Generalized Finite Element Methods

Stability, Preconditioning and Mass Lumping

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Meshfree Multiscale Methods

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01. July 2015

Overview

Generalized Finite Element Methods

Stability

Preconditioning & Fast Solvers

Variational Mass Lumping

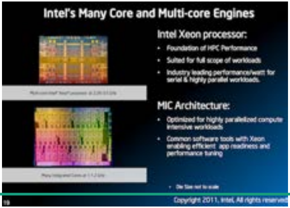
Motivation

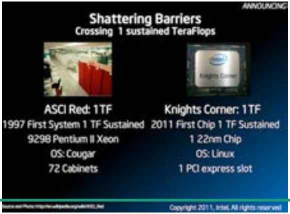
Why "new" methods?

Complex geometry, mesh generation, time-dependent adaptation of meshes.

Why "new" methods?

- Dramatic change in hardware design.
- Strong scaling / parallel speed-up $S_L(P) = \dfrac{T_L(1)}{T_L(P)}$
- Floating point operations "for free", memory transfers"expensive".
- Simple global data structures.
- Many operations per data (e.g. higher order methods).





An optimal method

- Simple global data structure.
- Convergence properties independent of regularity of solution u.
- Optimal basis functions Φ_i^u.

$$u_N(x) = \sum_{i=1}^{N} c_i^u \Phi_i^u(x)$$

- Basis functions are solution-dependent.
- Number of basis functions vs. quality of basis functions.

Few data (Dof), many local operations!

Generalized Finite Element Methods

$$-\nabla \kappa \nabla u = f, \quad \rho \ddot{u} = \operatorname{div} \boldsymbol{\sigma}(u) - f$$

Classical Approximation

- Choose atom, dilation & shift
- Study approximation space
- Identify with smoothness space
- PDE regularity results
- hp-adaptive refinement

Complex data, regularity determines convergence.

Optimal Approximation

- Choose PDE
- Local expansion/regularity
- NO dilation & shift
- Application-dependent basis
- Uniform refinement

Simple data, convergence independent of regularity.

Generalized Finite Element Methods

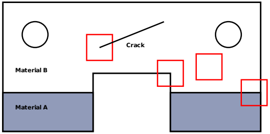
Identify optimal local basis with respect to required global accuracy measure. Merge and solve.

Decomposition of $u \in H^s(\Omega)$

$$u = u_{\text{smooth}} + u_{\text{jump}} + u_{\text{singular}}$$

Efficient approximation of u

- Higher order polynomials for u_{smooth}.
- Discontinuous basis functions for u_{jump}.
- Singular basis functions for u_{singular}.



Localization by partition of unity

- Consider a partition of unity (PU) $\{\varphi_i\}$ with $\omega_i := \operatorname{supp}(\varphi_i)$

$$u = \sum_{i=1}^{N} \varphi_i u = \sum_{i=1}^{N} \left(\varphi_i u_{\text{smooth}} + \varphi_i u_{\text{jump}} + \varphi_i u_{\text{singular}} \right).$$

- Localization of approximation: $u|_{\omega_i} \approx u_i \in V_i(\omega_i) = \operatorname{span}\langle \vartheta_i^k \rangle$.
- Smooth splicing of local spaces

$$V^{\text{PU}} := \sum_{i=1}^{N} \varphi_i V_i(\omega_i) = \sum_{i=1}^{N} \varphi_i(\mathcal{P}^{p_i} + \mathcal{E}_i).$$

- *No* compatability restrictions as in FEM (local/parallel).
- Approximation by $V_i(\omega_i)$, functions φ_i just “glue”.

Approximation

PUM error estimate

Let $u \in H^1(\Omega)$, $u^{\mathrm{PU}} := \sum_{i=1}^N \varphi_i u_i$ with $u_i \in V_i(\omega_i)$, $\mathrm{supp}(\varphi_i) = \omega_i$ where φ_i is a *non-negative* admissible PU then

$$\|u - u^{\mathrm{PU}}\|_{L^2(\Omega)} \leq \sqrt{C_\infty}\Big(\sum_{i=1}^N \hat{\epsilon}_i^2\Big)^{1/2},$$

$$\|\nabla(u - u^{\mathrm{PU}})\|_{L^2(\Omega)} \leq \sqrt{2}\Big(\sum_{i=1}^N M\Big(\frac{C_\nabla}{\mathrm{diam}(\omega_i)}\Big)^2 \hat{\epsilon}_i^2 + C_\infty \tilde{\epsilon}_i^2\Big)^{1/2}.$$

with constants M, C_∞, and C_∇ independent of N.

Standard choice of local approximation spaces

Local polynomials $\mathcal{P}^{p_i}(\omega_i)$

- Complete polynomials (total degree), or tensor products
- Subspaces: anisotropic products, harmonic polynomials, ...

