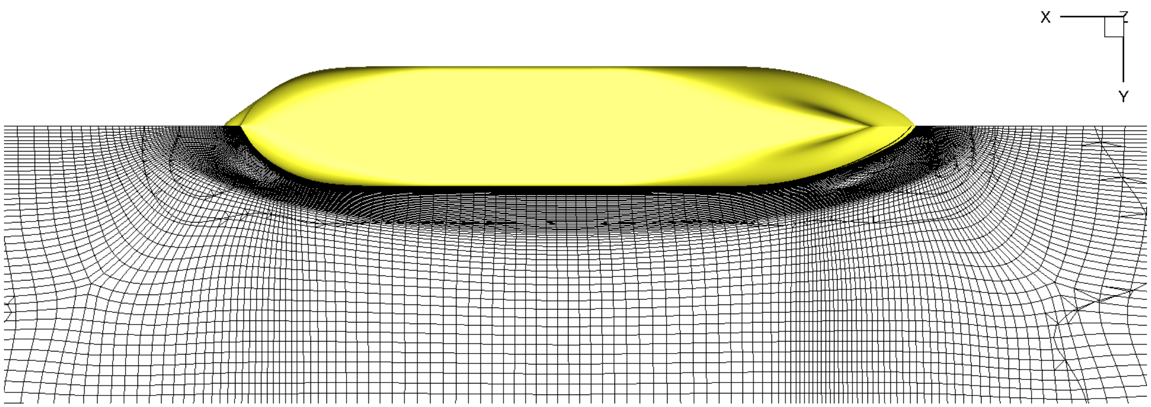
Model-scale:

$\mathrm{Re} = 1.3 \cdot 10^7$

13.3m cells

max aspect ratio $1 : 1600$

## Tanker (block-structured grid)

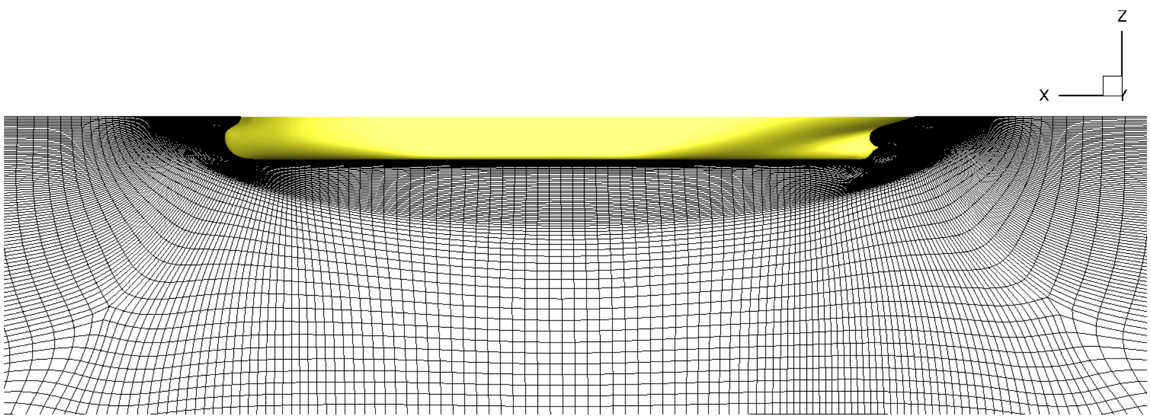



Model-scale:

