

Multigrid based preconditioners for the heterogeneous Helmholtz equation

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Abstract

In [1] we propose a class of preconditioner for the Helmholtz equation, based on a complex shift of the Laplace operator. There, we also give some spectral analysis of the preconditioned linear system. By using a particular choice of the shift we can make the spectrum bounded above by a value of order $O(1)$. We find also that the condition number of the preconditioned linear system solely depends on the lower bound of the spectrum, which in this case is of order $O(1/k)$, where k is the wavenumber.

In this talk, using this complex shifted Laplace operator, a multigrid based preconditioner can be designed to be used as the accelerator for Krylov subspace iterative methods. The standard multigrid components usually used for real-valued linear systems can be used for the complex-valued preconditioner operator without any adaptation. Analysis using local Fourier analysis is given and compared with numerical observation. Numerical test on various cases show the effectiveness and the robustness of the preconditioner.

Keywords: Helmholtz equation, Krylov subspace, complex multigrid preconditioner, local Fourier analysis.

AMS Subject Classification: 65N55, 65F10, 65N22, 78A45, 76Q05

References

- [1] Y.A. Erlangga, C. Vuik, C.W. Oosterlee, *On a class of preconditioner for solving the Helmholtz equation*, Appl. Numer. Math. 50 (2004), pp. 409–425.