Extra exercises

Section 2.2

1. (a) Suppose the value of the tangentfunction is known in 53° and 46°. Find the linear interpolation approximation in 50°.

   (b) What is the difference with the exact value?

α	$\tan\alpha$
46	1.035530
53	1.327045

2. (a) Suppose the value of $f(x) = e^x$ is known in $x = 0.98$ and $x = 1$. Find the linear interpolation approximation in $x = 0.99$.

   (b) What's the difference with the exact value?

x	e^x
0.98	2.664456
1	2.718282

Section 2.3

1. (a) Find the second order Lagrange polynomial of $f(x) = \frac{1}{1+x^2}$, using the nodes $x_0 = 0$, $x_1 = 1$, $x_2 = 4$

   (b) Approximate $f(2)$ and calculate the error.

2. (a) Find the second order polynomial of $f(x) = \frac{1}{x^2}$, using the nodes $x_0 = 1$, $x_1 = 3$, $x_2 = 4$.

   (b) Approximate $f(4.5)$ with extrapolation.

Section 2.4

1. (a) Use the following values to construct the Hermite interpolating polynomial.

k	x_k	$\sin x_k$	$[\sin x_k]' = \cos x_k$
0	0.40	0.3894	0.9211
1	0.42	0.4077	0.9131

   (b) Give an opproximation of sin(0.41)

Answers of the extra exercises

Section 2.2

1. (a) Take $f(x) = \tan(x)$ and $x_0 = 46°$, $x_1 = 53°$.
   The linear interpolation polynomial is now:

$$p(x) = f(x_0) + \frac{x - x_0}{x_1 - x_0} \cdot (f(x_1) - f(x_0))$$

$$p(50) = \tan(46) + \frac{50 - 46}{53 - 46} \cdot (\tan(53) - \tan(46))$$

$$= 1.035530 + \frac{4}{7}(1.327045 - 1.035530) = 1.202110$$

   (b) Difference: $|\tan(50°) - 1.202110| = 0.10 \cdot 10^{-1}$

2. (a) $f(x) = e^x$, $x_0 = 0.98$, $x_1 = 1$

$$p(0.99) = e^{0.98} + \frac{1}{2}(e^1 - e^{0.98}) = 2.691369$$

   (b) $\left|e^{0.99} - 2.691369\right| = 1.345277 \cdot 10^{-4}$

Section 2.3

1. (a) $L_2(x) = \sum_{k=0}^{2} f(x_k)L_{k,2}(x)$ is the Lagrange polynomial.

$$\begin{aligned}
L_{0,2}(x) &= \tfrac{(x-1)(x-4)}{(0-1)(0-4)} \\
&= \tfrac{x^2-5x+4}{4} \\
&= \tfrac{x(x-5)}{4} + 1 \\
L_{1,2}(x) &= \tfrac{(x-0)(x-4)}{(1-0)(1-4)} \\
&= \tfrac{x(x-4)}{-3} \\
L_{2,2}(x) &= \tfrac{(x-0)(x-1)}{(4-0)(4-1)} \\
&= \tfrac{x(x-1)}{12}
\end{aligned}$$

$$\begin{aligned}
f(x_0) &= f(0) = \tfrac{1}{1+0^2} = 1 \\
f(x_1) &= f(1) = \tfrac{1}{1+1^2} = \tfrac{1}{2} \\
f(x_2) &= f(4) = \tfrac{1}{1+4^2} = \tfrac{1}{17} \\
L_2(x) &= 1 \cdot \left(\tfrac{x(x-5)}{4} + 1\right) + \tfrac{1}{2}\left(\tfrac{x(x-4)}{-3}\right) + \tfrac{1}{17}\left(\tfrac{x(x-1)}{12}\right) \\
&= \tfrac{x(x-5)}{4} + 1 - \tfrac{x(x-4)}{6} + \tfrac{x(x-1)}{204} \\
&= \tfrac{x^2+5x}{4} + 1 - \tfrac{x^2-4x}{6} + \tfrac{x^2-x}{204} \\
&= \tfrac{51x^2-255x}{204} - \tfrac{34x^2-136x}{204} + \tfrac{x^2-x}{204} + 1 \\
&= \tfrac{18x^2-120x}{204} + 1 \\
&= \tfrac{3}{34}x^2 - \tfrac{10}{17}x + 1
\end{aligned}$$

   (b) $f(2) \approx L_2(2) = \frac{3}{34} \cdot 2^2 - \frac{10}{17} \cdot 2 + 1 = 0.176$

2. (a)

$$\begin{aligned}
L_{0,2}(x) &= \tfrac{x^2-7x+12}{6} \\
L_{1,2}(x) &= \tfrac{x^2-5x+4}{-2} \\
L_{2,2}(x) &= \tfrac{x^2-4x+3}{3}
\end{aligned}$$

$$\begin{aligned} f(1) &= \; = 1 \\ f(3) &= \tfrac{1}{9} \\ f(4) &= \tfrac{1}{16} \end{aligned}$$

$L_2(x) = \frac{19}{144}x^2 - \frac{35}{36}x + 1\frac{121}{144}$

(b) $f(4.5) \approx L_2(4.5) = 0.137$

Section 2.4

1. (a) $L_{01}(x) = \frac{x-0.42}{0.40-0.42} = -50x + 21$
   $L'_{01}(x) = -50$
   $L_{11}(x) = \frac{x-0.40}{0.42-0.40} = 50x - 20$
   $L'_{11}(x) = 50$

   $H_3(x) = f(x_0)H_{01}(x) + f'(x_0)\hat{H}_{01}(x) + f(x_1)H_{11}(x) + f'(x_1)\hat{H}_{11}(x)$
   met $H_{jn}(x) = [1 - 2(x - x_j)L'_{jn}(x_j)]L^2_{jn}(x)$
   en $\hat{H}_{jn}(x) = (x - x_j)L^2_{jn}(x)$

$$\begin{aligned} H_{01}(x) &= [1 - 2(x - 0.40)(-50)](-50x + 21)^2 \\ &= (1 + 100x - 40)(-50x + 21)^2 \\ &= (100x - 30)(-50x + 21)^2 \\ \hat{H}_{01}(x) &= (x - 0.40)(-50x + 21)^2 \\ H_{11}(x) &= [1 - 2(x - 0.42) \cdot 50](50x - 20)^2 \\ &= (1 - 100x + 42)(50x - 20)^2 \\ &= (43 - 100x)(50x - 20)^2 \\ \hat{H}_{11}(x) &= (x - 0.42)(50x - 20)^2 \end{aligned}$$

   So $H_3(x) = 0.3894(100x - 39)(-50x + 21)^2 + 0.9211(x - 0.40)(-50x + 21)^2$
   $+0.4077(43 - 100x)(50x - 20)^2 + 0.9131(x - 0.42)(50x - 20)^2$

   (b) $\sin(0.41) \approx H_3(0.41)$
   $= 0.3894(100\cdot0.41-39)(-50\cdot0.41+21)^2+0.9211(0.41-0.40)(-50\cdot0.41+21)^2$
   $+0.4077(43-100\cdot0.41)(50\cdot0.41-20)^2+0.9131(0.41-0.42)(50\cdot0.41-20)^2$
   $= 0.3986$