$\mathrm{Re} = 4.6 \cdot 10^6$

2.0m cells

max aspect ratio $1 : 7000$

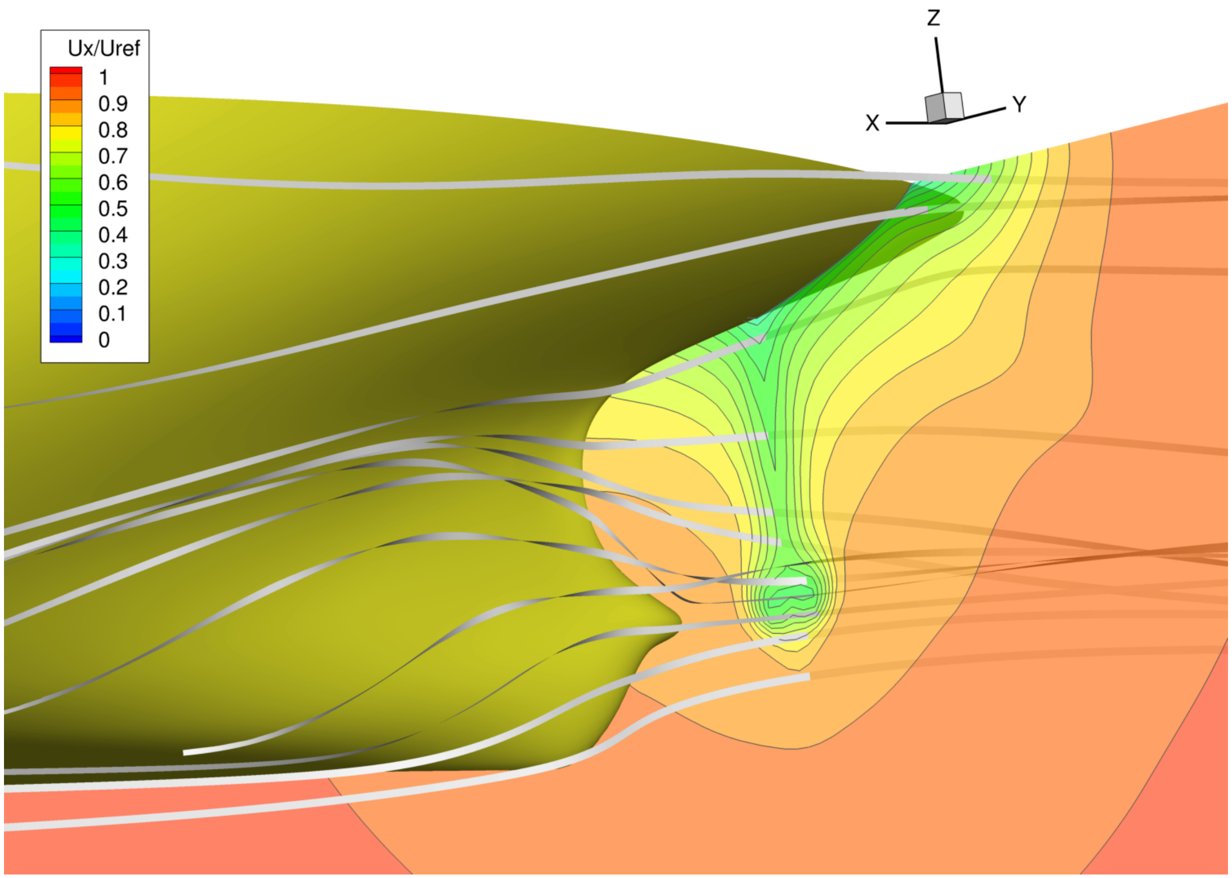



Full-scale:

$\mathrm{Re} = 2.0 \cdot 10^9$

2.7m cells

max aspect ratio $1 : 930\,000$




streamlines around the stern and the axial velocity field in the wake.

# Discretization

Co-located, cell-centered finite volume discretization of the steady Navier-Stokes equations with Picard linearization leads to linear system:

$$\begin{bmatrix} Q_1 & 0 & 0 & G_1 \\ 0 & Q_2 & 0 & G_2 \\ 0 & 0 & Q_3 & G_3 \\ D_1 & D_2 & D_3 & C \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ p \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ g \end{bmatrix} \qquad \text{for brevity: } \begin{bmatrix} Q & G \\ D & C \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$$

with $Q_1 = Q_2 = Q_3$.