Problem-dependent enrichment $\mathcal{E}_i(\omega_i) = \mathcal{E}|_{\omega_i}$

$$V_i = \mathcal{P}^{p_i} + \mathcal{E}_i = \mathrm{span}\langle\psi_i^t\rangle + \mathrm{span}\langle\eta_i^s\rangle = \mathrm{span}\langle\vartheta_i^k\rangle$$

$$u^{\mathrm{PU}}(x) := \sum_{i=1}^N \varphi_i(x) u_i(x) = \sum_{i=1}^N \varphi_i(x) \sum_{m=1}^{d_i} u_i^m \vartheta_i^m(x), \quad \tilde{u} := (u_i^m)_{i,m}$$

Fundamental Goal of PUM

General framework for application-dependent approximation.
Higher order approximation independent of regularity of solution.

$$V^{\mathrm{PU}} := \sum_{i=1}^{N} \varphi_i V_i(\omega_i) = \sum_{i=1}^{N} \varphi_i(\mathcal{P}^{p_i} + \mathcal{E}_i) = \sum_{i=1}^{N} (\varphi_i \mathcal{P}^{p_i} + \varphi_i \mathcal{E}_i).$$

Stability & Efficiency

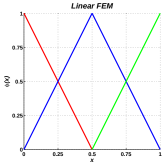
- Selection of local spaces $\mathcal{P}^{p_i}$ and $\mathcal{E}_i$ independent of neighbors.
- Construction of PU φ_i by Shepard approach, moving least squares.
- Adaptivity in p, h and enrichment $\mathcal{E}_i$ straight forward.
- Stability of global basis inherited from local stability (with flat-top).

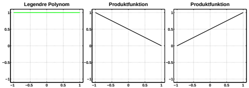
Selection of the PU - XFEM/GFEM

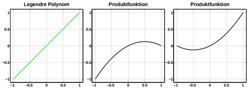
$$V^{PU} = \sum_{i=1}^{N} \varphi_i \; V_i = \sum_{i=1}^{N} \varphi_i \; \mathcal{P}^{p_i} + \sum_{i=1}^{N} \varphi_i \; \mathcal{E}_i$$

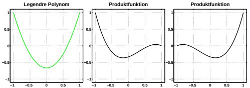
Linear FEM as PU

- Consider interval $[0, 1]$ mit $\varphi_{\mathbf{0}}^{\text{FEM}}$, $\varphi_{\mathbf{1}}^{\text{FEM}}$, $V_i = \{1, x\}$.
- Products of functions $\varphi_i^{\text{FEM}} \psi_i^n$ quadratic polynomials.
- Number of functions $\#\{\varphi_i^{\text{FEM}} \psi_i^n\} = 4$.







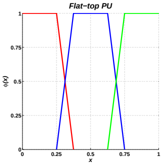


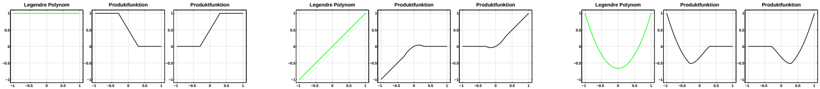
- Approximation benefits from higher reproducing properties of PU.
- Selection of local spaces *not* completely local (blending elements).
- Global stability *not* implied by local stability.
- Recently introduced: Stable GFEM (Babuška & Banerjee)

Selection of the PU - Meshfree

Flat-top PU

- Ensure $\varphi_i \equiv 1$ on $\omega_{i,\mathrm{FT}} \subset \omega_i = \mathrm{supp}(\varphi_i)$, $|\omega_{i,\mathrm{FT}}| \approx |\omega_i|$.
- Global independence implied by local independence on $\omega_{i,\mathrm{FT}}$.
- Supports are *smaller* than in FEM.





- Order of global approximation inherited from local orders.
- Complete independence of local spaces, no compatibility.
- Global stability implied by local stability.

$$K_1^{-1}\left(\sum_{i=1}^{N}\sum_{m=1}^{d_i}(u_i^m)^2\right)^{\frac{1}{2}} \leq h^{-\frac{d}{2}}\|u^{\mathrm{PU}}\|_{L^2(\Omega)} \leq K_2\left(\sum_{i=1}^{N}\sum_{m=1}^{d_i}(u_i^m)^2\right)^{\frac{1}{2}}$$

Numer. Math. **118** (2011)

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Selection of local enrichments

Enrichments

- Exact enrichments
  - Known singularitites (e.g. $\eta(x) = \|x - x_0\|^\alpha$),
  - Known discontinuities (e.g. $\eta(x) = \cos(\frac{\theta_c}{2})$)
- Approximate enrichments:
  - Singularities $\eta(x) = \|x - x_0\|^\beta$
  - Discontinuities $\eta(x) = H_\pm(x - c)$
  - Boundary layers $\eta(x) = \exp(1 - \text{dist}(x, c))$
  - Radial component of solution
- Numerical enrichments:
  - Cell problems (with/without global-local-approach)
  - Reconstruction of experimental data (or reduced order basis)
  - Eigenfunctions of local problems

Goals

- Optimal fine level approximation: Error minimization.
- Acceptable coarse level approximation: Fast & robust solution.
- Load-balancing in local and global operations.

Global stability & local preconditioning

Stability of local approximation spaces

- Orthogonal basis for local enrichment space $\mathcal{E}_i$.
- Elimination of $\mathcal{P}^{p_i}$ from enrichment space $\mathcal{E}_i$.

