Extra exercises

Section 1.4

1. Compute the absolute error and the relative error when we take β =10 and n=4.
   (a) π and $fl(\pi)$
   (b) $\sqrt{2}$ and $fl(\sqrt{2})$
   (c) e^{-4} and $fl(e^{-4})$
2. Perform the following computations, using β=10 and n=3 and compute the absolute error and the relative error.
   (a) 177 + 0.873
   (b) $\frac{3}{8} \cdot \frac{8}{7}$

Section 1.5

1. Let $f(x) = O(x^4)$ and $g(x) = O(x^7)$ for $x \to 0$. Compute the order of the following functions.
   (a) $2f(x) + 4g(x)$
   (b) $f(x)g(x)$
   (c) $\frac{g(x)}{|x|^2}$

Answers of the extra exercises

Section 1.4

1. (a) $x = \pi$=3.14159265
   $fl(x) = 0.3142 \cdot 10^1$
   Absolute error $= |fl(x) - x| = \left|0.3142 \cdot 10^1 - \pi\right| = 0.4073 \cdot 10^{-3}$
   Relative error $= \frac{|fl(x)-x|}{|x|} = \frac{\left|0.3142 \cdot 10^1 - \pi\right|}{\pi} = 0.1297 \cdot 10^{-3}$

   (b) $fl(\sqrt{2}) = 0.1414 \cdot 10^1$
   Absolute error $= \left|fl(\sqrt{2}) - \sqrt{2}\right| = 0.2136 \cdot 10^{-3}$
   Relative error $= \frac{\left|fl(\sqrt{2})-\sqrt{2}\right|}{\left|\sqrt{2}\right|} = 0.1510 \cdot 10^{-3}$

   (c) Absolute error $= 0.4361 \cdot 10^{-5}$
   Relative error $= 0.2381 \cdot 10^{-3}$

2. (a) Exact: 177 + 0.873 = 177.873
   Approximation: $fl(177 + 0.873) = 0.178 \cdot 10^3$
   Absolute error $= |fl(177 + 0.873) - 177.83| = \left|0.178 \cdot 10^3 - 177.873\right| = 0.127$
   Relative error $= \frac{|fl(177+0.873)-177.873|}{|177.873|} = \frac{|0.127|}{177.873} = 0.714 \cdot 10^{-3}$

   (b) Approximation: $fl(\frac{3}{8} \cdot \frac{8}{7}) = 0.429$
   Absolute error $= 0.429 \cdot 10^{-3}$
   Relative error $= 0.100 \cdot 10^{-2}$

Section 1.5

1. (a) $2f(x) + 4g(x)$ is $2 \cdot \mathcal{O}(x^4) + 4 \cdot \mathcal{O}(x^7)$ is $\mathcal{O}(x^4)$

   (b) $f(x)g(x)$ is $\mathcal{O}(x^4) \cdot \mathcal{O}(x^7)$ is $\mathcal{O}(x^{4+7})$ is $\mathcal{O}(x^{11})$

   (c) $\frac{g(x)}{|x|^2}$ is $\frac{\mathcal{O}(x^7)}{|x|^2}$ is $\mathcal{O}(x^{7-2})$ is $\mathcal{O}(x^5)$