$\Rightarrow$ Solve system with FGMRES and SIMPLE-type preconditioner

Turbulence equations ($k$-$\omega$ model) remain segregated

## Defect correction: cornerstone of FVM

Consider a lower-order scheme (e.g. the upwind scheme)

$$Q_{\text{UDS}}\, u = f_{\text{UDS}}$$

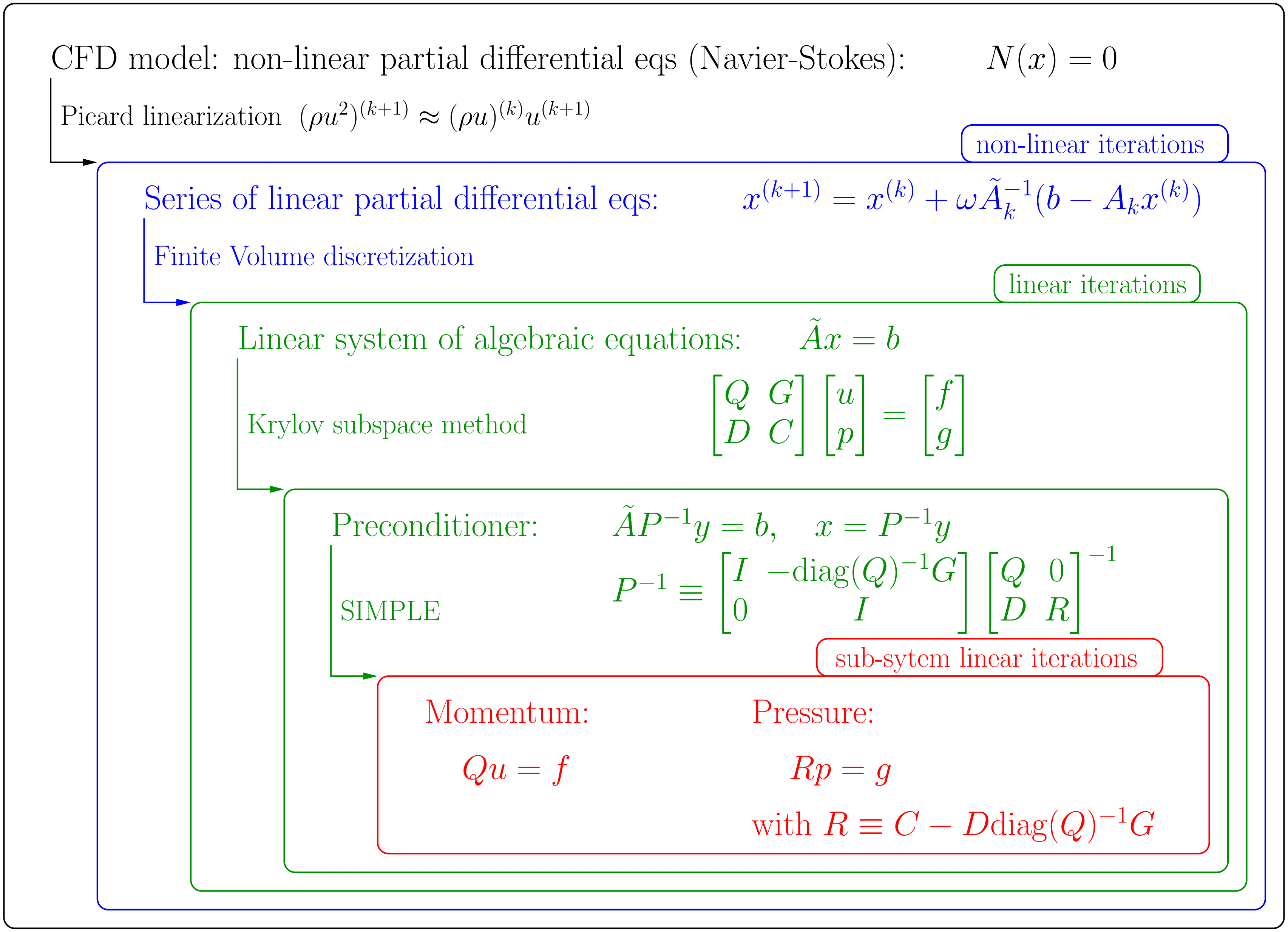
and a higher-order scheme (e.g. central or $\kappa$-scheme with limiter)

$$Q_{\text{CDS}}\, u = f_{\text{CDS}}$$

Then a single defect correction becomes

$$Q_{\text{UDS}}\, u^{k+1} = f_{\text{CDS}} - (Q_{\text{CDS}}\, u^{k} - Q_{\text{UDS}}\, u^{k})$$