Local preconditioner

Consider local mass matrix on patch ω_i (i.e. on $\omega_{i,\mathrm{FT}}$)

$$(M^i)_{n,m} := \int_{\omega_{i,\mathrm{FT}} \cap \Omega} \vartheta_i^n \vartheta_i^m \, dx \quad \text{für alle } m, n$$

$$M_i = \begin{pmatrix} M^i_{\mathcal{P},\mathcal{P}} & M^i_{\mathcal{P},\mathcal{E}} \\ M^i_{\mathcal{E},\mathcal{P}} & M^i_{\mathcal{E},\mathcal{E}} \end{pmatrix} \qquad \begin{aligned} O^T_{\mathcal{P}} M^i_{\mathcal{P},\mathcal{P}} O_{\mathcal{P}} &= D_{\mathcal{P}} \\ O^T_{\mathcal{E}} M^i_{\mathcal{E},\mathcal{E}} O_{\mathcal{E}} &= D_{\mathcal{E}} \end{aligned}$$

Stable basis for $V_i = \mathcal{P}^{p_i} + \mathcal{E}_i \approx \mathcal{P}^{p_i} \oplus \mathcal{D}_i$ with $\mathcal{D}_i \approx \mathcal{E} \setminus \mathcal{P}^{p_i}$ via

$$S_i^{\mathcal{E} \setminus \mathcal{P}} := \begin{pmatrix} D_{\mathcal{P}}^{-1/2} O_{\mathcal{P}}^T & 0 \\ -\tilde{D}_{\mathcal{D}}^{-1/2} \tilde{O}_{\mathcal{D}}^T M^*_{\mathcal{E},\mathcal{P}} D_{\mathcal{P}}^{-1/2} O_{\mathcal{P}}^T & \tilde{D}_{\mathcal{D}}^{-1/2} \tilde{O}_{\mathcal{D}}^T \tilde{D}_{\mathcal{E}}^{-1/2} \tilde{O}_{\mathcal{E}}^T \end{pmatrix}$$

Control of $K_{1,i}$ and $K_{2,i}$ during computation. (can be done for any norm)

Exact enrichments: Linear fracture mechanics

Goal

Error minimization of finest level (accuracy of SIF)

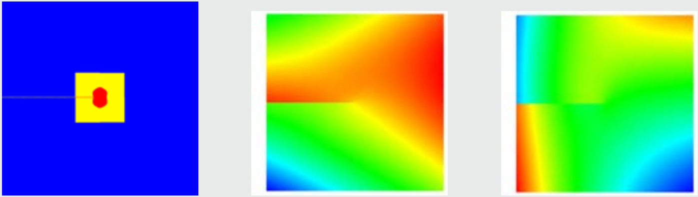
- Displacement discontinuous across crack

$$\mathcal{E}_i = \mathcal{P}^{p_i} \cdot H^C$$

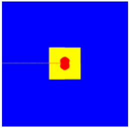
- Stress is singular at crack tip (i.e. gradient of displacement)

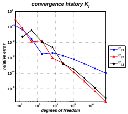
$$\mathcal{E} = \{\sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\sin\theta\sin\frac{\theta}{2}, \sqrt{r}\sin\theta\cos\frac{\theta}{2}\}.$$