$\Rightarrow$ matrix $Q_{\text{UDS}}$ is an M-matrix. Easy to solve. Eccentricity and non-orthogonality corrections also in defect correction form.


CFD model: non-linear partial differential eqs (Navier-Stokes): $N(x) = 0$
Picard linearization $(\rho u^2)^{(k+1)} \approx (\rho u)^{(k)} u^{(k+1)}$
non-linear iterations
Series of linear partial differential eqs: $x^{(k+1)} = x^{(k)} + \omega \tilde{A}_k^{-1}(b - A_k x^{(k)})$
Finite Volume discretization
linear iterations
Linear system of algebraic equations: $\tilde{A}x = b$
Krylov subspace method
$\begin{bmatrix} Q & G \\ D & C \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$
Preconditioner: $\tilde{A}P^{-1}y = b, \quad x = P^{-1}y$
SIMPLE
$P^{-1} \equiv \begin{bmatrix} I & -\mathrm{diag}(Q)^{-1}G \\ 0 & I \end{bmatrix} \begin{bmatrix} Q & 0 \\ D & R \end{bmatrix}^{-1}$
sub-sytem linear iterations
Momentum:
$Qu = f$
Pressure:
$Rp = g$
with $R \equiv C - D\mathrm{diag}(Q)^{-1}G$

## SIMPLE-method

Given $u^k$ and $p^k$:

1. solve $Qu^* = f - Gp^k$
2. solve $(C - DQ^{-1}G)p' = g - Du^* - Cp^k$
3. compute $u' = -Q^{-1}Gp'$
4. update $u^{k+1} = u^* + u'$ and $p^{k+1} = p^k + p'$

with the SIMPLE approximation $Q^{-1} \approx \mathrm{diag}(Q)^{-1}$.

$\Rightarrow$ "Matrix-free": only assembly and storage of $Q$ and $(C - DQ^{-1}G)$. For $D$, $G$ and $C$ the action suffices.

## SIMPLER: additional pressure prediction

Given $u^k$ and $p^k$, start with a pressure prediction:

1. solve
   $$(C - D\,\mathrm{diag}(Q)^{-1}G)p^* = g - Du^k - D\,\mathrm{diag}(Q)^{-1}(f - Qu^k)$$

2. continue with SIMPLE using $p^*$ instead of $p^k$

## Some practical constraints

Compact stencils are preferred on unstructured grids:

- neighbors of cell readily available; neighbors of neighbors not

Also preferred because of MPI parallel computation:

- domain decomposition, communication

Compact stencil?

✓ Matrix $Q_1 (= Q_2 = Q_3)$, thanks to defect correction

✗ Stabilization matrix $C$

$\Rightarrow$ modify SIMPLE(R) such that $C$ is not required on the l.h.s.

## Treatment of stabilization matrix

- In SIMPLE, neglect $C$ in l.h.s. of pressure correction equation

$$(C - D\text{diag}(Q)^{-1}G)p' = g - Du^* - Cp^k$$

$$\Downarrow$$

$$-D\text{diag}(Q)^{-1}Gp' = g - Du^* - Cp^k$$

- In SIMPLER, do *not* involve the mass equation when deriving the pressure prediction $p^*$

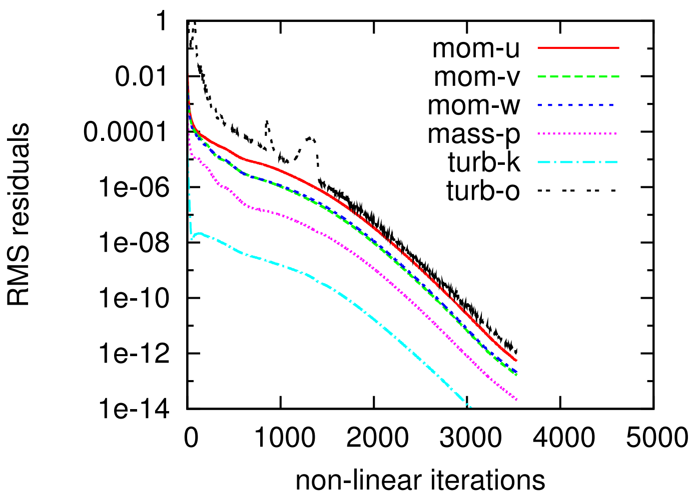
$$(C - D\,\text{diag}(Q)^{-1}G)p^* = g - Du^k - D\,\text{diag}(Q)^{-1}(f - Qu^k)$$

$$\Downarrow$$

$$-D\,\text{diag}(Q)^{-1}Gp^* = -D\,\text{diag}(Q)^{-1}(f - Qu^k)$$

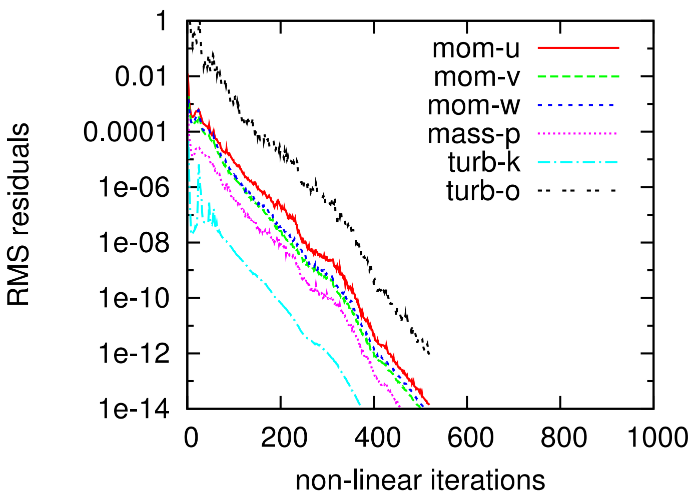
# Example of iterative convergence (tanker)

SIMPLE




$\omega_u = 0.2 \quad \omega_p = 0.1$

KRYLOV-SIMPLER




$\omega_u = 0.8 \quad \omega_p = 0.3$

# Container vessel

Tables show number of non-linear iterations and wall clock time needed to converge to machine precision, starting from uniform flow.

Model-scale $\mathrm{Re} = 1.3 \cdot 10^7$, max cell aspect ratio $1 : 1600$

| grid | CPU cores | SIMPLE | | KRYLOV-SIMPLER | |
|---|---|---|---|---|---|
| | | # its | Wall clock | # its | Wall clock |
| 13.3m | 128 | 3187 | 5h 26mn | 427 | 3h 27mn |

# Tanker

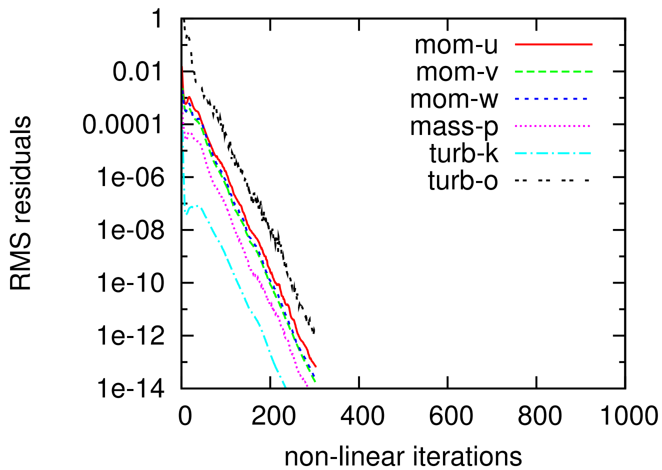
Model-scale $\mathrm{Re} = 4.6 \cdot 10^6$, max cell aspect ratio $1 : 7000$