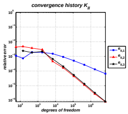
Enrichment zone & exact solution

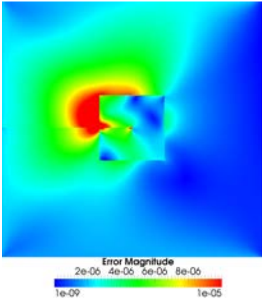


Exact enrichments: Linear fracture mechanics







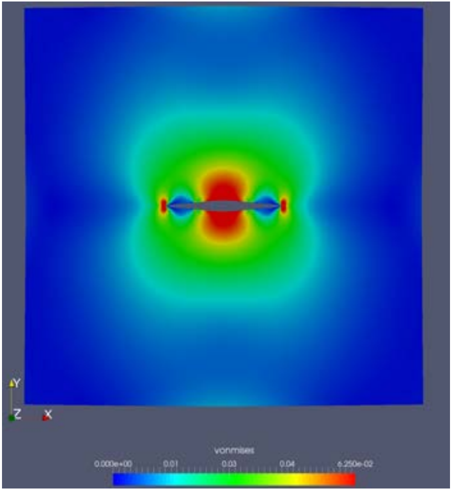


J	dof	N	e_{L^∞}	ρ_{L^∞}	e_{L^2}	ρ_{L^2}	e_{H^1}	ρ_{H^1}
				with respect to Ω				
4	1748	256	7.044_{-3}	0.90	3.425_{-3}	1.00	3.677_{-2}	0.56
5	6836	1024	2.349_{-3}	0.81	9.265_{-4}	0.96	1.795_{-2}	0.53
6	26996	4096	7.999_{-4}	0.78	2.410_{-4}	0.98	8.893_{-3}	0.51
7	107252	16384	2.751_{-4}	0.77	6.121_{-5}	0.99	4.508_{-3}	0.49
8	427508	65536	9.501_{-5}	0.77	1.535_{-5}	1.00	2.215_{-3}	0.51
9	1686716	262144	3.273_{-5}	0.78	3.820_{-6}	1.01	9.948_{-4}	0.58
				with respect to E_1				
4	236	16	5.745_{-2}	0.53	3.112_{-2}	0.66	1.050_{-1}	0.56
5	528	36	1.915_{-2}	1.36	7.083_{-3}	1.84	4.028_{-2}	1.19
6	1448	100	6.521_{-3}	1.07	1.594_{-3}	1.48	1.434_{-2}	1.02
7	4632	324	2.243_{-3}	0.92	3.639_{-4}	1.27	5.082_{-3}	0.89
8	16376	1156	7.747_{-4}	0.84	8.482_{-5}	1.15	1.802_{-3}	0.82
9	61368	4356	2.670_{-4}	0.81	2.020_{-5}	1.09	6.441_{-4}	0.78
				with respect to E_2				
4	56	4	6.977_{-2}	—	5.785_{-2}	—	9.873_{-2}	—
5	236	16	2.327_{-2}	0.76	1.435_{-2}	0.97	5.154_{-2}	0.45
6	528	36	7.923_{-3}	1.34	3.201_{-3}	1.86	1.984_{-2}	1.19
7	1448	100	2.725_{-3}	1.06	6.776_{-4}	1.54	7.085_{-3}	1.02
8	4632	324	9.410_{-4}	0.91	1.449_{-4}	1.33	2.518_{-3}	0.89
9	16376	1156	3.242_{-4}	0.84	3.227_{-5}	1.19	8.986_{-4}	0.82
				with respect to E_3				
5	56	4	3.072_{-2}	—	2.693_{-2}	—	4.728_{-2}	—
6	236	16	1.046_{-2}	0.75	6.846_{-3}	0.95	2.523_{-2}	0.44
7	528	36	3.597_{-3}	1.33	1.476_{-3}	1.91	9.733_{-3}	1.18
8	1448	100	1.242_{-3}	1.05	2.967_{-4}	1.59	3.481_{-3}	1.02
9	4632	324	4.279_{-4}	0.92	6.060_{-5}	1.37	1.244_{-3}	0.88

$e := \|u - u_l^{PU}\|, \qquad \rho := \log\left(\frac{e_l}{e_{l-1}}\right) / \log\left(\frac{\mathrm{dof}_l}{\mathrm{dof}_{l-1}}\right)$

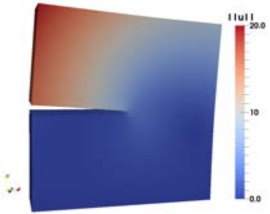
Optimal: $\rho_{L^2} = \frac{2}{2}$, $\rho_{H^1} = \frac{1}{2}$ classical (h^γ) via $d \cdot \rho = \gamma$

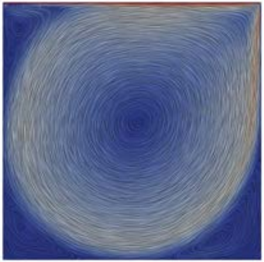
Hydraulic fracture

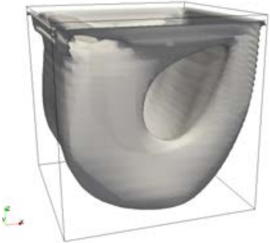


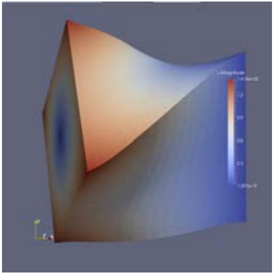
Quadratic polynomials, tip enrichment zone, Heaviside & signed distance enrichment.

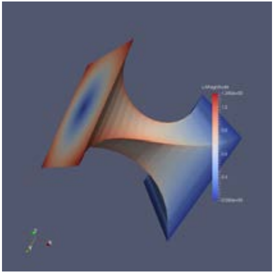
More examples

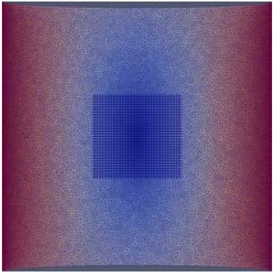












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Multilevel solver

Smoothing operator

Overlapping block-relaxation on $V_{i,k}$-blocks.

Transparent construction

Construction is directly applicable to any choice of enrichment.

Sequence of PUM spaces $V_k^{\mathrm{PU}} \not\supset V_{k-1}^{\mathrm{PU}}$

$$V_k^{\mathrm{PU}} := \sum_{i=1}^{N_k} \varphi_{i,k} V_{i,k} = \sum_{i=1}^{N_k} \varphi_{i,k} (\mathcal{P}^{p_{i,k}} + \mathcal{E}_{i,k}) = \sum_{i=1}^{N_k} \varphi_{i,k} (\mathcal{P}^{p_{i,k}} \oplus \mathcal{D}_{i,k})$$

from sequence of patches $\omega_{i,k}$ ($\omega_{j,k-1} \supseteq \omega_{i,k}$), e.g. PUs $\varphi_{i,k}$.

Interlevel transfer: Local L^2-projection

Block-diagonal prolongation: $V_{j,k-1} \to V_{i,k}$ (exact for $V_{j,k-1}$)

$$\widetilde{\Pi}_{k-1}^k := (\widetilde{M}_k^k)^{-1} (\widetilde{M}_{k-1}^k), \quad \tilde{\omega}_{i,k} := \omega_{i,k} \cap \Omega$$

$$(\widetilde{M}_k^k)_{n,m}^i := \langle \vartheta_{i,k}^m, \vartheta_{i,k}^n \rangle_{L^2(\tilde{\omega}_{i,k})} \quad (\widetilde{M}_{k-1}^k)_{n,m}^i := \langle \vartheta_{j,k-1}^m, \vartheta_{i,k}^n \rangle_{L^2(\tilde{\omega}_{i,k})}$$

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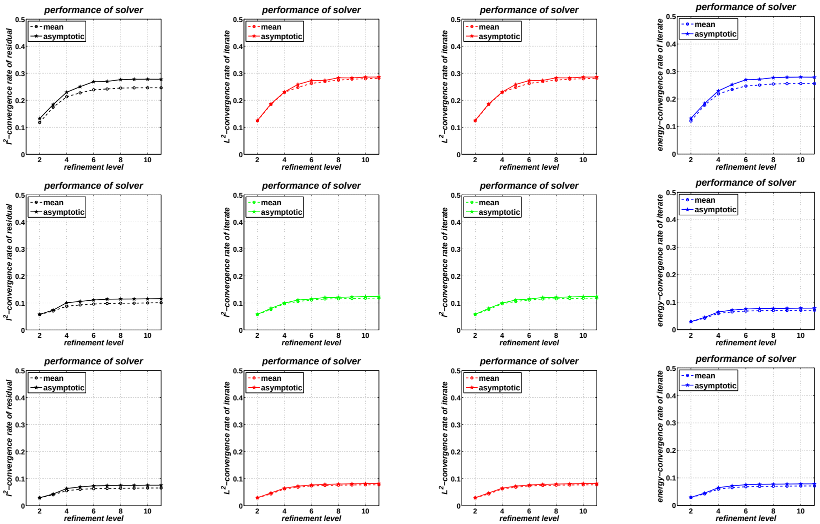
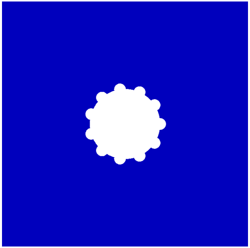
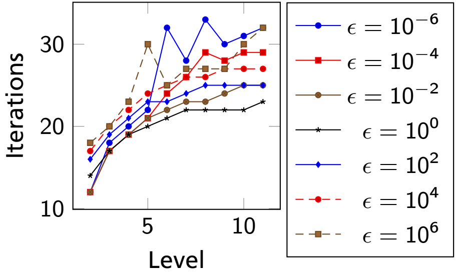
Solver efficiency: Polynomials

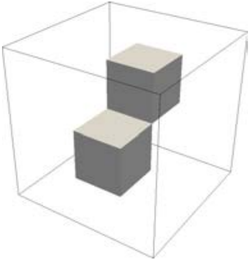


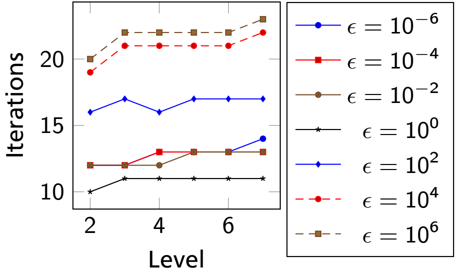
Convergence rates for the multilevel $V(s, s)$-cycles with $s = \mathbf{1}, \mathbf{3}, \mathbf{5}$ (top to bottom row) and block-Gauss–Seidel smoothing for Poisson problem with linear approximation spaces.

Solver efficiency: Approximate enrichments









Explicit dynamics

Model problem & Central differences in time

$$u_{tt}(x,t) = \Delta u(x,t) \quad (x,t) \in \Omega \times (0,T)$$

$$u(\cdot, t_{n+1}) = (\delta t)^2 \Delta u(\cdot, t_n) + 2u(\cdot, t_n) - u(\cdot, t_{n-1}) =: f(\cdot) \quad \text{in } \Omega.$$

Galerkin in space

Given $f \in L^2(\Omega)$ find $u^h \in V^h \subset L^2(\Omega)$ such that for all $v^h \in V^h$

$$\langle f - u^h, v^h \rangle_{L^2(\Omega)} = 0$$

Mass matrix problem

Let $\hat{f} = (f_i)$, $M = (M_{i,j})$ where $f_i = \langle f, \phi_i \rangle_{L^2(\Omega)}$, $M_{i,j} = \langle \phi_j, \phi_i \rangle_{L^2(\Omega)}$

$$M\tilde{u} = \hat{f}$$

L^2-projection onto V^{PU}

Global L^2-projection onto V^{PU}

$$\Pi_{L^2(\Omega)} : L^2(\Omega) \to V^{\mathrm{PU}}, \quad f \mapsto u^h, \quad M\tilde{u} = \hat{f}$$