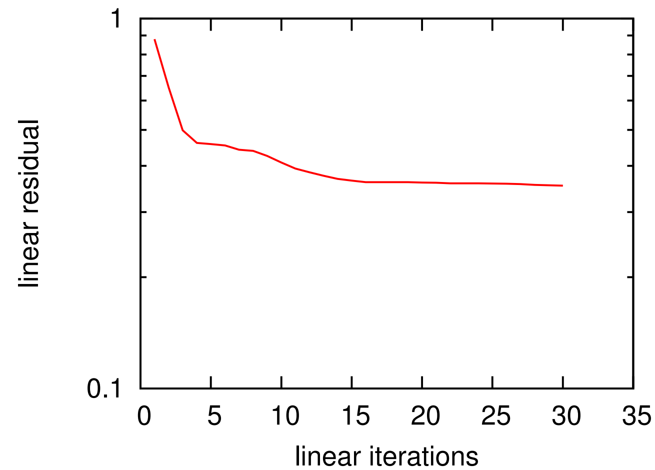
| grid | CPU cores | SIMPLE | | KRYLOV-SIMPLER | |
|---|---|---|---|---|---|
| | | its | Wall clock | its | Wall clock |
| 0.25m | 8 | 1379 | 25mn | 316 | 29mn |
| 0.5m | 16 | 1690 | 37mn | 271 | 25mn |
| 1m | 32 | 2442 | 57mn | 303 | 35mn |
| 2m | 64 | 3534 | 1h 29mn | 519 | 51mn |

Full-scale $\mathrm{Re} = 2.0 \cdot 10^9$, max cell aspect ratio $1 : 930\,000$

| grid | CPU cores | SIMPLE | | KRYLOV-SIMPLER | |
|---|---|---|---|---|---|
| | | its | Wall clock | its | Wall clock |
| 2.7m | 64 | 29 578 | 16h 37mn | 1330 | 3h 05mn |

# Remaining problems




Outer convergence... ...but inner stagnation(!)

- Larger nb of non-linear iters to compensate for stagnation of linear iter. Does not happen for academic cases (backward-facing step, lid-driven cavity, finite flat plate)

## Remaining problems (cont'd)

Main theoretical weakness is the approximation of the Schur complement $S \equiv C - DQ^{-1}G$

1. The SIMPLE approximation $Q^{-1} \approx \mathrm{diag}(Q)^{-1}$.
2. The stabilization matrix $C$ is moved to r.h.s
3. The matrix $-D\mathrm{diag}(Q)^{-1}G$ is approximated by a matrix $R$ with local stencil.

Other weaknesses are on the level of the discretization (Picard linearization, defect corrections, ...)

## Summary

- Coupled Navier-Stokes system has 10 blocks, we only assemble and store 2, for the others their action suffices.
- The stabilization matrix $C$ has a wide stencil, we changed SIMPLE(R) so that its assembly and storage is not needed.
- For maritime applications, we find that SIMPLE(R) as preconditioner reduces the number of non-linear iterations by 5 to 20 and the CPU time by 2 to 5. Greatest reduction found for most difficult case.

## Summary (cont'd)

C.M. Klaij and C. Vuik, *SIMPLE-type preconditioners for cell-centered, colocated finite volume discretization of incompressible Reynolds-averaged Navier-Stokes equations*, Int. J. Numer. Meth. Fluids 2013, 71(7):830–849.

Contains details on:

- academic benchmark cases (backward-facing step, lid-driven cavity, flat plate)
- choice of relaxation parameters
- choice of linear solvers and relative tolerances for sub-systems
- other variants (MSIMPLE and MSIMPLER)
- ...