Consistent mass matrix

$$M = (M_{(i,n),(j,m)}), \quad M_{(i,n),(j,m)} = \langle \varphi_j \vartheta_j^m, \varphi_i \vartheta_i^n \rangle_{L^2(\Omega)},$$

Moment-vector

$$\hat{f} = (f_{(i,n)}), \quad f_{(i,n)} = \langle f, \varphi_i \vartheta_i^n \rangle_{L^2(\Omega)}.$$

Re-interpretation of moments

$$\begin{aligned} f_{(i,n)} &= \langle f, \varphi_i \vartheta_i^n \rangle_{L^2(\Omega)} = \int_\Omega f \varphi_i \vartheta_i^n \, dx = \int_{\Omega \cap \omega_i} f \varphi_i \vartheta_i^n \, dx \\ &= \langle f | \varphi_i | \vartheta_i^n \rangle_{L^2(\Omega \cap \omega_i)} = \langle f, \vartheta_i^n \rangle_{L^2(\Omega \cap \omega_i, \varphi_i)} \end{aligned}$$

$$L^2(\Omega \cap \omega_i, \varphi_i) := \{ u \in L^2(\Omega) : \|u\|^2_{L^2(\Omega \cap \omega_i, \varphi_i)} := \int_{\Omega \cap \omega_i} \varphi_i |u|^2 \, dx < \infty \}$$

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L^2-projection onto V^{PU}: The Local Perspective

$$L^2(\Omega \cap \omega_i, \varphi_i) := \{u \in L^2(\Omega) : \|u\|^2_{L^2(\Omega\cap\omega_i,\varphi_i)} := \int_{\Omega\cap\omega_i} \varphi_i |u|^2 \, dx < \infty\}$$

Local L^2-projection onto V^{PU}

$$\bar{\Pi}_{L^2(\Omega)} : L^2(\Omega) \to V^{\mathrm{PU}}, \quad f \mapsto \bar{u}, \quad \bar{M}\tilde{\bar{u}} = \hat{f}$$

Localized mass matrix

$$\bar{M} = (\bar{M}_{(i,n),(j,m)}), \quad \bar{M}_{(i,n),(j,m)} = \begin{cases} 0 & i \neq j \\ \langle \vartheta_i^m | \varphi_i | \vartheta_i^n \rangle_{L^2(\Omega\cap\omega_i)} & i = j \end{cases}$$

- Construction is independent of local spaces (enrichments, order)
- Consistent right-hand side $\hat{f}$
- Block-diagonal matrix $\bar{M}$
- Symmetric positive definite $\bar{M}$

Consistent vs. Lumped Mass Matrix

Lemma

The approximation $\bar{u} \in V^{\text{PU}}$ obtained by local projection $\bar{\Pi}_{L^2(\Omega)}$ satisfies

$$\|f - \bar{u}\|_{L^2(\Omega)} \leq \sqrt{C_\infty} \left(\sum_{i=1}^{N} \hat{\epsilon}_i^2 \right)^{1/2}.$$

Moreover, the operator $\bar{M} - M$ is symmetric positive semi-definite.

Conservation $u = \Pi f = \bar{\Pi} f = \bar{u}$

For all $\tilde{w} \in \ker(\bar{M} - M)$ holds

$$\|w\|^2_{L^2(\Omega)} = \tilde{w}^T M \tilde{w} = \tilde{w}^T \bar{M} \tilde{w}.$$

If $w \in L^2(\Omega)$ such that $w|_{\Omega \cap \omega_i} \in V_i$ then $\tilde{w} \in \ker(\bar{M} - M)$.

Further Properties

Interpretation: Classical FEM

Linear FEM space $V^{\mathrm{FE}} = V^{\mathrm{PU}} = \mathrm{span}\langle \phi_i \rangle$ if $V_i = \mathrm{span}\langle 1 \rangle$. Thus,

$$\bar{M}_{i,i} = \int_\Omega \phi_i \, dx = \int_\Omega \sum_{j=1}^{N} \phi_j \phi_i \, dx = \sum_{j=1}^{N} \int_\Omega \phi_j \phi_i \, dx = \sum_{j=1}^{N} M_{i,j}.$$

Application to GFEM/XFEM

$\bar{M}$ always invertible if local basis stable with respect to $L^2(\Omega \cap \omega_i, \varphi_i)$.

Convergence: Discontinuous Galerkin

As $\varphi_i \to \chi_{\omega_i}$ we find $\bar{M}$ and M become the consistent mass matrix of the resulting discontinuous space $V = \sum_{i=1}^{N} \chi_{\omega_i} V_i$

Time-Stepping Results: Properties

Conservation

$$(\bar{M} - M)x = \lambda x, \quad \ker(\bar{M} - M) \supseteq \mathcal{P}^p$$

Critical time-step

$$Kx = \lambda M x, \quad Kx = \lambda \bar{M} x,$$

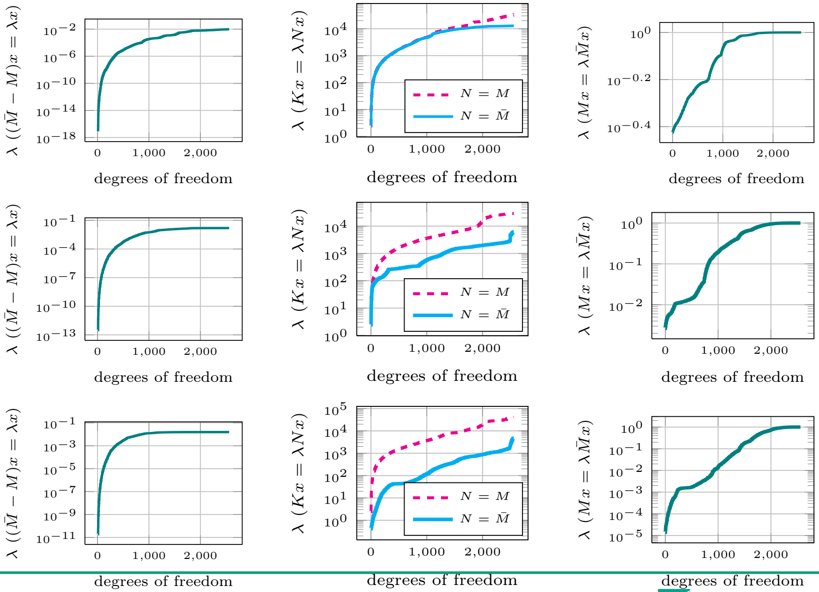
Stability limit on time-step:

$$\delta t_{\text{critical}} \leq \frac{2}{\sqrt{\lambda_{\text{max}}}},$$

Preconditioner

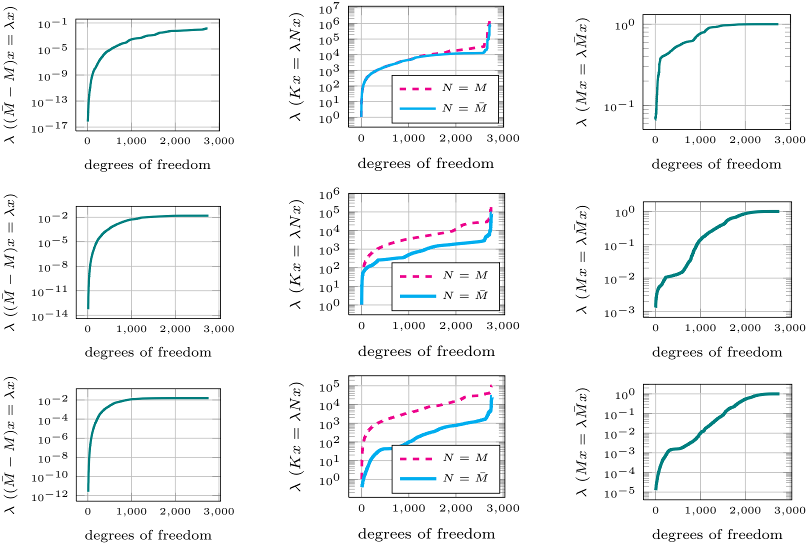
$$Mx = \lambda \bar{M} x, \quad \dim\{x : \lambda = 1\}$$

Time-Stepping Results: Properties



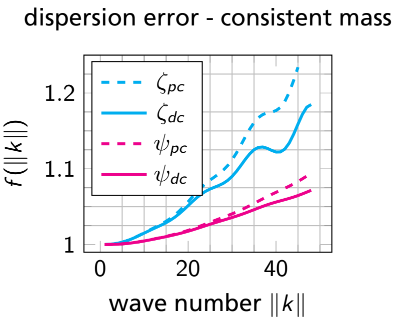
Smooth solution: $p = 3$ and $\alpha = 1.1, 1.5, 1.9$ (top to bottom)

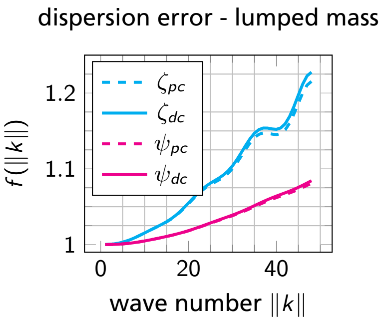
Time-Stepping Results: Properties



Singular solution: Enrichment, $p = 3$ and $\alpha = 1.1, 1.5, 1.9$ (top to bottom)

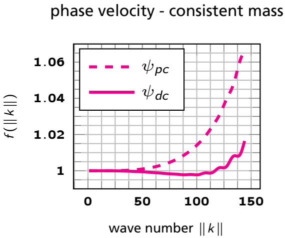
Dispersion error - Linear Approximation

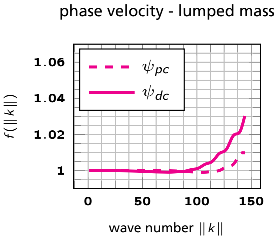


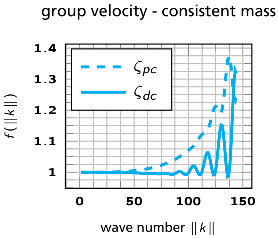


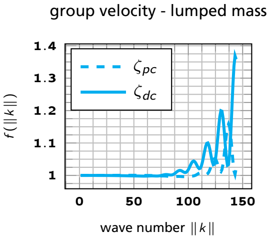
- Dispersion properties with lumped mass comparable to consistent mass.
- Results with lumped mass less sensitive to location of wave.
- Acceptable accuracy of $\leq 5\%$ error in phase velocity with ≈ 6 linear patches (small overlap) per wavelength.

Dispersion error - Cubic Approximation









Acceptable accuracy attained with single cubic patch (small overlap) per wavelength.

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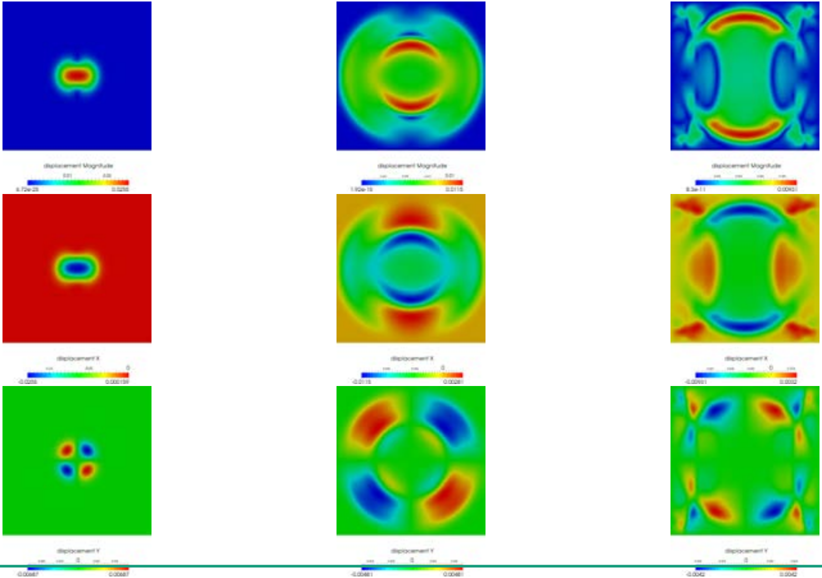
Elastic Wave 2D: Cubic Approximation, $t = 0.1, 0.4, 0.68$



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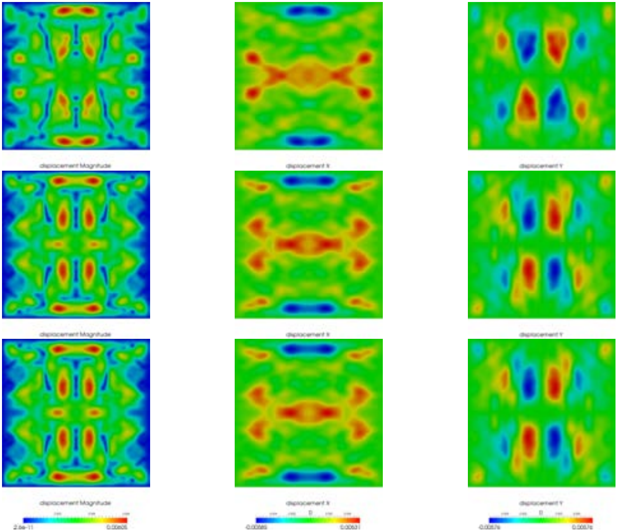
Snapshot comparison at $T = 4$

$M, \delta t = 0.9 \delta t_c^c$

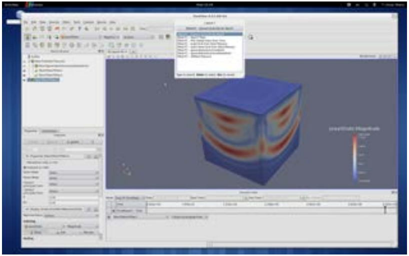
$\bar{M}, \delta t = 0.9 \delta t_c^c$

$\bar{M}, \delta t = 0.9 \delta t_c^l$



Software framework PaUnT

- CAD interface
- Polynomials of arbitrary degree
- User-definable enrichment functions
- Automatic construction of well-conditioned basis
- Multilevel solver, Newton solver, interfaces to external solvers
- Implicit & explicit time stepping schemes (consistent/lumped mass)
- Data export: VTK, Matlab
- Post-Processing: ParaView Plugin



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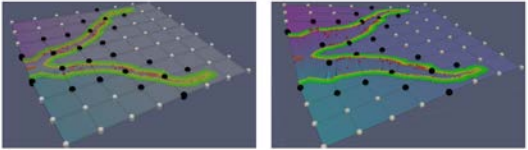
Upcoming event

Eighth International Workshop

Meshfree Methods for Partial Differential Equations



DEDICATION:	To the memory of Ted Belytschko
DATE:	SEPTEMBER 7-9, 2015
LOCATION:	BONN, GERMANY
SPONSORS:	Sonderforschungsbereich 1060 Hausdorff Center for Mathematics
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